

# A comparison of alternative models for estimating firm's growth rate

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**Abstract** The growth rate plays an important role in determining a firm's asset and equity values, nevertheless the basic assumptions of the growth rate estimation model are less well understood. In this paper, we demonstrate that the model makes strong assumptions regarding the financing mix of the firm. In addition, we discuss various methods to estimate firms' growth rate, including arithmetic average method, geometric average method, compound-sum method, continuous regression method, discrete regression method, and inferred method. We demonstrate that the arithmetic average method is very sensitive to extreme observations, and the regression methods yield similar but somewhat smaller estimates of the growth rate compared to the compound-sum method. Interestingly, the ex-post forecast shows that arithmetic average method (compound-sum method) yields the best (worst) performance with respect to estimating firm's future dividend growth rate. Firm characteristics, like size, book-to-market ratio, and systematic risk, have significant influence on the forecast errors of dividend and sales growth rate estimation.

**Keywords** Equity valuation · Estimation of growth rate · Gordon's growth model · Determinants of growth forecast errors

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**JEL Classification** G31 · G35**1 Introduction**

In security analysis and portfolio management, growth rate estimates of earnings, dividends, and price per share are important factors in determining the value of an investment or a firm. One of the most highly used valuation models is that developed by Gordon and Shapiro (1956) and Gordon (1962) known as the dividend growth model. These publications demonstrate that the growth rate is found by taking the product of the retention rate and the return on equity. In this paper, we evaluate several methods suggested in the literature on estimating growth rates (e.g. Lee et al. 2009, 2012, 2013; Ross et al. 2012).

We examine various methods to estimate firms' growth rate, including arithmetic average method, geometric average method, compound-sum method, continuous regression method, discrete regression method, and inferred method. To compare various estimation methods, we empirically obtain historical dividend growth rates of all dividend paying companies in U.S. using the various estimation methods suggested in the literature. We find that the arithmetic average method is sensitive to extreme values and has an upward bias, resulting in a larger estimated dividend growth rates in comparison to all of the other methods. We also estimate sales growth rates for companies listed in three major U.S. stock exchanges and find that internal growth model and sustainable growth model yield relative conservative estimations.

The fact that one obtains varying estimates from different estimation methods indicates that choosing an appropriate method to estimate a firm's growth rate can yield a more precise estimation and be helpful for the security analysis and valuation. However, all of these methods use historical information to obtain growth estimate. To the extent that the future may differ from the past, we will ultimately determine the efficacy of any of these methods. We therefore conduct an ex-post forecast to examine the performance of various estimation methods in predicting firm's dividend growth rates and sales growth rates. Results of the ex-post forecast show that, for dividend growth rate estimations, the arithmetic average method is consistently and significantly superior to the other methods, and the continuous regression method performs the worst during the sample period between 1980 and 2012. For sales growth rate estimations, arithmetic average method, internal growth and sustainable growth model perform well, while compound sum method yields the worst estimation. We also find that applying those estimation methods to forecast dividend growth rate faces a challenge in recent years after 2005. We further investigate the determinants of the forecast errors and find that estimation models fail to forecast dividend growth rate and sales growth rate for small firms, illiquid firms, value firms, and firms with higher systematic risk.

This paper is organized as follows. Section 2 reviews the growth estimation literature beginning with the Gordon and Shapiro model (1956). The inherent assumptions of the model and implied methods to estimate the growth rate are discussed. Section 3 empirically applies various methods to estimate growth rates. Section 4 presents ex-post forecasts to compare forecast errors for various estimation methods. Concluding remarks appear in Sect. 5.

## 2 Literature review

In this section, we review both the theoretical and empirical methods to estimate growth according to the literature.<sup>1</sup> The traditional academic approach to evaluate a firm's equity is based upon the constant discount rate method. The value of equity can be directly found by discounting the dividends per share by the cost of equity, or more formally:

$$\text{Value of Common Stock } (P_0) = \sum_{t=1}^{\infty} \frac{d_t}{(1+r)^t}, \quad (1)$$

where  $d_t$  is the dividend per share at time  $t$ . Boudreaux and Long (1979) and Chambers et al. (1982) demonstrate that assuming a constant discount rate as assumed in Eq. (1) requires a financial strategy whereby the level of debt is a constant percentage of the value of the firm.<sup>2</sup>

If we assume that dividends per share grow at a constant rate  $g$ , then Eq. (2) is reduced to the basic dividend growth model<sup>3</sup>:

$$P_0 = \frac{d_1}{(r-g)}. \quad (2)$$

Gordon and Shapiro (1956) demonstrates that if  $b$  is the fraction of earnings retained within the firm, and  $r$  is the rate of return the firm will earn on all new investments (defined as ROE), then  $g = br$ .<sup>4</sup> Generally, practitioners define ROE as the ratio of the Net Income to the end of year Stockholders Equity. Here we are defining ROE as the ratio of the Net Income to the beginning of the year Stockholders Equity.<sup>5</sup> An investor can use Eq. (2) to obtain the theoretical stock price assuming the investor can empirically estimate next year's dividend per share, the firm's long-term growth rate, and the rate of return stockholders require (perhaps using the CAPM to estimate  $r$ ) for holding the stock. Stocks that have theoretical prices above actual price are candidates for purchase; those with theoretical prices below their actual price are candidates for sale or for short sale.

The internal growth model assumes that the firm can only finance its growth by its internal funds. Consequently, the cash to finance growth must come from only retained earnings, and the use of cash represented by the increase in assets must equal the change in the level of retained earnings. The internal growth rate can therefore be presented as<sup>6</sup>:

$$g = \frac{b \times ROA}{1 - b \times ROA}, \quad (3)$$

where  $ROA$  is the return on assets. The internal growth rate is the maximum growth rate that can be achieved without debt or equity kind of external financing. However, as Brick et al. (2014) note, this assumption of not issuing new debt or common stock to finance

<sup>1</sup> For a more detailed survey of the literature, see Brick et al. (2014).

<sup>2</sup> See Brick and Weaver (1984, 1997) concerning the magnitude of error in the valuation using a constant discount rate when the firm does not maintain a constant market based leverage ratio.

<sup>3</sup> Gordon and Shapiro's (1956) model assume that dividends were paid continuously and hence  $P_0 = d_1/(r-g)$ .

<sup>4</sup> Earnings in this model are defined using the cash-basis of accounting and not on an accrual basis.

<sup>5</sup> Baucus et al. (1993) and Brick et al. (2012) demonstrate that the practitioner's definition is one of the sources for the Bowman Paradox reported in the Organization Management literature.

<sup>6</sup> For a clear presentation of the internal growth rate, see Ross et al. (2010) and Brick et al. (2014).

growth is inconsistent with the basic assumption of the constant discount rate models that the firm maintains a constant market based leverage ratio. Hence, this model should not be used to estimate the growth rate and be employed by the Gordon's growth model.

Higgins (1977, 1981, 2008) developed a sustainable growth model assuming that firms can generate new funds by using retained earnings or issuing debt, but not issuing new shares of common stock. Assuming a company is not raising new equity, the cash to finance growth must come from retained earnings and new borrowings. Further, because the company wants to maintain a target debt-to-equity ratio equal to  $L$ , each dollar added to the owners' equity enables it to increase its indebtedness by  $\$L$ . The use of cash represented by the increase in assets must equal the two sources of cash (retained earnings and new borrowings). Higgins (1977, 1981, 2008) demonstrates that one can estimate the growth rate as equal to the growth rate of sales. Consequently,

$$g = \frac{\Delta S}{S} = \frac{pb(1+L)}{T - pb(1+L)}, \quad (4)$$

where  $T$  is the ratio of total assets to sales. In Eq. (4),  $\Delta S/S$  or  $g$  is the firm's sustainable growth rate assuming no infusion of new equity.

Growth and its management present special problems in financial planning. According to Eq. (4), a company's growth rate in sales must equal the indicated combination of four ratios,  $p$ ,  $b$ ,  $L$ , and  $T$ . In addition, if the company's growth rate differs from  $g$ , one or more of the ratios must change. For example, suppose a company grows at a rate in excess of  $g$ , then it must either use its assets more efficiently, or it must alter its financial policies. Efficiency is represented by the profit margin and asset-to-sales ratio. It therefore would need to increase its profit margin ( $p$ ) or decrease its asset-to-sales ratio ( $T$ ) in order to increase efficiency. Financial policies are represented by payout or leverage ratios. In this case, a decrease in its payout ratio ( $1 - b$ ) or an increase in its leverage ( $L$ ) would be necessary to alter its financial policies to accommodate a different growth rate. From a financial perspective, growth is not always a blessing. Rapid growth can put considerable strain on a company's resources, and unless management is aware of this effect and takes active steps to control it, rapid growth can lead to bankruptcy. In other words, it should be noted that increasing efficiency is not always possible and altering financial policies are not always wise.

If we divide both numerator and denominator of Eq. (4) by  $T$  and rearrange the terms, then we can show that the sustainable growth rate can be shown as

$$g = \frac{\Delta S}{S} = \frac{pb(1+L)/T}{1 - pb(1+L)/T} = \frac{b \times ROE}{1 - b \times ROE}. \quad (5)$$

Please note that, in the framework of internal growth rate and sustainable growth rate presented above, the source of cash are taken from the end of period values of assets and assumed that the required financing occurs at the end of the period. However, Ross et al. (2010) show that if the source of cash is from the beginning of the period, the relationship between the use and the source of cash can be expressed for the internal growth model as

$\Delta ST = pSb$  and for the sustainable growth model,  $\Delta ST = pbS + pbSL$ . Such relationship will result an internal growth rate of  $b \times ROA$  and a sustainable growth rate of  $b \times ROE$ .

Note that the intent of the Higgins' sustainable growth rate allows only internal source and external debt financing. Chen et al. (2013) incorporate Higgins (1977) and Lee et al. (2011) frameworks, allowing company to use both external debt and equity, and derive a generalized sustainable growth rate as

$$g(t) = \frac{b \times ROE}{1 - b \times ROE} + \frac{\lambda \times \Delta n \times P/E}{1 - b \times ROE}, \tag{6}$$

where  $\lambda$  is the degree of market imperfection,  $\Delta n$  is the number of shares of new equity issued,  $P$  is price per share of new equity issued, and  $E$  represents the total equity. Comparing to Eq. (5), the generalized sustainable growth rate has an additional positive term,  $\frac{\lambda \times \Delta n \times p/E}{1 - b \times ROE}$ , when the new equity issue is taken into account. Therefore, Chen et al. (2013) show that Higgins' (1977) sustainable growth rate is underestimated because of the omission of the source of the growth related to new equity issue.

In addition, Chen et al. (2013) theoretically show the existence of specification error of dividend per share when introducing stochastic growth rate. If a firm's asset growth rate is not deterministic, the estimated dividend payouts is measured with error. Their empirical results show the importance of covariance between the profitability and the growth rate in determining dividend payouts and provide an alternative explanation of the fact of disappearing dividends over decades.

Instead of relying on financial ratios to estimate firm's growth rates, one may use statistical methods to determine firm's growth rates. A simple growth rate can be estimated by calculating the percentage change in earnings over a time period, and taking the arithmetic average. For instance, the growth rate in earnings over one period can be expressed as:

$$g_t = \frac{E_t - E_{t-1}}{E_{t-1}}. \tag{7}$$

The arithmetic average is given by

$$\bar{g} = \frac{1}{n} \sum_{t=1}^n g_t. \tag{8}$$

The arithmetic average growth rate method ignores compounding. Consequently, we can obtain an estimate of the growth rate by solving for the compounded growth rate:

$$X_t = X_0(1 + g)^t, \tag{9}$$

or

$$g = \left(\frac{X_t}{X_0}\right)^{1/t} - 1, \tag{10}$$

where  $X_0$  is the measure in the current period (measure can be sales, earnings, or dividends); and  $X_t$  is the measure in period  $t$ . This method is called the (*discrete*) *compound-sum* method of growth-rate estimation. For this approach to be consistent with the dividend growth model, the duration of each period (e.g., quarterly or yearly) must be consistent with the compounding period used in the dividend growth model.

Another method of estimating the growth rate uses the continuous compounding process. The concept of continuous compounding process can be expressed mathematically as

$$X_t = X_0 e^{gt}. \tag{11}$$

Equation (10) describes a discrete compounding process and Eq. (11) describes a continuous compounding process. The relationship between Eqs. (10) and (11) can be illustrated by using an intermediate expression such as:

$$X_t = X_0 \left(1 + \frac{g}{m}\right)^{mt}, \tag{12}$$

where  $m$  is the frequency of compounding in each year. If  $m = 4$ , Eq. (12) implies a quarterly compounding process; if  $m = 365$ , it describes a daily process; and if  $m$  approaches infinity, it describes a continuous compounding process. Thus Eq. (11) can be derived from Eq. (12) based upon the definition

$$\lim_{m \rightarrow \infty} \left(1 + \frac{1}{m}\right)^m = e. \tag{13}$$

Then the continuous analog for Eq. (12) can be rewritten as

$$\begin{aligned} \lim_{m \rightarrow \infty} X_t &= \lim_{m \rightarrow \infty} X_0 \left(1 + \frac{g}{m}\right)^{mt} \\ &= X_0 \lim_{m \rightarrow \infty} \left(1 + \frac{1}{m/g}\right)^{\left(\frac{m}{g}\right)gt} \\ &= X_0 e^{gt}. \end{aligned} \tag{14}$$

Therefore, the growth rate estimated by continuous compound-sum method can be expressed by

$$g = \frac{1}{t} \ln \frac{X_t}{X_0}. \tag{15}$$

If you estimate the growth rate via Eq. (15), you are implicitly assuming the dividends are growing continuously consistent with the Gordon and Shapiro's (1956) growth model,  $P_0 = d_0 / (r - g)$ .

To use all the information available to the security analysts, two regression equations can be employed. The first regression equation can be derived from Eq. (10) by taking the logarithm on both sides of equation yields:

$$\ln X_t = \ln X_0 + t \ln(1 + g). \tag{16}$$

If Eq. (16) can be used to estimate the growth rate, then the antilog of the regression slope estimate would equal the growth rate. The second regression equation is based upon the continuous growth Eq. (11). Taking the logarithm on both sides of the equation yields:

$$\ln X_t = \ln X_0 + gt. \tag{17}$$

Both Eqs. (16) and (17) indicate that  $X_n$  is linearly related to  $t$ ; and the growth rate can be estimated by the ordinary least square (OLS) regression. For example, growth rates for *EPS* and *DPS* can be obtained from an OLS regression by using

$$\ln\left(\frac{EPS_t}{EPS_0}\right) = a_0 + a_1T + \varepsilon_{1t}, \quad (18)$$

and

$$\ln\left(\frac{DPS_t}{DPS_0}\right) = b_0 + b_1T + \varepsilon_{2t}, \quad (19)$$

where  $EPS_t$  and  $DPS_t$  are earnings per share and dividends per share, respectively, in period  $t$ , and  $T$  is the time indicators (i.e.,  $T = 1, 2, \dots, n$ ). We denote  $\hat{a}_1$  and  $\hat{b}_1$  as the estimated coefficients for Eqs. (18) and (19). The estimated growth rates for  $EPS$  and  $DPS$ , therefore, are  $\exp(\hat{a}_1) - 1$  and  $\exp(\hat{b}_1) - 1$  in terms of discrete compounding process and  $\hat{a}_1$  and  $\hat{b}_1$  in terms of continuous compounding process.

Finally, Gordon and Gordon (1997) suggest that one can infer the growth rate using the dividend growth model. In particular, the practitioner can use regression analysis to calculate the beta of the stock and use the *CAPM* to estimate the cost of equity. Since

$$P_0 = \frac{d_0(1+g)}{(r-g)}, \quad (20)$$

and the price of the stock is given by the market, the cost of equity is obtained using the *CAPM*, and  $d_0$  and the current dividend is known, one can infer the growth rate using Eq. (20). If the inferred growth rate is less than the practitioner's estimate, then the recommendation will be to buy the stock. On the other hand, if the inferred growth is greater than the practitioner's estimate, the recommendation will be to sell the stock. However, it should be noted that the explanatory power of the *CAPM* to explain the relationship between stock returns and risk has been extensively questioned in the literature. See for example, Fama and French (1992).

### 3 Estimations for various growth rates

In this section, we provide empirical tests to assess the accuracy of the methods listed in the previous section in estimating the growth rate. We first will provide an example of two companies to provide additional insights into the estimation problems that might occur. Next, we summarize the results of estimating the growth rates for public stocks on US exchanges. Finally, we present the results of an ex-post forecast to examine how well of each growth model in predicting firm's dividend growth rates.

#### 3.1 Data

To examine the efficacy of various growth rate estimation methods, we collect firm information from Compustat, including cash dividends, sales, net income, total asset, total equity, etc. We also collect price and turnover data from CRSP to obtain firm's size, turnover, and beta. Companies listed on NYSE, AMEX, or NASDAQ with data available from Compustat and CRSP are included in our sample. The sample period is from 1981 through 2012. For the dividend growth rate estimation, we include companies paying cash dividends at least for ten consecutive years during the sample period. For the sales growth rate estimation, we include companies with at least ten consecutive positive annual sales and earnings during the sample period.

### 3.2 An example

To compare various estimation methods, we empirically obtain dividends per share, sales, and net income for Pepsico and Wal-Mart during the period from 1981 to 2010 and estimate their dividend growth rates and sales growth rates by various estimation methods. Table 1 shows that Pepsico's dividend payments range from \$0.72 to \$4.51, while Wal-Mart's dividend payments, between \$1.02 and \$4.20, exhibiting a steady increase during the period. In addition, Wal-Mart experiences a faster sales growth than Pepsico does

**Table 1** Dividend behavior of firms Pepsico and Wal-Mart

Year	T	Pepsico		Wal-Mart	
		Dividend per share (\$)	Sales (\$MM)	Dividend per share (\$)	Sales (\$MM)
1981	1	3.61	7027	1.73	2445
1982	2	2.40	7499	2.50	3376
1983	3	3.01	7896	1.82	4667
1984	4	2.19	7699	1.40	6401
1985	5	4.51	8057	1.91	8451
1986	6	1.75	9291	1.16	11,909
1987	7	2.30	11,485	1.59	15,959
1988	8	2.90	13,007	1.11	20,649
1989	9	3.40	15,242	1.48	25,811
1990	10	1.37	17,803	1.90	32,602
1991	11	1.35	19,608	1.14	43,887
1992	12	1.61	21,970	1.4	55,484
1993	13	1.96	25,021	1.74	67,345
1994	14	2.22	28,472	1.02	82,494
1995	15	2.00	30,421	1.17	93,627
1996	16	0.72	31,645	1.33	104,859
1997	17	0.98	20,917	1.56	117,958
1998	18	1.35	22,348	1.98	137,634
1999	19	1.40	20,367	1.25	165,639
2000	20	1.51	20,438	1.41	192,003
2001	21	1.51	26,935	1.49	218,529
2002	22	1.89	25,112	1.81	245,308
2003	23	2.07	26,971	2.03	257,157
2004	24	2.45	29,261	2.41	286,103
2005	25	2.43	32,562	2.68	313,335
2006	26	3.42	35,137	2.92	345,977
2007	27	3.48	39,474	3.17	375,376
2008	28	3.26	43,251	3.36	402,298
2009	29	3.81	43,232	3.73	406,103
2010	30	3.97	57,838	4.20	420,016

This table provides dividends per share and annual sales of Pepsico and Wal-Mart during the period from 1981 to 2010



**Table 2** Estimated growth rates for Pepsico and Wal-Mart

	Pepsico (%)	Wal-Mart (%)
<i>Panel A: Dividend growth rate</i>		
Arithmetic average	4.64	8.99
Geometric average	0.99	5.45
Compound-sum method	0.99	5.30
Regression method (continuous)	0.56	7.04
Regression method (discrete)	0.56	7.29
Gordon's growth model	9.92	3.59
<i>Panel B: Sales growth rate</i>		
Arithmetic average	8.32	19.97
Geometric average	3.21	8.01
Compound-sum method	3.05	7.45
Regression method (continuous)	2.70	7.60
Regression method (discrete)	2.73	7.70
Internal growth model	6.56	2.40
Sustainable growth model	11.20	3.74

This table presents estimated dividend growth rates and sales growth rates for Pepsico and Wal-Mart. Estimation periods is from 1981 to 2010. Dividend growth rates are estimated by arithmetic average method, geometric average method, compound-sum method, regression methods in terms of discrete and continuous compounding processes, and Gordon's growth model. Sales growth rates are estimated by arithmetic average method, geometric average method, compound-sum method, regression methods in terms of discrete and continuous compounding processes, internal growth model, and sustainable growth model

during the sample period. Using the data in Table 1 for Pepsico and Wal-Mart, we can estimate the growth rates for their respective dividend and sales streams.

Table 2 presents the estimated the dividend growth rates and sales growth rates for Pepsico and Wal-Mart by arithmetic average method, geometric average method, compound-sum method, and regression methods in terms of discrete and continuous compounding processes. We also estimate their dividend growth rates by Gordon's growth model and sales growth rates by internal growth model and sustainable growth model. Panel A shows estimations of dividend growth rate for Pepsico and Wal-Mart by various estimation methods. Estimations of dividend growth rates for Pepsico range from 0.56 to 9.92 %, while estimations for Wal-Mart range from 3.59 to 8.99 %. For example, the arithmetic average estimates the dividend growth rate for Pepsico to be 4.64 %. The geometric average and compound-sum method provide an estimate growth rate of 0.99 % for Pepsico. The continuous and discrete regression methods estimate the dividend growth rate for Pepsico to be 0.56 %.<sup>7</sup> Finally, the Gordon's growth model provides an estimate of 9.92 % for Pepsico. Panel B presents estimations of sales growth rate for Pepsico and Wal-Mart. Similar to the estimation of dividend growth rate, we find different methods may result in different sales growth estimations.

There are varying attributes some complications to be aware of when employing the various empirical estimating methods discussed in the previous section. For example, the

<sup>7</sup> The growth rates for Pepsico obtained by continuous and discrete regression methods are not statistically different from zero.

arithmetic average is quite sensitive to extreme values. The arithmetic average, therefore, has an upward bias that increases directly with the variability of the data. Consider the following situation. Dividends in years 1, 2, and 3 are \$2, \$4, and \$2. The arithmetic average of growth rate is 25 % but the geometric average of growth rate is 0 %. The difference in the two average techniques will be greater when the variability of the data is larger. Therefore, it is not surprising that we find differences in the estimated growth rates using arithmetic average and geometric average methods for Pepsico and Wal-Mart in Table 2.

The regression method uses more available information than the geometric average, discrete compounding, and continuous compounding methods in that the other methods only take into account the observed growth rates between the first and last period of the sample. A null hypothesis test can be used to determine whether the growth rate obtained from the regression method is statistically significantly different from zero or not.<sup>8</sup> However, logarithms cannot be taken with zero or negative numbers. Under this circumstance the arithmetic average will be a better alternative.

Instead of using historical dividend payments or sales to estimate firm's dividend growth rate or sales growth rate, Gordon's growth model, internal growth model, and sustainable growth model use information other than dividend and sales, such as retention rate, ROA, and ROE, to estimate a firm's growth rates. However, such growth rate estimations are theoretical and all assume that the firm has a constant efficiency and leverage level.

### 3.3 Empirical estimates of growth for Public U.S. companies

We further estimate dividend growth rates for companies listed in NYSE, AMEX, or NSDAQ by various estimation methods including arithmetic average method, geometric average method, compound-sum method, regression methods in terms of discrete and continuous compounding processes, Gordon's growth model, internal growth model, and sustainable growth model.<sup>9</sup> Companies included in our sample should have cash dividends for at least 10 consecutive years during the period from 1981 to 2012. Table 3 shows the summary statistics for those major characteristics of sample firms, including total assets, total liability, dividends, sales, net income, ROE, ROA, market capitalization, book-to-market ratio, and beta coefficient. Panel A of Table 3 presents mean, standard deviation, maximum, third-quartile, median, first-quartile, minimum of characteristics for all firm-years during the period from 1981 to 2012. Panel B of Table 3 provide cross-sectional averages of each characteristics by year.

<sup>8</sup> The growth rates for Pepsico obtained by continuous and discrete regression methods are not statistically different from zero.

<sup>9</sup> When applying Gordon's growth model, internal growth model, and sustainable model, additional information should be obtained, including the required rate of return for equity holders, ROE, ROA, and the retention rate. As defined by Eq. (2), ROE and ROA are obtained from the ratios of net income of the year to the book value of common equity and the book value of total assets at the beginning of year. Retained earnings are computed as one subtract to the ratio of cash dividends to the income before extraordinary items. We apply Gordon's growth model with cost of equity by CAPM, in that the CAPM-based cost of equity is calculated as individual firm's beta times market risk premium plus risk-free rate. Individual firm's beta is estimated over the past three years monthly returns and risk-free rate and market risk premium is retrieved from Kenneth French's website, [http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html).

**Table 3** Summary statistics of sample firm characteristics

	Total asset (\$MM)	Total liability (\$MM)	Dividend (\$MM)	Net income (\$MM)	Sales (\$MM)	Retention Rate (%)	ROE (%)	ROA (%)	ME (\$MM)	BM	Beta
<i>Panel A: Summary statistics of sample firm characteristics for all firm-years</i>											
Mean	5977.76	5042.91	63.37	2203.40	113.20	32.04	1.91	1.13	635.10	0.93	1.16
Std. Dev.	40960.59	37905.43	306.49	8728.82	583.95	270.50	56.97	18.07	3692.08	1.62	0.78
Max	1369301.67	1266749.85	6276.67	183357.51	15470.61	13280.81	132.18	19.90	177715.47	48.79	8.06
Q3	1817.64	1394.27	17.32	1047.49	44.58	37.95	4.56	9.28	271.93	1.09	1.55
Median	334.32	270.92	1.24	222.17	4.36	6.25	1.83	5.65	66.04	0.68	1.09
Q1	53.33	43.15	0.002	40.55	-0.57	0.00	-2.10	1.17	18.12	0.39	0.67
Min	0.16	0.18	0	-0.03	-6449.16	-7802.93	-155.16	-149.97	0.30	0.01	-5.1
Total firm-years	5818	5818	5818	5818	5818	5818	5818	5818	5818	5818	5818
<i>Panel B: Cross-sectional averages of firm characteristics by year</i>											
Year	Total assets (\$MM)	Total liability (\$MM)	Dividend (\$MM)	Sales (\$MM)	Net income (\$MM)	Retention rate (%)	ROE (%)	ROA (%)	ME (\$MM)	BM	Beta
1981	1429.94	1068.64	27.948	1187.78	60.27	31.63	7.56	9.09	518.50	1.047	1.14
1982	1553.99	1226.95	27.07	1140.81	49.99	33.35	6.85	5.15	484.95	1.01	1.12
1983	1611.24	1286.25	28.43	1119.00	53.937	27.93	6.31	6.14	475.79	1.00	1.12
1984	1676.06	1375.47	28.74	1167.02	61.00	27.79	5.16	4.90	483.08	1.00	1.11
1985	1806.32	1543.21	28.06	1189.70	52.28	34.80	0.70	3.15	488.07	0.97	1.11
1986	1925.50	1677.30	29.54	1154.60	50.93	35.47	3.33	2.28	526.98	0.96	1.12
1987	2198.81	1859.70	30.71	1252.96	62.66	41.43	6.75	2.99	506.71	0.95	1.11
1988	2460.07	2164.06	34.28	1366.28	74.09	37.64	1.95	3.46	517.74	0.94	1.09
1989	2644.12	2371.62	35.06	1446.09	74.32	55.68	2.45	3.10	519.12	0.95	1.09
1990	2828.48	2565.83	36.02	1561.10	63.63	29.20	-4.10	2.46	501.36	0.94	1.08
1991	2866.43	2615.30	34.42	1525.11	46.60	48.07	-1.15	2.35	527.77	0.92	1.09
1992	2930.32	2717.92	34.73	1513.45	31.52	17.49	2.14	2.69	523.01	0.90	1.09

Table 3 continued

Year	Total assets (\$MM)	Total liability (\$MM)	Dividend (\$MM)	Sales (\$MM)	Net income (\$MM)	Retention rate (%)	ROE (%)	ROA (%)	ME (\$MM)	BM	Beta
1993	3046.27	2867.77	33.67	1450.21	48.10	40.61	6.98	3.20	521.24	0.89	1.09
1994	3232.19	3022.15	34.48	1543.07	73.33	23.46	6.80	2.98	521.76	0.88	1.09
1995	3444.30	3320.66	35.54	1613.60	77.61	108.22	4.03	2.77	524.01	0.86	1.10
1996	3740.46	3594.51	39.20	1689.96	95.42	21.65	1.73	3.08	405.71	1.03	1.09
1997	4213.57	4138.95	43.19	1820.10	99.74	26.12	2.10	1.98	927.86	0.96	1.07
1998	4982.65	4961.71	45.76	1978.19	103.90	19.94	3.91	0.51	744.65	0.89	1.09
1999	5763.17	5679.67	50.69	2209.93	132.88	25.72	3.71	0.68	1173.92	0.96	1.06
2000	6607.42	6569.70	52.59	2591.34	151.53	20.58	2.08	0.40	955.99	0.93	1.01
2001	7201.89	7226.85	55.22	2675.30	46.55	29.19	-1.51	-2.75	583.22	1.01	0.96
2002	7850.57	7978.48	57.03	2746.34	30.54	33.73	-5.27	-2.15	782.10	1.02	1.01
2003	8858.90	9094.11	66.17	3070.79	178.42	30.93	3.49	-0.55	224.91	0.96	0.90
2004	10,156.65	10,548.35	84.19	3522.99	226.60	18.22	6.01	1.51	2137.22	0.92	0.97
2005	10,824.70	11,253.19	98.23	3751.71	255.91	-10.32	5.51	0.96	1285.34	0.86	0.89
2006	12,609.28	13,084.57	121.44	4303.83	354.44	20.74	6.52	1.09	833.36	0.84	1.00
2007	15,018.70	15,789.24	147.40	4838.92	344.94	40.19	5.42	1.16	1571.54	0.91	1.04
2008	15,333.77	16,266.79	141.71	5047.84	103.07	58.18	-5.22	-1.74	1123.53	0.85	1.12
2009	15,656.16	16,033.30	126.44	4720.51	228.49	38.19	-8.75	-1.24	595.11	0.87	1.07
2010	18,194.95	18,658.94	137.71	5389.54	359.17	35.97	1.30	1.80	833.34	0.80	1.03
2011	19,596.91	21,378.17	164.16	6059.95	400.45	61.31	4.11	1.67	1122.70	0.90	1.03
2012	21,447.26	21,106.25	188.97	6431.73	390.39	57.11	3.27	1.22	3834.76	0.76	1.00
Avg.	6990.97	7032.68	65.59	2596.24	136.96	35.01	2.63	2.01	836.73	0.93	1.06

This table presents the descriptive statistics for major characteristics of sample firms. Sample includes those firms listed on NYSE, AMEX, and NASDAQ with at least 10 years of data available to compute the average total assets, dividend, sales, net income, retention rate, return on assets (ROA), return on equity (ROE), total liability, firms size (ME), book-to-market ratios (BM), and beta. ROE is obtained as the ratio of the net income to stockholders equity of the beginning of year and ROA is ratio of net income to total assets of the beginning of year. Beta is estimated from the regression of capital asset pricing model by using prior 3 years monthly returns. Panel A lists the summary statistics for all firm-years, while Panel B lists cross-sectional averages of each variable by year

**Table 4** Estimated dividend growth rates for U.S. companies

	All firms (2287 firms)	Firms with positive dividend growth (1898 firms)	Firms with negative dividend growth (389 firms)
<i>Panel A. Period: 1981–2012</i>			
Arithmetic average	5.13 % (4.92)	9.39 % (4.53)	−3.50 % (−3.21)
Geometric average	2.02 % (9.36)	3.15 % (7.35)	−0.51 % (−1.46)
Compound-sum method	1.26 % (4.24)	3.07 % (8.86)	−2.06 % (−3.79)
Regression method (continuous)	−0.84 % (−4.34)	−0.40 % (−1.61)	−2.98 % (−35.01)
Regression method (discrete)	−0.84 % (−4.34)	−0.40 % (−1.61)	−2.94 % (−35.01)
Gordon's growth model	2.48 % (5.01)	7.87 % (11.80)	−3.08 % (−4.20)
<i>Panel B. Period: 1981–1995</i>			
Arithmetic average	4.42 % (2.97)	10.53 % (2.67)	−4.80 % (−2.39)
Geometric average	0.26 % (2.07)	2.31 % (2.96)	−1.76 % (−3.32)
Compound-sum method	0.32 % (2.00)	0.75 % (4.05)	−0.58 % (−1.99)
Regression method (continuous)	−1.71 % (−8.10)	−1.05 % (−4.42)	−1.92 % (−8.74)
Regression method (discrete)	−1.69 % (−8.10)	−1.05 % (−4.42)	−1.90 % (−8.74)
Gordon's growth model	5.11 % (7.01)	10.53 % (8.23)	−1.75 % (−5.50)
<i>Panel C. Period: 1996–2012</i>			
Arithmetic average	5.79 % (3.90)	8.31 % (5.00)	−2.30 % (−2.40)
Geometric average	2.78 % (29.46)	3.95 % (12.80)	0.67 % (3.37)
Compound-sum method	0.04 % (0.31)	0.85 % (5.79)	−2.44 % (−8.87)
Regression method (continuous)	1.55 % (4.48)	1.95 % (4.63)	0.38 % (1.64)
Regression method (discrete)	1.56 % (4.48)	1.97 % (4.63)	0.38 % (1.64)
Gordon's growth model	0.35 % (0.52)	5.29 % (12.27)	−3.97 % (−3.30)

This table presents the averages of estimated dividend growth rates for companies by arithmetic average method, geometric average method, compound-sum method, and the regression methods in terms of discrete and continuous compounding processes. Sample companies are listed in NYSE, AMEX, or NASDAQ, and pay cash dividends at least for ten consecutive years during the period from 1981 to 2012. Sample companies are further divided into positive dividend growth firms and negative dividend growth firms in terms of their arithmetic average growth rates. Panel A exhibits the estimations of whole sample period, while Panels B and C cover the estimations of the sample period from 1981 to 1995 and 1996 to 2012, respectively. Values denoted in parentheses are *t*-statistics on the null hypothesis that the average number is different from zero

Table 4 shows averages of estimated dividend growth rates for 2287 companies by each of the different methods described in Sect. 2. Table 4 also dichotomizes the companies by whether or not they exhibit positive or negative growth firms as measured by their arithmetic average growth rates. As summarized by Table 4, the arithmetic average provides the highest estimated growth rates compared to the other methods. This is consistent with our discussion in the previous section, whereby we noted that the arithmetic average is sensitive to extreme values and has an upward bias. We also find, for positive growth companies, estimates from regression methods are less positive (even yield negative numbers) relative to estimates from arithmetic average, geometric average, and compound-sum methods, while estimates for negative growth companies from regression methods are

less negative relative to those from other methods. We therefore demonstrate that, on average, regression methods yield relatively conservative dividend growth rate estimates as compared to estimates obtained by arithmetic average, geometric average, and compound-sum methods. Similar results can be found in two-sub-periods, 1981–1995 and 1996–2012, presented in Panels B and C of Table 4 respectively.

In addition to estimating dividend growth rates, we can estimate sales growth rates using the different procedures outlined in Sect. 2. We estimate sales growth rates for companies listed in NYSE, AMEX, or NSDAQ during the period from 1981 to 2012. We also estimate sales growth rates for subsamples with positive/negative growth rates. As shown in Table 5, similar to the results of Table 4, the arithmetic averages yield highest sales growth rates, geometric average and regression methods obtain relatively lower growth rates, whereas internal growth model and sustainable growth model have the lowest sales growth estimations.

#### 4 Ex-post forecasts

We further conduct an ex-post forecast to examine how accurate each of the growth estimation procedures in predicting firm's dividend growth rates. For each growth rate procedure, we forecast the following year's growth rate of each firm based on previous 10 years data. For example, we estimate the dividend growth rates for individual companies in 1980 fiscal year sample by using their dividend payments from 1970 to 1979. Same procedures are used to estimate dividend growth rates for each firm-years until 2012. Table 6 presents the estimated forecast errors and mean square errors for each approach during the year from 1981 to 2012. As can be discerned by Table 6, the arithmetic error on average is unbiased. The average error for the entire period is  $-0.008$  and this average is not significantly different from zero. The continuous regression estimation model exhibits the greatest average forecast error (0.157) and the error is significantly different from zero. The significantly positive averages of forecast errors for geometric average method and continuous regression method show that geometric average method and continuous regression method tend to underestimate a firm's dividend growth rate.

The mean square errors of arithmetic average and geometric average methods are relatively small during 1980s, 1990s and early 2000s, and increase after 2006 indicating that arithmetic average and geometric average methods, on average, can estimate dividend growth rate for most years, but lose their efficacy during a period of a financial crisis (post 2006). Mean square errors of compound-sum method are relatively stable, though the magnitude is larger, during the period between 1980 and 2012, but getting worse after 2003. In contrast, continuous regression and discrete regression methods, except 2012, yield stable mean square errors for the all sample period. The results of time-series comparison present a serious challenge in recent years for those methods, except for perhaps the regression methods, in predicting a firm's dividend growth rate.

Table 7 presents forecast errors and mean square errors in estimating sales growth rates. Note that we use the sustainable and internal growth models to estimate sales growth rate and not dividend growth rate. As shown by Brick et al. (2014), the sustainable growth model is based upon the growth rate of sales. Internal growth rate model will yield the identical rate as the Gordon's growth model if one assumes the dividend payout ratio is constant. However, the internal growth model can also be derived by the framework of Higgin's sustainable growth model if we assume there is no external fund can be generated by the company. In that case, the internal growth rate is the sales growth rate. The

**Table 5** Estimated sales growth rates for U.S. companies

	All firms (5,818 firms)	Firms with positive sales growth (5,390 firms)	Firms with negative sales growth (428 firms)
<i>Panel A. Period: 1981–2012</i>			
Arithmetic average	20.88 % (25.70)	22.17 % (26.18)	-5.79 % (-3.14)
Geometric average	5.33 % (25.71)	6.06 % (29.78)	-8.98 % (-11.89)
Compound-sum method	6.40 % (42.04)	6.96 % (47.53)	-5.31 % (-7.59)
Regression method (continuous)	6.73 % (41.16)	7.34 % (46.48)	-5.29 % (-7.44)
Regression method (discrete)	7.32 % (41.47)	7.94 % (45.91)	-4.87 % (-7.36)
Internal growth model	1.27 % (64.91)	2.19 % (75.31)	-0.03 % (-1.25)
Sustainable growth model	1.82 % (73.88)	3.17 % (84.96)	-0.10 % (-3.93)
<i>Panel B. Period: 1981–1995</i>			
Arithmetic average	27.45 % (19.06)	29.17 % (19.39)	-6.40 % (-2.09)
Geometric average	13.38 % (25.07)	14.90 % (27.73)	-16.65 % (-11.68)
Compound-sum method	8.06 % (32.98)	8.60 % (35.34)	-2.94 % (-2.37)
Regression method (continuous)	8.26 % (30.57)	8.82 % (33.03)	-2.75 % (-1.72)
Regression method (discrete)	9.64 % (29.17)	10.16 % (32.73)	-0.59 % (-0.20)
Internal growth model	1.55 % (54.85)	2.42 % (59.63)	0.16 % (4.87)
Sustainable growth model	2.00 % (56.96)	3.17 % (63.48)	0.10 % (2.41)
<i>Panel C. Period: 1996–2012</i>			
Arithmetic average	14.39 % (18.84)	15.60 % (19.27)	-5.15 % (-2.65)
Geometric average	5.14 % (23.74)	5.79 % (27.36)	-7.58 % (-6.57)
Compound-sum method	4.22 % (20.78)	4.75 % (23.82)	-6.97 % (-6.05)
Regression method (continuous)	4.30 % (17.41)	4.95 % (21.12)	-8.51 % (-4.56)
Regression method (discrete)	5.15 % (20.79)	5.74 % (23.41)	-6.62 % (-5.02)
Internal growth model	1.04 % (38.79)	1.99 % (48.11)	-0.20 % (-5.29)
Sustainable growth model	1.68 % (48.95)	3.18 % (58.08)	-0.24 % (-7.33)

This table presents the averages of estimated dividend growth rates for companies by arithmetic average method, geometric average method, compound-sum method, and the regression methods in terms of discrete and continuous compounding processes. Sample companies are listed in NYSE, AMEX, or NASDAQ, and have at least ten-year sales or earnings data during the period from 1981 to 2012. Sample companies are further divided into positive sales growth firms and negative sales growth firms in terms of their arithmetic average growth rates. Panel A shows estimated sales growth rates, and Panel B shows estimated earnings growth rates by various estimation methods. Values denoted in parentheses are *t*-statistics on the null hypothesis that the average number is different from zero

arithmetic average, the internal growth model and sustainable growth models have the smallest average prediction error during the entire sample period. The mean arithmetic average forecasting error is  $-0.001$  and the mean forecasting error of the internal growth model is  $-0.002$ . The average prediction error for the sustainable growth model is  $0.003$ . All of these means are not statistically different from zero. All of the other approaches yield average prediction errors that are statistically significant. In addition, negative forecast errors for compound-sum method indicate that this method consistently underestimates sales growth rates. In contrast, geometric average method and regression methods consistently overestimate sales growth rates over the testing period. Except for the

**Table 6** Ex-post forecast: Alternative statistical methods to estimate dividend growth rates

Year	Arithmetic Average		Geometric Average		Compound-sum		Continuous Regression		Discrete Regression		Gordon's Growth Model	
	Error	MSE	Error	MSE	Error	MSE	Error	MSE	Error	MSE	Error	MSE
1981	-0.069	0.122	-0.03	0.098	-0.358	0.712	0.051	0.235	-0.03	0.195	-0.050	0.074
1982	-0.078	0.078	-0.039	0.063	-0.395	0.668	-0.004	0.222	-0.208	0.284	0.062	0.092
1983	-0.145	0.114	-0.105	0.088	-0.508	0.872	0.047	0.556	-0.003	0.623	-0.008	0.447
1984	0.041	0.155	0.078	0.151	-0.227	0.904	0.102	0.297	0.050	0.355	-0.026	0.030
1985	-0.035	0.119	0.003	0.111	-0.233	0.728	0.159	0.323	0.103	0.348	0.008	2.408
1986	-0.036	0.149	0.003	0.138	-0.180	0.746	0.102	0.381	0.039	0.364	-0.003	0.060
1987	0.045	0.153	0.088	0.127	-0.015	0.726	0.215	0.264	0.163	0.275	-0.032	0.048
1988	0.366	0.338	0.412	0.313	0.435	0.874	0.586	0.629	0.527	0.643	0.012	0.078
1989	0.258	0.258	0.311	0.192	0.348	0.683	0.405	0.543	0.346	0.574	0.177	1.637
1990	0.039	0.265	0.098	0.225	0.189	0.688	0.231	0.515	0.155	0.525	-0.079	0.096
1991	-0.060	0.312	0.004	0.202	0.116	0.731	0.091	0.311	-0.034	0.327	0.043	0.877
1992	-0.090	0.171	-0.027	0.098	0.130	0.650	0.061	0.257	-0.017	0.243	-0.048	1.428
1993	0.042	0.178	0.117	0.123	0.294	0.755	0.225	0.474	0.150	0.528	-0.063	1.696
1994	0.053	0.202	0.120	0.106	0.281	0.745	0.197	0.383	0.383	0.492	-0.091	0.506
1995	-0.040	0.125	0.031	0.064	0.165	0.577	0.135	0.430	0.030	0.519	0.024	1.741
1996	0.042	0.236	0.109	0.161	0.244	0.797	0.151	0.348	0.062	0.459	-0.042	0.980
1997	-0.190	0.275	-0.013	0.159	0.061	0.745	0.078	0.255	0.010	0.345	-0.053	1.281
1998	-0.073	0.268	-0.003	0.154	0.107	0.737	0.195	0.323	0.049	0.420	0.007	1.177
1999	-0.103	0.258	0.070	0.160	0.234	0.811	0.065	0.281	-0.412	0.384	-0.038	0.612
2000	0.055	0.293	0.130	0.104	0.322	0.864	0.153	0.227	-0.220	0.276	-0.072	0.229
2001	-0.160	0.145	0.014	0.083	0.216	0.684	0.048	0.413	-0.376	0.371	-0.101	2.862
2002	-0.116	0.190	0.082	0.094	0.264	0.732	0.108	0.242	0.008	0.283	-0.031	0.269
2003	-0.089	0.331	0.100	0.280	0.268	1.085	0.140	0.449	0.064	0.455	-0.040	2.965
2004	0.096	0.470	0.182	0.364	0.301	1.169	0.233	0.608	0.150	0.596	-0.016	0.167



**Table 6** continued

Year	Arithmetic Average		Geometric Average		Compound-sum		Continuous Regression		Discrete Regression		Gordon's Growth Model	
	Error	MSE	Error	MSE	Error	MSE	Error	MSE	Error	MSE	Error	MSE
2005	0.130	0.377	0.216	0.255	0.267	1.068	0.292	0.494	0.140	0.489	-0.016	0.096
2006	0.009	0.402	0.116	0.162	0.114	0.905	0.174	0.293	0.016	0.280	-0.016	0.949
2007	0.191	0.480	0.289	0.39	0.268	1.197	0.316	0.484	0.203	0.493	0.015	0.371
2008	-0.021	0.494	0.09	0.333	-0.060	1.040	0.135	0.482	-0.367	0.463	-0.105	1.373
2009	-0.215	0.425	-0.104	0.225	-0.311	1.125	-0.08	0.274	-0.128	0.326	0.075	0.910
2010	-0.039	0.498	0.146	0.358	0.065	1.281	0.178	0.522	0.145	0.522	0.053	0.511
2011	0.021	0.415	0.132	0.258	0.034	1.480	0.176	0.404	0.114	0.389	-0.006	0.247
2012	-0.077	0.244	-0.028	0.194	-0.473	1.215	0.048	0.347	0.030	0.332	-0.009	2.486
Avg.	-0.008	0.267	0.081	0.182	0.061	0.875	0.157	0.383	0.036	0.412	-0.015	0.897
<i>t</i> -stats	-0.35	12.06	3.99	11.05	1.32	22.45	7.12	18.14	0.99	19.67	-1.48	2.45

This table presents averages of forecast error and mean square errors for the estimation of the dividend growth rate by various estimation methods. The forecast period is from 1981 to 2012. Dividend growth rate of each company is estimated in each year based on prior 10-year data by various estimation methods including arithmetic average method, geometric average method, compound-sum method, continuous regression method, discrete regression method, and Gordon's growth model. Forecast error is the difference between the real dividend growth rate and the estimated dividend growth rate. The cross-sectional averages of forecast errors and the mean square errors for various estimation methods are presented in each year. Time-series averages of averages of forecast errors, mean square errors, and associated *t*-statistics are also presented

**Table 7** Ex-post forecast: Alternative statistical methods to estimate sales growth rates

Year	Arithmetic average		Geometric average		Compound-sum		Continuous regression		Discrete regression		Internal growth model		Sustainable growth model	
	Error	MSE	Error	MSE	Error	MSE	Error	MSE	Error	MSE	Error	MSE	Error	MSE
1981	0.011	0.343	0.103	0.107	-0.131	1.534	0.396	0.597	0.329	0.654	0.051	0.135	0.036	0.304
1982	-0.100	0.349	-0.013	0.127	-0.303	1.544	0.156	0.341	0.12	0.355	0.019	0.075	0.025	0.344
1983	-0.046	0.277	0.035	0.090	-0.214	1.447	0.397	0.686	0.038	0.723	0.023	0.114	0.020	0.271
1984	0.077	0.254	0.150	0.109	-0.092	1.349	0.411	0.484	0.341	0.502	0.006	0.059	0.014	0.272
1985	0.002	0.356	0.089	0.164	-0.199	1.468	0.297	0.461	0.256	0.496	0.011	0.144	-0.010	0.251
1986	-0.008	0.438	0.079	0.237	-0.198	1.527	0.413	0.623	0.366	0.680	0.012	0.310	0.016	0.358
1987	0.014	0.313	0.092	0.151	-0.157	1.423	0.457	0.632	0.366	0.728	-0.002	0.126	0.014	0.262
1988	0.022	0.361	0.098	0.162	-0.203	1.540	0.312	0.461	0.262	0.481	-0.039	0.104	0.022	0.355
1989	-0.011	0.326	0.059	0.167	-0.272	1.544	0.224	0.400	0.175	0.409	0.018	0.188	0.041	0.352
1990	-0.043	0.375	0.017	0.179	-0.346	1.598	0.167	0.363	-0.144	0.361	0.003	0.095	0.031	0.214
1991	-0.082	0.320	-0.022	0.147	-0.437	1.505	0.091	0.269	0.041	0.349	-0.035	0.452	0.043	0.258
1992	0.016	0.246	0.070	0.110	-0.282	1.277	0.182	0.239	0.116	0.267	-0.032	0.159	0.015	0.390
1993	0.053	0.315	0.088	0.158	-0.304	1.378	0.213	0.316	0.171	0.374	-0.033	0.248	0.017	0.288
1994	0.061	0.358	0.124	0.184	-0.268	1.328	0.221	0.282	0.149	0.322	-0.033	0.120	0.013	0.296
1995	0.090	0.378	0.123	0.194	-0.263	1.408	0.239	0.315	0.139	0.347	-0.024	0.217	0.053	0.314
1996	0.040	0.500	0.121	0.298	-0.305	1.626	0.191	0.344	0.151	0.405	-0.024	0.147	0.047	0.235
1997	0.062	0.597	0.149	0.342	-0.291	1.682	0.245	0.494	0.131	0.524	-0.012	0.150	0.010	0.299
1998	0.021	0.572	0.118	0.269	-0.356	1.710	0.193	0.446	0.144	0.500	-0.028	0.281	-0.019	0.247
1999	-0.019	0.499	0.064	0.225	-0.402	1.698	0.133	0.354	-0.002	0.383	-0.065	0.112	-0.057	0.276
2000	0.122	0.568	0.198	0.347	-0.269	1.842	0.283	0.449	0.230	0.452	-0.020	0.124	-0.002	0.312
2001	-0.064	0.55	0.044	0.248	-0.495	1.979	0.132	0.331	0.092	0.366	0.024	0.175	-0.003	0.170
2002	-0.102	0.562	-0.033	0.237	-0.557	2.075	0.022	0.294	-0.008	0.299	0.018	0.206	-0.007	0.259
2003	-0.008	0.483	0.089	0.187	-0.400	1.787	0.118	0.215	0.072	0.220	-0.011	0.386	-0.050	0.285
2004	0.033	0.410	0.129	0.166	-0.363	1.674	0.189	0.218	0.165	0.229	0.038	0.184	-0.010	0.248

**Table 7** continued

Year	Arithmetic average		Geometric average		Compound-sum		Continuous regression		Discrete regression		Internal growth model		Sustainable growth model	
	Error	MSE	Error	MSE	Error	MSE	Error	MSE	Error	MSE	Error	MSE	Error	MSE
2005	0.016	0.490	0.128	0.235	-0.398	1.767	0.143	0.244	0.118	0.254	0.002	0.236	0.088	0.338
2006	0.034	0.434	0.114	0.190	-0.444	1.760	0.153	0.216	0.13	0.223	0.012	0.169	-0.012	0.432
2007	0.006	0.508	0.109	0.210	-0.461	1.826	0.144	0.228	0.119	0.222	-0.013	0.154	-0.048	0.616
2008	-0.002	0.500	0.066	0.172	-0.494	1.743	0.088	0.228	0.065	0.240	0.006	0.186	-0.032	0.384
2009	-0.185	0.585	-0.088	0.203	-0.633	1.990	-0.044	0.315	-0.069	0.341	0.003	0.226	-0.003	0.356
2010	0.027	0.466	0.084	0.151	-0.33	1.576	0.153	0.216	0.132	0.228	0.027	0.269	-0.079	0.372
2011	0.020	0.423	0.097	0.168	-0.306	1.422	0.164	0.212	0.129	0.211	0.03	0.237	-0.047	0.526
2012	-0.091	0.278	-0.007	0.094	-0.459	1.393	0.083	0.154	0.062	0.177	0.014	0.268	-0.018	0.859
Avg.	-0.001	0.42	0.077	0.188	-0.332	1.607	0.205	0.357	0.137	0.385	-0.002	0.189	0.003	0.336
<i>t</i> -stats	-0.09	18.18	6.77	13.72	-12.6	36.03	9.95	12.30	7.13	12.08	-0.31	12.48	0.45	12.95

This table presents averages of forecast error and mean square errors for the estimation of sales growth rate by various estimation methods. The forecast period is from 1981 to 2012. Sales growth rate of each company is estimated in each year based on prior 10-year data by various estimation methods including arithmetic average method, geometric average method, compound-sum method, continuous regression method, discrete regression method, internal growth model and sustainable growth model. Forecast error is the difference between the real sales growth rate and the estimated sales growth rate. The cross-sectional averages of forecast errors and the mean square errors for various estimation methods are presented in each year. Time-series averages of averages of forecast errors, mean square errors, and associated *t*-statistics are also presented

**Table 8** Determinants of forecast error for dividend growth estimation

	Arithmetic Average	Geometric Average	Compound-sum	Continuous Regression	Discrete Regression	Gordon's model Growth Model
<i>Panel A. Fixed effect regressions with 2-digit SIC industry dummies and year dummies</i>						
Intercept	-0.004 (-0.01)	0.093 (0.28)	-0.564 (-1.34)	0.109 (0.34)	-0.134 (-0.28)	-0.024 (-0.23)
Size	-0.018 <sup>a</sup> (-7.33)	-0.022 <sup>a</sup> (-9.44)	-0.050 <sup>a</sup> (-16.98)	-0.021 <sup>a</sup> (-9.29)	0.009 <sup>a</sup> (2.69)	0.005 (1.16)
Turnover	0.022 <sup>b</sup> (2.30)	-0.017 <sup>c</sup> (-1.87)	0.168 <sup>a</sup> (13.53)	0.005 (0.57)	0.066 <sup>a</sup> (4.92)	-0.008 (-0.47)
BM	0.048 <sup>a</sup> (4.64)	0.092 <sup>a</sup> (9.20)	0.025 <sup>b</sup> (2.20)	0.064 <sup>a</sup> (6.70)	-0.007 (-0.45)	-0.004 (-0.22)
D/E	-0.001 <sup>a</sup> (-3.11)	-0.001 <sup>a</sup> (-3.38)	<0.001 (-1.36)	-0.001 <sup>a</sup> (-4.15)	<0.001 (0.45)	<0.001 (0.12)
Beta	0.0207 <sup>b</sup> (2.09)	0.0393 <sup>a</sup> (4.09)	-0.0514 <sup>a</sup> (-4.32)	0.0224 <sup>a</sup> (2.47)	-0.0551 <sup>a</sup> (-4.04)	0.032 (1.35)
Industry Dummy	Yes	Yes	Yes	Yes	Yes	Yes
Year Dummy	Yes	Yes	Yes	Yes	Yes	Yes
Adj. R <sup>2</sup>	0.0209	0.0177	0.0377	0.0189	0.0176	0.002

**Table 8** continued

	Arithmetic Average	Geometric Average	Compound-sum	Continuous Regression	Discrete Regression	Gordon's model Growth Model
<i>Panel B. Fixed effect regressions with Fama-French industry dummies and year dummies</i>						
Intercept	-0.275 (-0.97)	-0.469 (-1.72)	0.025 (0.08)	0.197 (0.6)	-3.139 (-0.67)	-0.034 (-0.28)
Size	-0.014 <sup>b</sup> (-2.01)	-0.009 (-1.34)	-0.019 <sup>a</sup> (-2.63)	-0.027 <sup>a</sup> (-3.97)	0.008 (0.08)	0.005 (1.05)
Turnover	0.032 (1.14)	0.034 (1.26)	0.029 (0.93)	0.063 (1.36)	0.625 (1.47)	0.011 (0.66)
BM	0.018 (0.60)	-0.0010 (-0.35)	0.037 (1.27)	0.008 (0.27)	-0.467 (-1.03)	-0.024 (-1.31)
D/E	-0.001 <sup>c</sup> (-1.74)	-0.001 <sup>c</sup> (-1.80)	-0.001 (-1.03)	-0.001 (-1.67)	-0.001 (-0.10)	0.000 (0.14)
Beta	0.094 <sup>b</sup> (2.46)	0.0689 <sup>c</sup> (1.82)	0.0448 (1.10)	0.0591 <sup>c</sup> (1.64)	0.0938 (0.15)	0.032 (1.27)
F-F Industry Dummy	Yes	Yes	Yes	Yes	Yes	Yes
Year Dummy	Yes	Yes	Yes	Yes	Yes	Yes
Adj. R <sup>2</sup>	0.006	0.007	0.015	0.008	0.002	0.003

This table presents results of the regression in determining the dividend growth forecast error. The dependent variable is the measure of forecast error, and the independent variables are firm characteristics including firm size, turnover rate, book-to-market ratio, debt-to-equity ratio, and firm's beta. Dividend growth rate of each company are estimated in each year based on prior 10 years data by various estimation methods including arithmetic average, geometric average, compound-sum, continuous regression, and discrete regression. Forecast error is the difference between the real dividend growth rate and the estimated dividend growth rate. Panel A presents results of fixed-effect regressions with year dummies and 2-digit SIC industries dummies. Panel B presents results of fixed-effect regressions with year dummies and Fama-French industries dummies. The adjusted R<sup>2</sup>s are presented, and the associated t-statistics are in parentheses. <sup>a</sup> Significant at the 1 % level, <sup>b</sup> significant at the 5 % level, and <sup>c</sup> significant at the 10 % level

**Table 9** Determinants of forecast error for sales growth estimation

	Arithmetic average	Geometric average	Compound-sum	Continuous regression	Discrete regression	Internal growth model	Sustainable growth model
<i>Panel A. Fixed effect regressions with 2-digit SIC industry dummies and year dummies</i>							
Intercept	-0.359 (-0.48)	-0.194 (-0.29)	-0.472 (-0.53)	0.984 (0.78)	0.714 (0.34)	-0.020 (-0.02)	-0.072 (-0.10)
Size	-0.001 (-0.33)	-0.019 <sup>a</sup> (-6.80)	-0.084 <sup>a</sup> (-23.03)	-0.025 <sup>a</sup> (-6.13)	-0.015 <sup>b</sup> (-2.22)	0.005 (1.41)	0.007 <sup>b</sup> (2.20)
Turnover	-0.007 (-0.64)	0.002 (0.21)	-0.02 (-1.38)	0.040 <sup>a</sup> (2.64)	0.042 <sup>c</sup> (1.64)	0.001 (0.65)	0.005 (0.42)
BM	0.031 <sup>b</sup> (2.54)	0.030 <sup>a</sup> (2.74)	0.031 <sup>b</sup> (2.08)	0.018 (1.08)	0.019 (0.71)	-0.015 (-0.97)	-0.009 (-0.74)
D/E	<0.001 (0.06)	<0.001 (0.18)	>-0.001 (-0.12)	<0.001 (1.27)	<0.001 (0.80)	>-0.001 (-0.11)	<0.001 (0.63)
Beta	-0.005 (0.38)	0.034 <sup>a</sup> (2.82)	0.084 <sup>a</sup> (5.18)	0.116 <sup>a</sup> (6.87)	0.060 <sup>b</sup> (2.13)	0.0007 (0.36)	-0.011 (-0.79)
Industry dummy	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year dummy	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adj. R <sup>2</sup>	0.002	0.004	0.019	0.004	0.002	0.004	0.003

**Table 9** continued

	Arithmetic average	Geometric average	Compound-sum	Continuous regression	Discrete regression	Internal growth model	Sustainable growth model					
<i>Panel B. Fixed effect regressions with Fama-French industry dummies and year dummies</i>												
Intercept	-0.3291 (-0.44)	-0.3078 (-0.20)	-0.1287 (-0.20)	-0.3251 (-0.37)	-0.4016 (-0.45)	-0.0481 (-0.04)	-0.1175 (-0.09)	-0.0704 (-0.03)	-0.0285 (-0.03)	-0.0334 (-0.04)	-0.0261 (-0.04)	-0.0060 (-0.01)
Size	>-0.0000 (0.01)	-0.0002 (-0.06)	-0.0166 <sup>a</sup> (-6.06)	-0.0803 <sup>a</sup> (-22.17)	-0.0806 <sup>a</sup> (-21.98)	-0.0214 <sup>a</sup> (-5.19)	-0.0205 <sup>a</sup> (-4.92)	-0.0167 <sup>b</sup> (-2.40)	0.0052 (1.38)	0.0052 (1.35)	0.0048 (1.62)	0.0054 <sup>c</sup> (1.83)
Turnover	-0.0083 (-0.74)	-0.0070 (-0.62)	0.0008 (0.08)	0.0010 (0.10)	-0.0212 (-1.57)	0.0381 <sup>b</sup> (2.56)	0.0339 <sup>b</sup> (2.26)	0.0416 <sup>c</sup> (1.69)	0.0079 (0.56)	0.0080 (0.55)	0.0030 (0.25)	0.0039 (0.30)
BM	0.0304 <sup>b</sup> (2.52)	0.0309 <sup>b</sup> (2.51)	0.0319 <sup>a</sup> (2.96)	0.0294 <sup>a</sup> (2.66)	0.0328 <sup>b</sup> (2.21)	0.0113 (0.70)	0.0020 (0.12)	0.0175 (0.65)	0.0145 (0.53)	-0.0147 (-0.92)	-0.0063 (-0.56)	-0.0049 (-0.44)
D/E	<0.0001 (0.16)	<0.0001 (0.15)	<0.0001 (0.23)	<0.0001 (0.21)	>-0.0001 (-0.09)	<0.0001 (1.31)	<0.0001 (1.32)	<0.0001 (0.83)	<0.0001 (0.83)	>-0.0001 (-0.12)	>-0.0001 (0.56)	0.0001 (0.54)
Beta	-0.0061 (-0.45)	-0.0061 (-0.45)	0.0244 <sup>b</sup> (1.99)	0.0244 <sup>b</sup> (1.99)	0.0664 <sup>a</sup> (4.03)	0.0664 <sup>a</sup> (4.03)	0.0880 <sup>a</sup> (5.12)	0.0272 (0.95)	0.0272 (0.95)	0.0007 (0.04)	0.0007 (0.04)	-0.0176 (-1.26)
F-F industry dummy	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year dummy	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adj. R <sup>2</sup>	0.003	0.003	0.006	0.006	0.021	0.006	0.006	0.002	0.002	0.004	0.003	0.003

This table presents results of the regression in determining the sales growth forecast error. The dependent variable is the measure of forecast error, and the independent variables are firm characteristics including firm size, turnover rate, book-to-market ratio, debt-to-equity ratio, and firm's beta. Dividend growth rate of each company are estimated in each year based on prior 10 years data by various estimation methods including arithmetic average, geometric average, compound-sum, continuous regression, and discrete regression. Forecast error is the difference between the real dividend growth rate and the estimated dividend growth rate. Panel A presents results of fixed-effect regressions with year dummies and 2-digit SIC industries dummies. Panel B presents results of fixed-effect regressions with year dummies and Fama-French industries dummies. The adjusted R<sup>2</sup>s are presented, and the associated t-statistics are in parentheses. <sup>a</sup> Significant at the 1 % level, <sup>b</sup> significant at the 5 % level, and <sup>c</sup> significant at the 10 % level

internal growth model and sustainable growth model, mean square errors of all models are stable during the period between 1980 and 2012.

Results of ex-post forecast show that the forecast errors of dividend and sales growth rates exist in various estimation models. We further investigate the determinants of the forecast errors and examine whether forecast errors of different estimation models are related to different firm characteristics. Table 8 presents results of fixed-effect regressions in which the dependent variable is the dividend growth forecast error for each firm-year, and the independent variables are firm characteristics including firm size, turnover rate, book-to-market ratio, debt-to-equity ratio, and firm's beta. Each of the regressions employed include year dummies and industry dummies. Panel A reports the regression results using industry dummies defined by 2-digit SIC code. Panel B reports the regression result using Fama–French industry dummies.

To conserve space, we primarily summarize the results for the arithmetic average method because that method yields the overall best forecast. As can be seen from Panel A of Table 8, the dividend growth forecast error is negatively associated to size and leverage ratio, positively associated to turnover, book-to-market ratio, and beta. In other words, an upward forecasted bias is likely when we use the arithmetic average model to forecast dividend growth rate for small, low leveraged, relatively undervalued firms, value with higher turnover and higher systematic risk. In comparison to other forecasting models, the forecast error for geometric method is more sensitive to book-to-market ratio, and compound-sum method is highly affected by firm size and stock turnover.

Table 9 presents results of the regression in determining the sales growth forecast error. Sales growth forecast error is negatively associated to size and positively associated to book-to-market ratio, and the systematic risk. In summary, earnings and sales growth rate estimations can perform well on large companies with less systematic risk.

## 5 Conclusion

In this paper, we explain that the traditional dividend growth model makes strong assumptions regarding the financing mix of the firm. In addition, we discuss several methods suggested in the literature on estimating growth rates and analyze whether these approaches are consistent with the use of using a constant discount rate to evaluate the firm's assets and equity. In particular, we show that the underlying assumptions of the internal growth model (whereby no external funds are used to finance growth) are incompatible with the constant discount rate model of valuation. We also introduce various statistical estimation methods suggested in the literature, including arithmetic average method, compound-sum method, and/or regression methods. We also discussed the inferred method suggested by Gordon and Gordon (1997) to estimate the growth rate. To compare various estimation methods, we empirically obtain historical dividend growth rates of all dividend paying companies in U.S. using the various estimation methods suggested in the literature. We find that the arithmetic average method is sensitive to extreme values and has an upward bias, resulting in a larger estimated dividend growth rates in comparison to all of the other methods. To determine the efficacy of these methods, we conduct an ex-post forecast and find that, in terms of forecast error, arithmetic average method is superior to the other methods and the compound-sum and the continuous regression methods yield the worst estimations in predicting firm's dividend and sales growth rates. The forecast errors are positively related to book-to-market ratio and firm's systematic risk and negatively related to firm size.



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