Examining the non-monotonic relationship between risk and security returns using the quantile regression approach

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Keywords: Systematic risk; idiosyncratic risk; CAPM; quantile regression

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ABSTRACT
While incorporating the quantile regression (QR) technique, this work examines the non-monotonic relationship between risk and returns in the cross-section of stock returns. The empirical data include U.S. S&P500 firm stocks listed during 1998-2007. The present empirical results indicate that the systematic (idiosyncratic) risk-return relationship is significant (insignificant) for stocks with stable price movements, as demonstrated by the CAPM theory, whereas the model’s theoretical risk-return relation is failed when stocks with volatile price changes are considered. Further, our empirical results could satisfactorily explain the longstanding risk-return puzzles in earlier studies.

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Keywords: Systematic risk; idiosyncratic risk; CAPM; quantile regression
1. Introduction

In this paper, we reexamine the risk-return relationship in the cross-section of stock returns. Our ideas are based on the following notions. First, we consider that the security risk should be classified into two categories, systematic against idiosyncratic. Second, the capital asset pricing model (CAPM) theory suggests that stock prices should reflect systematic risk only, with no compensation given to diversifiable risk (i.e., idiosyncratic risk). Third, the empirical findings regarding the link between risk and security returns are inconclusive and it has posed a longstanding problem in research field.

In particular, while the CAPM risk-return relationship has been widely tested empirically, the empirical evidence is generally mixed (e.g., Lintner, 1965; Black et al., 1972; Levy, 1978; Amihud et al., 1992; Jagannathan and Wang, 1993), and most studies reveal weak or no support at all for the CAPM. Further, Fama and French (1992), an influential and widely cited study, strongly rejects the CAPM. Further, although financial theory suggests that idiosyncratic risk should not be priced in the capital markets, Barberis and Huang (2001), Malkiel and Xu (2002) and Jones and Rhodes-Kropf (2003) find that idiosyncratic risk is positively related to stock returns, whereas Ang et al. (2006) indicates stocks with high idiosyncratic risk have low average returns.

Although these inconsistent empirical findings might be attributed to the small sample size problems, or the quality of the data used, we posit that the risk-return relationship is possibly not uniform cross-sectionally, and sometimes the relation is not even monotonic. In particular, the CAPM risk-return relationship is derived under certain assumptions. We argue that it is possible that some of the CAPM assumptions are not held for certain stocks, and therefore the model’s theoretical risk-return link is
not consistent across various stock observations. Given the above discussion, this work is one of the first studies to examine the non-monotonic risk-return relationship in the cross-section of stock returns using quantile regression (hereinafter referred to as QR).

As a statistical term, the quantile is a statistical term describing a division of observations into certain defined intervals based upon the values of the data, and the return quantile of a specific stock could show the relative magnitude of it return in comparison with the entire set of stock observations. Moreover, the median value of stock returns, i.e. the 0.5 return quantile, approaches zero, and thus higher and lower than the 0.5 return quantiles can be defined as a rise and fall in price, respectively. Accordingly, this work is concerned with whether the CAPM risk-return relationship is valid for specific stocks which are experiencing volatile price movements, i.e. stocks with an extremely high/low return quantile condition. The opposite scenario occurs for moderate quantiles, i.e. a stability in stock price.

The rest of this paper is organized as follows. Section 2 details the method for calculating the systematic and idiosyncratic risk. Section 3 presents the underlying models for examining the risk-return relationship in the cross-section of stock returns, including the QR and conventional regression approaches. Section 4 shows the empirical results, and Section 5 concludes the paper.

2. Calculation of systematic and idiosyncratic risk

Both systematic and idiosyncratic risks are considered in this study, and the risk-return relationship in stocks is examined. To calculate the two types of risk, the CAPM regression is employed in this study and expressed as follows:
\[ R_{i,t} = \alpha_i + \beta_i \times R_{m,t} + \epsilon_{i,t} \]  

(1)  

where \( R_{i,t} \) and \( R_{m,t} \) denote the daily excess returns of the \( i \)-th stock and the market at time \( t \), respectively. To avoid ad hoc problems of market index selection, this study employs the S&P500 market index to calculate the market returns. Further, to convert returns to excess returns, the daily US three-month Treasury bill rate is subtracted from the raw returns.

Undertaking variance calculation on both sides of Eq. 1, we have

\[ \sigma_i^2 = \beta_i^2 \times \sigma_m^2 + \sigma^2(\epsilon_{i,t}) \]  

(2)  

where \( \sigma_i \), \( \sigma_m \) and \( \sigma(\epsilon_{i,t}) \) are the standard deviation of \( R_{i,t} \) (i.e., the excess returns of the \( i \)-th stock), \( R_{m,t} \) (i.e., the excess returns of the S&P500 market index) and the residual term \( \epsilon_{i,t} \), respectively. Next, we define \( \beta_i \) and \( \sigma(\epsilon_{i,t}) \) as the systematic and idiosyncratic risk for the \( i \)-th stock, respectively. It should be noted that we collect one-year daily data to run Eq. 1, and thus the two types of risk for each stock are calculated.

3. Model specifications for examining the risk-return relationship

3.1 No quantile models: OLS and LAD

Once the two types of risk for each stock are calculated, the risk-return relation in the cross-section of stock returns is examined as follows. Let \((y_{i,t}, x_{it}), i=1, 2, \ldots, N\) and \(t=1, 2, \ldots, T\) be a sample population, where subscript \(i\) denotes the \( i \)-th stock and \(t\) denotes the \( t \)-th period. The explained variable \( y_{i,t} \) represents the annual excess stock return of the \( i \)-th stock at period \( t \), and \( x_{it} \) is a (2X1) vector in which two types of risk (systematic vs. idiosyncratic) for the \( i \)-th stock are involved. Since our data have a
panel structure, the fixed effects model is thus used and expressed as follows:

\[ y_{it} = \alpha_i + x_{it} \cdot \beta + u_{it}, \quad (3) \]

where \( \alpha_i (i=1, 2\ldots, N) \) and \( \beta \) (2X1 vector) are unknown parameters to be estimated.

As part of this study’s focus on the dynamic relationships between \( y_{it} \) and \( x_{it} \) (i.e., the \( \beta \) parameters) we take the ‘group difference’ between variables and redefine Eq. 3 as follows:

\[ y_{it}^* = x_{it}^* \cdot \beta + u_{it}^*, \quad (4) \]

where * denotes variables deviated from the group mean, that is, \( y_{it}^* = y_{it} - \bar{y}_i \), \( x_{it}^* = x_{it} - \bar{x}_i \), \( u_{it}^* = u_{it} - \bar{u}_i \), and \( \bar{y}_i \), \( \bar{x}_i \) and \( \bar{u}_i \) are the means of \( y \), \( x \) and \( u \) of stock \( i \), respectively.

It should be noted that the setting in Eq. 4 (i.e., the non-quantile model) is potentially limited owing to the use of a constant parameter in each risk variable (i.e., \( x_{it} \)) for stock returns (i.e., \( y_{it} \)). Specifically, once the final result is a model of Eq. 2, the values of all the elements in the (KX1) vector \( \beta \) are fixed between the stocks experiencing a fall and rise in price.

By using the mathematical optimization technique of OLS, the estimator vector of \( \beta \) is obtained from:

\[ \min \sum_i 1 \times (u_{it}^*)^2 = \sum_i 1 \times (y_{it}^* - x_{it}^* \cdot \beta)^2. \quad (5) \]
Further, the sum of absolute errors can be minimized to get the \( \beta \) estimate of LAD:

\[
\min \sum_i |u_{it}^*| = \sum_i |y_{it}^* - x_{it}^* \cdot \beta|.
\]  

(6)

The constant term one in Eqs. 5 and 6 represents that the error terms are averaged by equal weight; thus, \( x_{it}^* \cdot \beta \) represents the conditional mean and the conditional median functions in the optimization technique of OLS and LAD, respectively. One key limitation of the OLS and LAD estimates is that they provide only one measure of the central distribution tendency of the dependent variable, and tail behaviors are not considered.

### 3.2 QR model

To resolve the potential shortcoming of a uniform risk-return relationship for stock returns, this study adopts the QR technique developed by Koenker and Basset (1978) to establish the following model specification:

\[
y_{it}^* = x_{it}^* \cdot \beta_{\theta} + u_{it}^*
\]

\[
Quant_{\theta}(y_{it}^*|x_{it}^*) \equiv \inf \{y : F_{\theta}(y|x)\theta = x_{it}^*</ \cdot \beta_{\theta} ,
\]

\[
Quant_{\theta}(u_{it}^*|x_{it}^*) = 0
\]

where \( Quant_{\theta}(y_{it}^*|x_{it}^*) \) denotes the \( \theta \)-th conditional quantile of \( y_{it}^* \) on the regressor vector \( x_{it}^* \); \( \beta_{\theta} \) is the unknown vector of parameters to be estimated for different values of \( \theta \) in \( (0,1) \); \( u_{it}^* \) is the error term assumed to be continuously differentiable c.d.f.
(cumulative density function) of $F_{u|x}(.|x)$ and a density function $f_{u|x}(.|x)$. The value $F_{u|x}(.|x)$ denotes the conditional distribution of the dependent variable conditional on $x$. Varying the value of $\theta$ from 0 to 1 reveals the entire distribution of $y$ conditional on $x$.

The estimator for $\beta_\theta$ is obtained from:

$$\min \sum_{u:u_0 > 0} \theta \times |u_{it}^*| + \sum_{u:u_0 < 0} (1-\theta) \times |u_{it}^*|$$

$$= \sum_{u:y_0 - x_0 \cdot \beta_\theta > 0} \theta \times |y_{it}^* - x_{it}^* \cdot \beta_\theta| + \sum_{u:y_0 - x_0 \cdot \beta_\theta < 0} (1-\theta) \times |y_{it}^* - x_{it}^* \cdot \beta_\theta|.$$  \hspace{1cm} (8)

Notably, the estimators do not have an explicit form, but the resulting minimization problem can be solved by linear programming techniques.\(^1\)

Restated, one feature of the QR is the ability to trace the entire distribution of dependent variables conditional on the independent variable. Comparing Eq. 8 with Eqs. 5 and 6 reveals a key feature of the QR technique: the estimator vector of $\beta_\theta$ varies with $\theta$. Moreover, by comparing the behaviors with different $\theta$, one could thus characterize the dynamic estimator vector, namely $\beta_\theta$, in various return-quantile regimes. In addition, a comparison of Eq. 6 with Eq. 8 reveals that the LAD estimator is a special case of the quantile-varying estimator with a quantile of 0.5. Further, one key limitation of the OLS and LAD estimators is that only a single measure of the central distribution tendency is provided, without considering tail behaviors.

4. Data and empirical results

Data from S&P500 firms are analyzed over a 10-year period of 1998 to 2007. Financial firms are excluded from the data sample, since their liabilities and capital

\(^1\) See Koenker (2000) and Koenker and Hallock (2001) for the related discussions.
structure intrinsically differ from those of non-financial firms. The overall sample consists of 392 firms and 3,794 annual observations. All data are obtained from the Datastream and Compustat databases.

It must be noted that we adopt a two-step procedure to examine the risk-return link in the cross-section of stock returns. In particular, we first estimate the two types of risk (systematic and idiosyncratic) using Eqs. 1 and 2, and then examine the risk-return relationship using Eq. 3 to Eq. 8. Table 1 summarizes the definitions of stock return and risk variables selected for this study, and the descriptive statistics of those variables are presented in Table 2.

The following two tables summarize the estimation results of the proposed QR model, with the OLS results also presented for comparison purposes. Notably, a multiple regression approach is adopted when implementing the QR model, in which the two explanatory variables (systematic and idiosyncratic risk) for stock returns are included simultaneously.

Table 3 summarizes the estimation results of the QR model to illustrate how systematic risk and stock returns are related. Figure 1 displays the corresponding graph. First, using the 5% level as a criterion, the OLS estimate of the systematic variable is insignificant. This result indicates the systematic risk-return relationship is not presented in our data. Next, the non-monotonic risk-return link derived by the QR model is examined. Importantly, while the systematic risk-return relationship deep outside the central region (i.e., 0.05 to 0.20 and 0.70 to 0.95) is insignificant, it is significant when the moderate quantiles from 0.25 to 0.45 are concerned.

The right columns of Table 3 list the $F$ test of the equality of slope parameters

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2 It must be noted that the specified financial data of some firms are not available for the entire ten-year period, and some firms do not continue to exist for ten consecutive years. Further, firms for which the annual stock returns exceed 500% and the beta estimates are larger than 6 in absolute value are excluded. Consequently, the panel data used in this work is unbalanced, and thus the number of available observations over the ten-year period is less than 3,920 (=392x10).
across various quantiles. It should be noted that the estimate at the 0.5 quantile is used as a benchmark. Notably, the inequality of slope estimates across various quantiles is significant (insignificant) at the tail (central) regions. In particular, near the central regions from 0.15 vs. 0.50 to 0.60 vs. 0.50, the inequality of estimates is insignificant. However, in the left-tailed (0.05 vs. 0.50) and right-tailed regions (0.95 vs. 0.50, 0.90 vs. 0.50, 0.85 vs. 0.50, 0.80 vs. 0.50, 0.75 vs. 0.50, 0.70 vs. 0.50 and 0.65 vs. 0.50), the difference is significant.

Table 4 lists the OLS and QR estimates for the idiosyncratic risk variable and Figure 2 displays the corresponding graph. Interestingly, the idiosyncratic risk variable is accompanied with an insignificant coefficient at moderate quantiles ranging from 0.45 to 0.80, as found in the OLS estimate, whereas the coefficient becomes significant for the left-tailed quantiles (0.05 to 0.40) and right-tailed quantiles (0.85 to 0.95). Further, the idiosyncratic risk is positively related to stock returns over the higher quantiles, from 0.85 to 0.95. Conversely, they are negatively connected for the lower quantiles, from 0.05 to 0.40. In addition, the right columns of Table 2 depict the $F$ tests of the equality of slope parameters across various quantiles: the differences between slope estimates at the $\theta$ and $(1-\theta)$ quantiles. Interestingly, differences across various quantiles are significant, except for the central region: 0.45 vs 0.55. Consequently, the observed nonlinearities derived from conditional QR reveal considerable differences in the idiosyncratic risk-return link with different levels of return quantile.

The aforementioned findings require further clarification. First, we state that situations involving the extreme return quantiles, i.e. $\theta=0.95$ and 0.05, closely correspond to volatile price conditions. Accordingly, the inverted U-shaped estimates in Figure 1 show that the CAPM systematic risk-return connection is valid for stocks
with a relatively stable price, i.e. stocks with moderate return quantiles. The opposite scenario occurs for tail quantiles, i.e. a large change in stock price. Prior studies (e.g. Basak, 2005; Levy et al., 2006; Fama and French, 2007; Berrada, 2008) invariably indicate that some of the assumptions for the CAPM do not hold, which is why the traditional CAPM has failed to explain the variation in equity prices. We further reveal that the invalid CAPM risk-return link is mainly composed of stocks with volatile price movements.

Further, although the link between idiosyncratic risk and returns is insignificant over moderate quantiles, as shown in Table 4 and Figure 2, they are significantly related when the tail quantiles are encountered. Malkiel and Xu (2002) and Jones and Rhodes-Kropf (2003) indicate that investors demand compensation for not being able to diversify risk. Some studies (see Barberis and Huang, 2001) employ behavioral models and demonstrate that stocks involved with higher idiosyncratic volatility earn higher returns. Conversely, Jiang, Xu, and Yao (2005), Zhang (2006), Bali and Cakici (2008) and Ang, et al. (2006, 2009) show that stocks with high idiosyncratic volatility have low returns. Importantly, these inconsistent empirical results involved in earlier studies could be satisfactorily accounted for by this study. In particular, as shown in Table 4 and Figure 2, the relationship between idiosyncratic risk and return is significantly positive over the higher quantiles, from 0.85 to 0.95, whereas their relation is significantly negative for the lower quantiles, from 0.05 to 0.40.

5. Conclusions

The CAPM risk-return relationship in stocks has long been of great interest to researchers, since the CAPM is one of the most fundamental models in economics and finance. While adopting large panel data over the period of 1998-2007, this work is
one of the first to examine the changing distribution in stock returns, across stocks and over time by using a conditional QR. The empirical results obtained with the QR system show that the risk-return link in stocks is not uniform cross-sectionally and occasionally, the relations are not even monotonic.

Our empirical results further demonstrate that the CAPM risk-return relationship is valid when stocks with stable price movements are concerned. Conversely, for stocks with volatile price changes, the model’s theoretical risk-return link is invalid. Importantly, this study demonstrates that the conventionally adopted OLS optimization approach elucidates central behaviors only, and misidentifies the risk-return connection, particularly for stocks significantly rising and falling in price. Finally, the non-monotonic relationship between risk and stock returns addressed by this study could provide a meaningful solution to the longstanding risk-return puzzles in earlier studies.
References


### Table 1 Definition of dependent/independent variables

<table>
<thead>
<tr>
<th>Variables</th>
<th>Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent variable ( y_{it} )</td>
<td>The annual excess stock returns</td>
</tr>
<tr>
<td>Independent variables ( x_{it} )</td>
<td>Systematic risk – The firm’s beta estimated from the CAPM regression using one-year historical data</td>
</tr>
<tr>
<td></td>
<td>Idiosyncratic risk: The standard deviation of the residual of the CAPM regression</td>
</tr>
</tbody>
</table>

Notes:
1. Data from S&P500 firms are analyzed over the 1998-2007 period. Financial firms are excluded from the data sample. The overall sample consists of 392 firms and 3,794 annual observations.
2. The specified financial data of some firms are not available for the entire ten-year period, and some firms do not continue to exist for ten consecutive years. Further, the firms in which the annual stock returns exceed 500% and the beta estimates are larger than 6 in the absolute value are excluded. Accordingly, the panel data used in this work is unbalanced, and thus the available observations over the ten-year period are less than 3,920 (=392x10).

### Table 2 Descriptive statistics of dependent/independent variables

<table>
<thead>
<tr>
<th></th>
<th>The annual stock returns</th>
<th>Systematic risk (beta)</th>
<th>Idiosyncratic risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>16.54</td>
<td>0.94</td>
<td>2.59</td>
</tr>
<tr>
<td>Median</td>
<td>8.96</td>
<td>0.89</td>
<td>1.85</td>
</tr>
<tr>
<td>S.D.</td>
<td>48.78</td>
<td>0.55</td>
<td>3.96</td>
</tr>
<tr>
<td>Skewness</td>
<td>2.41</td>
<td>1.05</td>
<td>9.14</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>11.34</td>
<td>8.49</td>
<td>105.58</td>
</tr>
<tr>
<td>Maximum</td>
<td>424.11</td>
<td>5.42</td>
<td>64.56</td>
</tr>
<tr>
<td>Minimum</td>
<td>-95.37</td>
<td>-3.19</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Notes:
1. This table summarizes certain descriptive statistics of the dependent/independent variables selected for this study.
2. The data source is same as in Table 1.
Table 3 The relationship between systematic risk and stock returns across various quantile levels

<table>
<thead>
<tr>
<th>Quantile</th>
<th>Estimated results of quantile regression</th>
<th>Statistic tests of the equality of slope estimates across various quantiles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Quantile Estimate (p-value)</td>
<td>Quantile Estimate (p-value)</td>
</tr>
<tr>
<td>0.05</td>
<td>-8.03 (0.069)</td>
<td>0.95</td>
</tr>
<tr>
<td>0.10</td>
<td>0.01 (0.998)</td>
<td>0.90</td>
</tr>
<tr>
<td>0.15</td>
<td>5.01 (0.141)</td>
<td>0.85</td>
</tr>
<tr>
<td>0.20</td>
<td>6.51 (0.055)</td>
<td>0.80</td>
</tr>
<tr>
<td>0.25</td>
<td>8.48 (0.005)**</td>
<td>0.75</td>
</tr>
<tr>
<td>0.30</td>
<td>8.05 (0.000)**</td>
<td>0.70</td>
</tr>
<tr>
<td>0.35</td>
<td>7.51 (0.000)**</td>
<td>0.65</td>
</tr>
<tr>
<td>0.40</td>
<td>7.08 (0.000)**</td>
<td>0.60</td>
</tr>
<tr>
<td>0.45</td>
<td>7.56 (0.000)**</td>
<td>0.55</td>
</tr>
<tr>
<td>0.50</td>
<td>7.02 (0.000)**</td>
<td>OLS</td>
</tr>
</tbody>
</table>

Notes:
1. The ** and * denote significance at the 1% and 5% levels, respectively.
2. The right columns of this table present the F tests of the equality of slope parameters across various quantiles. Notably, the estimate at the 0.5 quantile is used as a benchmark and the inequality of slope estimates across various quantiles is examined.
3. The data source is same as in Table 1.
### Table 4 The relationship between idiosyncratic risk and stock returns across various quantile levels

<table>
<thead>
<tr>
<th>Quantile</th>
<th>Estimate (p-value)</th>
<th>Quantile</th>
<th>Estimate (p-value)</th>
<th>Statistic tests of the equality of slope estimates across various quantiles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Quantile</td>
</tr>
<tr>
<td>0.05</td>
<td>-2.40 (0.000)**</td>
<td>0.95</td>
<td>2.50 (0.000)**</td>
<td>0.05 vs 0.95</td>
</tr>
<tr>
<td>0.10</td>
<td>-2.28 (0.000)**</td>
<td>0.90</td>
<td>1.26 (0.005)**</td>
<td>0.10 vs 0.90</td>
</tr>
<tr>
<td>0.15</td>
<td>-2.05 (0.000)**</td>
<td>0.85</td>
<td>0.79 (0.000)**</td>
<td>0.15 vs 0.85</td>
</tr>
<tr>
<td>0.20</td>
<td>-2.14 (0.000)**</td>
<td>0.80</td>
<td>0.63 (0.101)</td>
<td>0.20 vs 0.80</td>
</tr>
<tr>
<td>0.25</td>
<td>-1.48 (0.003)**</td>
<td>0.75</td>
<td>0.24 (0.498)</td>
<td>0.25 vs 0.75</td>
</tr>
<tr>
<td>0.30</td>
<td>-1.48 (0.000)**</td>
<td>0.70</td>
<td>0.11 (0.742)</td>
<td>0.30 vs 0.70</td>
</tr>
<tr>
<td>0.35</td>
<td>-1.33 (0.000)**</td>
<td>0.65</td>
<td>-0.08 (0.832)</td>
<td>0.35 vs 0.65</td>
</tr>
<tr>
<td>0.40</td>
<td>-1.28 (0.002)**</td>
<td>0.60</td>
<td>-0.25 (0.572)</td>
<td>0.40 vs 0.60</td>
</tr>
<tr>
<td>0.45</td>
<td>-0.96 (0.067)</td>
<td>0.55</td>
<td>-0.47 (0.364)</td>
<td>0.45 vs 0.55</td>
</tr>
<tr>
<td>0.50</td>
<td>-0.62 (0.266)</td>
<td><strong>OLS</strong></td>
<td>-0.03 (0.901)</td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**
1. The ** and * denote significance at the 1% and 5% levels, respectively.
2. The right columns of this table present the \( F \) tests of the equality of slope parameters across various quantiles. In particular, the difference between slope estimates at the \( \theta \) and \( (1-\theta) \) quantiles is examined in this table.
3. The data source is same as in Table 1.
Figures

Figure 1 The relationship between systematic risk and stock returns: QR estimates with 95% confidence intervals versus OLS estimate

Figure 2 The relationship between idiosyncratic risk and stock returns: QR estimates with 95% confidence intervals versus OLS estimate