**CHAPTER 11 PERFORMANCE-MEASURE APPROACHES FOR SELECTING OPTIMUM PORTFOLIOS**

1.

a) Short Selling - Involves borrowing a stock which is not owned and selling it in hopes of repurchasing it at a lower price.

b) Single Index Model - is a portfolio selection model which assumes that the securities are related through some market index. This assumption greatly reduces the number of inputs necessary for portfolio selection. The equation for the single index model



c) Treynor Measure - performance measure which is the ratio of the excess return of the portfolio over the risk free rate to the portfolio beta.



d) Sharpe Measure - performance measure which is the ratio of the excess return of a portfolio over the risk free rate to the total risk of the portfolio



2.

a) 

b) 

c) The difference between the two measures is measure of risk which is used. If the portfolio is well diversified, then the two measures will give similar results. Here, while the portfolio has a high total, risk as measured by σP, it also has a low systematic risk as measured by β*.* Treynor measure is better because it uses the systematic risk rather than the total risk.

3.

Using the Treynor measure approach of Elton, et al. (1976) in section 11.3, we need to find C\* first to compute Hi for each security i in Equation (11.12) and then we can obtain the optimal weights of standard method by $W\_{i}=\frac{H\_{i}}{\sum\_{i=1}^{n}H\_{i}}$ and obtain the optimal weights of Lintner’s method by $W\_{i}=\frac{H\_{i}}{\sum\_{i=1}^{n}\left|H\_{i}\right|}$

To find C\*, first step is to rank security by Treynor performance measure

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Security | $$\overbar{R\_{i}}$$ | $$\overbar{R\_{i}}-R\_{f}$$ | $$β\_{i}$$ | $$σ\_{ϵi}^{2}$$ | $$(\overbar{R\_{i}}-R\_{f})/β\_{i}$$ | $$β\_{i}/σ\_{ϵi}^{2}$$ |
| D | 7 | 3 | 0.6 | 10 | 5 | 0.06 |
| A | 10 | 6 | 1.3 | 30 | 4.615385 | 0.043333 |
| B | 9 | 5 | 1.2 | 20 | 4.166667 | 0.06 |
| C | 8 | 4 | 1.7 | 50 | 2.352941 | 0.034 |

By equation (11.15),
$C^{\*}=\frac{σ\_{m}^{2}\sum\_{j=1}^{d}\frac{(\overbar{R\_{i}}-R\_{f})β\_{i}}{σ\_{ϵi}^{2}}}{1+σ\_{m}^{2}\sum\_{j=1}^{d}\frac{β\_{i}^{2}}{σ\_{ϵi}^{2}}}$ where d is the set which contains all securities with positive $H\_{i}$, that is,
$$\frac{\overbar{R\_{i}}-R\_{f}}{β\_{i}}-C^{\*}\geq 0 for i\in set d$$

Given $σ\_{m}^{2}=10$,

|  |  |  |  |
| --- | --- | --- | --- |
| Security | $$\frac{σ\_{m}^{2}(\overbar{R\_{i}}-R\_{f})β\_{i}}{σ\_{ϵi}^{2}}$$ | $$σ\_{m}^{2}\frac{β\_{i}^{2}}{σ\_{ϵi}^{2}}$$ | $$\frac{σ\_{m}^{2}\sum\_{j=1}^{d}\frac{(\overbar{R\_{i}}-R\_{f})β\_{i}}{σ\_{ϵi}^{2}}}{1+σ\_{m}^{2}\sum\_{j=1}^{d}\frac{β\_{i}^{2}}{σ\_{ϵi}^{2}}}$$ |
| D | 1.8 | 0.36 | 1.323529 |
| A | 2.6 | 0.563333 | 2.287695 |
| B | 3 | 0.72 | **2.799496 (**$C^{\*}$**)** |
| C | 1.36 | 0.578 | 2.719371 |

$C^{\*}$=**2.799496** because $\frac{\overbar{R\_{i}}-R\_{f}}{β\_{i}}-C^{\*}\geq 0$ for securities D, A, B to get positive $H\_{i}$. If we choose C\*=2.719371, $\frac{\overbar{R\_{C}}-R\_{f}}{β\_{C}}-C^{\*}<0$ is negative. Therefore, the optimal $C^{\*}$=2.799496 where the set d includes securities D, A, and B.

Then we can obtain the optimal weights by using standard method and Lintner’s method as follows:

|  |  |  |  |
| --- | --- | --- | --- |
| Security | $$H\_{i}=\frac{β\_{i}}{σ\_{ϵi}^{2}}(\frac{\overbar{R\_{i}}-R\_{f}}{β\_{i}}-C^{\*})$$ | $$W\_{i}=\frac{H\_{i}}{\sum\_{i=1}^{n}H\_{i}}$$ | $$W\_{i}=\frac{H\_{i}}{\sum\_{i=1}^{n}\left|H\_{i}\right|}$$ |
| D | 0.13203 | 0.475671 | 0.428764 |
| A | 0.078689 | 0.283495 | 0.255539 |
| B | 0.08203 | 0.295534 | 0.266391 |
| C | -0.01518 | -0.0547 | -0.04931 |

The weights by using standard method and Lintner’s method are different are due to the different definitions of short selling discussed earlier. The standard method assumes that the investor has the proceeds of the short sale, while Lintner’s method assumes that the short seller does not receive the proceeds and must provide funds as collateral.

4. 





substituting values from table:

10 – 6 = HA(10)2 + HB(10)(6)(.5) + HC(10)(5)(.3)

8 – 6 = HA(10)(6)(.5) + HB(6)2 + HC(6)(5)(.8)

7 – 6 = HA(10) (5)(.3) + HB(6)(5)(.8) + HC(5) 2

simplifying:

4 = 100HA + 30HB + 15HC

2 = 30HA + 36HB + 24HC

1 = 15HA + 24HB + 25HC

Writing in matrix form, we get:



|  |  |  |
| --- | --- | --- |
|  | Hi | Wi |
| A | 0.0297 | 0.5541 |
| B | 0.0444 | 0.8280 |
| C | –0.0205 | –0.3822 |
| Total | 0.0537 | 1.0000 |

H1, H2, and H3 can be solved by Cramer’s rule (see page 404 of text) or by inverting S matrix.



5. Markowitz Model requires estimates of the covariance between each pair of assets and, therefore, is the most difficult portfolio selection model to implement.

Both the single index and multi-index models greatly reduce the number of inputs necessary for conducting portfolio analysis.

The performance measure approaches allow us to use information computed by the single index model to form optimal portfolios.