**Chain Rule**

The chain rule is similar to deal with [differentiating](https://www.wyzant.com/resources/lessons/math/calculus/differentiation) compositions of functions. A [composite function](https://www.wyzant.com/resources/lessons/math/algebra/functions) is a function that contains another function. The chain rule can be thought of as taking the derivative of the outer function (applied to the inner function) and multiplying it times the derivative of the inner function. The chain rule is arguably the most important rule of differentiation. It is commonly where most students tend to make mistakes, by forgetting to apply the chain rule when it needs to be applied, or by applying it improperly. Try to keep that in mind as you take derivatives.

**For a composite function** $y=g\left[f\left(x\right)\right]$**,our goal is to find the derivative.**

****

**based on our knowledge of the functions *f* and *g*. Now, we know that**

****

[**Leibniz's**](http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Leibniz.html)**differential notation suggests that perhaps derivatives can be treated as fractions, leading to the speculation that**

****

**This leads to the (possible) chain rule:**

****

**Before use the chain rule, you should remember the derivatives of some common functions.**

|  |  |  |
| --- | --- | --- |
| **Common Functions** | **Function** | **Derivative** |
| Constant | c | 0 |
| Line | x | 1 |
|   | ax | a |
| Square | x2 | 2x |
| Square Root | √x | (½)x-½ |
| Exponential | ex | ex |
|   | ax | ln(a) ax |
| Logarithms | ln(x) | 1/x |
|   | loga(x) | 1 / (x ln(a)) |
| Trigonometry (x is in [radians](https://www.mathsisfun.com/geometry/radians.html)) | sin(x) | cos(x) |
|   | cos(x) | −sin(x) |
|   | tan(x) | sec2(x) |
| **Rules** | **Function** | **Derivative** |
| Multiplication by constant | cf | cf’ |
| Power Rule | xn | nxn−1 |
| Sum Rule | f + g | f’ + g’ |
| Difference Rule | f - g | f’ − g’ |
| Product Rule | fg | f g’ + f’ g |
| Quotient Rule | f/g | (f’ g − g’ f )/g2 |
| Reciprocal Rule | 1/f | −f’/f2 |

**Some simple Examples:**

|  |  |
| --- | --- |
| **a.f(x) = e2x             f´(x) = e2x·2** | **Differentiating the exponential function leaves it unchanged , The derivative of 2x is 2.** |
| **b. f(x) = esin x  f´(x) = esin x·cos x** | **Differentiating the exponential function leaves it unchanged ,the derivative of sin x is cos x.** |

**c. f(x) = ln(x2 + 1).**



**d.f(x) = x·ln x – x + 5**

**f´(x) = 1·lnx + x·1/x – 1 = ln x**

**Example**

**The function sin(2*x*) is the composite of the functions sin(*u*) and *u*=2*x*. Then,**

****

**Example**



**Example**



**Reference**

MIT Open course

<http://www.rasmus.is/uk/t/F/Su64k05.htm>

https://sydney.edu.au/stuserv/documents/maths\_learning\_centre/compositefunctionrule.pdf