**Chapter 46**

**Alternative Methods for Estimating Firm’s Growth Rate\***

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**Alternative Methods for Estimating Firm’s Growth Rate**

Abstract

The most common valuation model is the dividend growth model. The growth rate is found by taking the product of the retention rate and the return on equity. What is less well understood are the basic assumptions of this model. In this paper, we demonstrate that the model makes strong assumptions regarding the financing mix of the firm. In addition, we discuss several methods suggested in the literature on estimating growth rates and analyze whether these approaches are consistent with the use of using a constant discount rate to evaluate the firm’s assets and equity. The literature has also suggested estimating growth rate by using the average percentage change method, compound-sum method, and/or regression methods. We demonstrate that the average percentage change is very sensitive to extreme observations. Moreover, on average, the regression method yields similar but somewhat smaller estimates of the growth rate compared to the compound-sum method. We also discussed the inferred method suggested by Gordon and Gordon (1997) to estimate the growth rate. Advantages, disadvantages, and the interrelationship among these estimation methods are also discussed in detail.

*Keywords*: Compound sum method, Discount cash flow model, Growth rate, Internal growth rate, Sustainable growth rate

*JEL Classification*: G31, G35

* 1. **Introduction**

One of the more highly used valuation models is that developed by Gordon and Shapiro (1956) and Gordon (1962) known as the dividend growth model. In security analysis and portfolio management, growth rate estimates of earnings, dividends, and price per share are important factors in determining the value of an investment or a firm. These publications demonstrate that the growth rate is found by taking the product of the retention rate and the return on equity. What less well understood are the basic assumptions of this model. In this paper, we demonstrate that the model makes strong assumptions regarding the financing mix of the firm.

In addition, we will also discuss several methods suggested in the literature on estimating growth rates. We will analyze whether these approaches are consistent with the use of using a constant discount rate to evaluate the firm’s assets and equity. In particular, we will demonstrate that the underlying assumptions of the internal growth rate model (whereby no external funds are used to finance growth) is incompatible with the constant discount rate model of valuation.

The literature has also suggested estimating growth rate by taking the average of percentage change of dividends over a sample period, taking the geometric average of the change in dividends or using regression analysis to estimate the growth rate (e.g. Le et al., 2009; Lee et al., 2012; Lee et al., 2000; and Ross et al., 2010). Gordon and Gordon (1997) suggest first using the Capital Asset Pricing Model (*CAPM*) to determine the cost of equity of the firm and then using the dividend growth model to infer the growth rate. Advantages, disadvantages, and the interrelationship among these estimation methods are also discussed in detail.

This paper is organized as follows. In section 2 we present the Gordon and Shapiro model (1956). We discuss the inherent assumptions of the model and its implied method to estimate the growth rate. Section 3 analyzes the internal growth rate and sustainable growth rate models. Section 4 describes leading statistical methods for estimating firm’s growth rates. We will also present the inferred method suggested by Gordon and Gordon (1997) to estimate the growth rate. Concluding remarks appear in Section 5.

* 1. **The Discounted Cash Flow Model and the Gordon Growth Model**

The traditional academic approach to evaluate a firm’s equity is based upon the constant discount rate method. One approach uses the after-tax weighted average cost of capital as a discount rate. This model is expressed as:

 , (46.1)

where  is the expected unlevered cash flow of the firm at time *t* and is the market value of debt outstanding.  equals  where  is the market value proportion of debt,  is the corporate tax rate,  is the cost of debt and  is the cost of equity. The first term on the right hand side of Eq. (46.1) is the value of the assets. Subtracting out the value of debt yields the value of equity. The price per share is therefore the value of equity divided by the number of shares outstanding. Alternatively, the value of equity can be directly found by discounting the dividends per share by the cost of equity, or more formally:

 , (46.2)

where  is the dividend per share at time *t*. Boudreaux and Long (1979), and Chambers et al. (1982) demonstrate the equivalence of these two approaches assuming that the level of that the level of debt is a constant percentage of the value of the firm.[[1]](#footnote-1) Accordingly:

  (46.3)

If we assume that dividends per share grow at a constant rate *g*, then Eq. (46.2) is reduced to the basic dividend growth model:[[2]](#footnote-2)

 . (46.4)

Gordon and Shapiro (1956) demonstrates that if *b* is the fraction of earnings retained within the firm, and *r* is the rate of return the firm will earn on all new investments, then . Let denote the level of new investment at time *t*. Because growth in earnings arises from the return on new investments, earnings can be written as:

 , (46.5)

where  is the earnings in period *t*.[[3]](#footnote-3) If the firm’s retention rate is constant and used in new investment, then the earnings at time *t* is

 . (46.6)

Growth rate in earnings is the percentage change in earnings and can be expressed as

 . (46.7)

If a constant proportion of earnings is assumed to be paid out each year, the growth in earnings equals the growth in dividends, implying . It is worthwhile to examine the implication of this model for the growth in stock prices over time. The growth in stock price is

 . (46.8)

Recognizing that and can be defined by Eq. (46.4), while noting that  is equal to then:

 . (46.9)

Thus, under the assumption of a constant retention rate, for a one-period model, dividends, earnings, and prices are all expected to grow at the same rate.

 The relationship between the growth rate, *g*, the retention rate, *b*, and the return on equity, *r*, can be expanded to a multi-period setting as the following numerical examples illustrates. In the first example, we assume that the book value of the firm’s assets equal the market value of the firm. We will assume that the growth rate of the firm sales and assets is 4% and the tax rate is equal to 40%. The book value of the assets at time 0 is $50 and we assume a depreciation rate of 10% per annum. The amount of debt outstanding is $12.50 and amount of equity outstanding is $37.50. We assume that the cost of debt, , is 12% and the cost of equity, *r*, is 25%, implying an *ATWACOC* of 20.55%. The expected dividend at *t* = 1, *d1*, must satisfy Equation (46.4). That is, .

**Table 46.1 The Book Value of the Firm’s Assets Equal the Market Value of the Firm (Growth Rate is 4%)**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 | 2 | 3 | 4 | 5 |
| Assets | $50.00 | $52.00 | $54.08 | $56.24 | $58.49 | $60.83 |
| Debt | $12.50 | $13.00 | $13.52 | $14.06 | $14.62 | $15.21 |
| Equity | $37.50 | $39.00 | $40.56 | $42.18 | $43.87 | $45.62 |
| Rd | 0.12 | 0.12 | 0.12 | 0.12 | 0.12 | 0.12 |
| r | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 |
| ATWACOC | 0.2055 | 0.2055 | 0.2055 | 0.2055 | 0.2055 | 0.2055 |
| Asset Turnover |  | 1.7 | 1.7 | 1.7 | 1.7 | 1.7 |
| GPM |  | 0.26029 | 0.26029 | 0.26029 | 0.26029 | 0.26029 |
| Sales |  | $85.00 | $88.40 | $91.94 | $95.61 | $99.44 |
| Cost  |  | $62.88 | $65.39 | $68.01 | $70.73 | $73.55 |
| Depreciation |  | $5.00 | $5.20 | $5.41 | $5.62 | $5.85 |
| Interest Exp. |  | $1.50 | $1.56 | $1.62 | $1.69 | $1.75 |
| EBT |  | $15.63 | $16.25 | $16.90 | $17.58 | $18.28 |
| Tax |  | $6.25 | $6.50 | $6.76 | $7.03 | $7.31 |
| NI |  | $9.38 | $9.75 | $10.14 | $10.55 | $10.97 |
| DIV |  | $7.88 | $8.19 | $8.52 | $8.86 | $9.21 |
| New Debt |  | $.50 | $0.52 | $0.54 | $0.56 | $0.59 |
| CFu |  | $8.28 | $8.61 | $8.95 | $9.31 | $9.68 |
| FirmValue | $50.00 | $52.00 | $54.08 | $56.24 | $58.49 | $60.83 |
| Investment |  | $7.00 | $7.28 | $7.57 | $7.87 | $8.19 |
| Vequity | $37.50 | $39.00 | $40.56 | $42.18 | $43.87 | $45.62 |
| RE |  | $1.50 | $1.56 | $1.62 | $1.69 | $1.75 |
| ROE |  | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 |
| 1-b |  | 0.84 | 0.84 | 0.84 | 0.84 | 0.84 |
| g  |  | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 |

The unlevered cash flow is defined as Sales less Costs (excluding the depreciation expense) less Investment less the tax paid. Tax paid is defined as the tax rate (which we assume to be 40%) times Sales minus Costs minus the Depreciation Expense. Recognizing that the value of the firm is given by , if firm value is $50, *g* = 4% and *ATWACOC* is 20.55%, then the expected unlevered cash flow is at time 1 is $8.28. We assume that the asset turnover ratio is 1.7. Hence, if assets at time 0 is $50, the expected sales at time 1 is $85. To obtain the level of investment, note that the depreciation expense at time 1 is $5. If the book value of assets equals $52, then the firm must invest $7. To obtain an expected unlevered cash flow at *t* = 1 of $8.28, the Gross Profit Margin is assumed to be approximately 26.03%, resulting in expected costs at time 1 of $62.88. The interest expense at time 1, is the cost of debt times the amount of debt outstanding at time zero, or $1.50. The Earnings Before Taxes (*EBT*) is defined as Sales – Costs – Interest Expense – Depreciation Expense, which equals $15.63 at time 1. 40% of *EBT* is the taxes paid or $6.25 resulting in a net income (*NI*) of $9.38. *ROE*, which equals Net Income/Book Value of Equity at the beginning of the period is 25%. Since the aggregate level of dividends at time 1 is $7.88, then the dividend payout ratio () is 84%. Note that *b* is therefore equal to 16% and .[[4]](#footnote-4)

Further note that the firm will increase its book value of equity via retention of *NI* by $1.50 (*RE* in the table). In order to maintain a leverage ratio of 25%, the firm must increase the level of debt from time 0 to time 1 by $0.50. The entries for time periods 2 – 5 follow the logical extension of the above discussion, and as shown in the table, the retention rate *b* is 16% and *ROE* = 25% for each period. Again the product of *b* and *ROE* results in the expected growth rate of 4%. Further note, that  imply that sales, costs, book value of asset, depreciation, unlevered cash flow, cash flow to stockholders, value of debt and value of equity to increase by 4% per annum.

Investors may use a one-period model in selecting stocks, but future profitability of investment opportunities plays an important role in determining the value of the firm and its *EPS* and dividend per share. The rate of return on new investments can be expressed as a fraction, *c* (perhaps larger than 1), of the rate of return security holders require (*r*):

 . (46.10)

Substituting this into the well-known relationship that  and rearranging, we have

 . (46.11)

If a firm has no extraordinary investment opportunities (), then  and the rate of return that security holders require is simply the inverse of the stock’s price to earnings ratio. In our example of Table 46.1, *NI* at time 1 is $9.38 and the value of equity at time 0 is $37.50. The ratio of these two numbers (which is equivalent to *EPS/P*) is *ROE* or 25%.

On the other hand, if the firm has investment opportunities that are expected to offer a return above that required by the firm’s stockholders (), the earnings to price ratio at which the firm sells will be below the rate of return required by investors. To illustrate consider the following example whereby market value of the firm and equity is greater than its book value. This example is depicted in Table 46.2. The basic assumptions of the model is as follows: We will

**Table 46.2 The Market Value of the Firm and Equity is Greater than its Book Value**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 | 2 | 3 | 4 | 5 |
| Assets | $50.00 | $52.00 | $54.08 | $56.24 | $58.49 | $60.83 |
| Firm Value | $60.00 | $62.40 | $64.90 | $67.49 | $70.19 | $73.00 |
| Debt | $12.50 | $13.00 | $13.52 | $14.06 | $14.62 | $15.21 |
| Equity | $47.50 | $49.40 | $51.38 | $53.43 | $55.57 | $57.79 |
| Rd | 0.12 | 0.12 | 0.12 | 0.12 | 0.12 | 0.12 |
| r | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 |
| ATWACOC | 0.2129 | 0.2129 | 0.2129 | 0.2129 | 0.2129 | 0.2129 |
| Asset Turnover |  | 1.7 | 1.7 | 1.7 | 1.7 | 1.7 |
| GPM |  | 0.3093 | 0.3093 | 0.3093 | 0.3093 | 0.3093 |
| Sales |  | $85.00 | $88.40 | $91.94 | $95.61 | $99.44 |
| Cost  |  | $58.71 | $61.06 | $63.50 | $66.04 | $68.68 |
| Depreciation |  | $5.00 | $5.20 | $5.41 | $5.62 | $5.85 |
| Interest Exp. |  | $1.50 | $1.56 | $1.62 | $1.69 | $1.75 |
| EBT |  | $19.79 | $20.58 | $21.41 | $22.26 | $23.15 |
| Tax |  | $7.92 | $8.23 | $8.56 | $8.91 | $9.26 |
| NI |  | $11.88 | $12.35 | $12.84 | $13.36 | $13.89 |
| DIV |  | $9.98 | $10.37 | $10.79 | $11.22 | $11.67 |
| New Debt |  | $.50 | $0.52 | $0.54 | $0.56 | $0.59 |
| CFu |  | $10.38 | $10.79 | $11.22 | $11.67 | $12.14 |
| FirmValue | $60.00 | $62.40 | $64.90 | $67.49 | $70.19 | $73.00 |
| Investment |  | $7.40 | $7.70 | $8.00 | $8.32 | $8.66 |
| Vequity | $47.50 | $49.40 | $51.38 | $53.43 | $55.57 | $57.79 |
| RE |  | $1.90 | $1.98 | $2.06 | $2.14 | $2.22 |
| Market Based ROE |  | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 |
| 1-b |  | 0.84 | 0.84 | 0.84 | 0.84 | 0.84 |
| g  |  | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 |

assume that the growth rate of the firm sales and book value of the assets is 4%. The book value of the assets at time 0 is again $50 and we assume a depreciation rate of 10% per annum. However, note that the market value of the firm is $60. The entries for Debt and Equity represent market values. The amount of debt outstanding is $12.50 and amount of equity outstanding is now $47.50. We assume that the cost of debt, , is 12% and the cost of equity, *r*, is 25%, implying an *ATWACOC* of 21.29%. For the valuation of the firm to be internally consistent, the unlevered cash flow at time 1 is $10.38. Similarly, the value of equity to be internally consistent, the expected dividends at t = 1 is $9.98. Note that net income is $11.88 implying a dividend payout ratio of 84% and a retention rate of 16%. The *book value based* *ROE*, *k*, is found by taking the net income divided by the book value of equity. In our example, implied book value of equity is $37.50. Hence, *k* = 31.68%, implying that the book value ROE is greater than the cost of equity which is the required rate of return. But *g* is given by the *market value based* *ROE* which is defined as Net Income over market value of equity. That is *r* = 25%. Note again,  is 4%.

An investor could predict next year’s dividends, the firm’s long-term growth rate, and the rate of return stockholders require (perhaps using the *CAPM* to estimate *r*) for holding the stock. Eq. (46.4) could then be solved for the theoretical price of the stock that could be compared with its present price. Stocks that have theoretical prices above actual price are candidates for purchase; those with theoretical prices below their actual price are candidates for sale or for short sale.

* 1. **Internal Growth Rate and Sustainable Growth Rate Models**

The internal growth rate model assumes that the firm can only finance its growth by its internal funds. Consequently, the cash to finance growth must come from only retained earnings. Therefore, retained earnings can be expressed as

 , (46.12)

where

the profit margin on all sales;

annual sales; and

the increase in sales during the year.

Because retained earnings is the only source of new funds, the use of cash represented by the increase in assets must equal the retained earnings:

 

 

 ,

 ,

 , (46.13)

where the ratio of total assets to sales. If we divide both numerator and denominator of Eq. (46.13) by *T* and make rearrange the terms, then we can show that the internal growth rate is: , (46.14)

where *ROA* is the return on assets. The internal growth rate is the maximum growth rate that can be achieved without debt or equity kind of external financing. But note this assumption of not issuing new debt or common stock to finance growth is inconsistent with the basic assumption of the constant discount rate models that the firm maintains a constant market based leverage ratio. Hence, this model cannot be used to estimate the growth rate and be employed by the Gordon Growth Model.

 Higgins (1977, 1981, and 2008) has developed a sustainable growth rate under assumption that firms can generate new funds by using retained earnings or issuing debt, but not issuing new shares of common stock. Growth and its management present special problems in financial planning. From a financial perspective, growth is not always a blessing. Rapid growth can put considerable strain on a company’s resources, and unless management is aware of this effect and takes active steps to control it, rapid growth can lead to bankruptcy. Assuming a company is not raising new equity, the cash to finance growth must come from retained earnings and new borrowings. Further, because the company wants to maintain a target debt-to-equity ratio equal to *L*,each dollar added to the owners’ equity enables it to increase its indebtedness by *$L*. Since the owners’ equity will rise by an amount equal to retained earnings, the new borrowing can be written as:

 **.

The use of cash represented by the increase in assets must equal the two sources of cash (retained earnings and new borrowings)[[5]](#footnote-5):

 

 

 

 

 . (46.15)

In Eq. (46.15) the  or *g* is the firm’s sustainable growth rate assuming no infusion ofnew equity. Therefore, a company’s growth rate in sales must equal the indicated combination of four ratios, *p,* b, *L,* and *T.* In addition, if the company’s growth rate differs from *g*,one or more of the ratios must change. For example, suppose a company grows at a rate in excess of *g*, then it must either use its assets more efficiently, or it must alter its financial policies. Efficiency is represented by the profit margin and asset-to-sales ratio. It therefore would need toincrease its profit margin (*p*)or decrease its asset-to-sales ratio (*T*) in order toincrease efficiency. Financial policies are represented by payout or leverage ratios. In this case, a decrease in its payout ratio (1-*b*)or an increase in its leverage (*L*) wouldbe necessary to alter its financial policies toaccommodate a different growth rate. It should be noted that increasing efficiency is not always possible and altering financial policies are not always wise.

If we divide both numerator and denominator of Eq. (46.15) by *T* and rearrange the terms, then we can show that the sustainable growth rate can be shown as

 . (46.16)

Please note that, in the framework of internal growth rate and sustainable growth rate presented above, the source of cash are taken from the end of period values of assets and assumed that the required financing occurs at the end of the period. However, Ross et al. (2010) show that if the source of cash is from the beginning of the period, the relationship between the use and the source of cash can be expressed for the internal growth rate model as  and for the sustainable growth rate model, . Such relationship will result an internal growth rate of  and a sustainable growth rate of . For example, Table 46.3 assumes identical assumptions to that of Table 46.1, but now we will assume a growth rate of 4.1667% and use total asset, total equity, and total debt from the beginning of the period balance sheet to calculate the net income. Recall that *ROE* is the net income divided by stockholders’ equity at the beginning of the period. Note that the product of ROE and b will yield 4.1667%.

**Table 46.3 The Book Value of the Firm’s Assets Equal the Market Value of the Firm (Sustainable Growth Rate is 4.1667%)**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 | 2 | 3 | 4 | 5 |
| Assets | $50.00 | $52.08 | $54.25 | $56.51 | $58.87 | $61.32 |
| Value | $50.00 | $52.08 | $54.25 | $56.51 | $58.87 | $61.32 |
| Debt | $12.50 | $13.02 | $13.56 | $14.13 | $14.72 | $15.33 |
| Equity | $37.50 | $39.06 | $40.69 | $42.39 | $44.15 | $45.99 |
| R | 0.12 | 0.12 | 0.12 | 0.12 | 0.12 | 0.12 |
| Re | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 |
| ATWACOC | 0.2055 | 0.2055 | 0.2055 | 0.2055 | 0.2055 | 0.2055 |
| Asset Turnover |  | 1.7 | 1.7 | 1.7 | 1.7 | 1.7 |
| GPM |  | 0.26029 | 0.26029 | 0.26029 | 0.26029 | 0.26029 |
| Sales |  | $85.00 | $88.54 | $92.23 | $96.07 | $100.08 |
| Cost  |  | $62.88 | $65.49 | $68.22 | $71.07 | $74.03 |
| Depreciation |  | $5.00 | $5.21 | $5.43 | $5.65 | $5.89 |
| Interest Exp |  | $1.50 | $1.56 | $1.63 | $1.70 | $1.77 |
| EBT |  | $15.63 | $16.28 | $16.95 | $17.66 | $18.40 |
| Tax |  | $6.25 | $6.51 | $6.78 | $7.06 | $7.36 |
| NI |  | $9.38 | $9.77 | $10.17 | $10.60 | $11.04 |
| DIV |  | $7.81 | $8.14 | $8.48 | $8.83 | $9.20 |
| new debt |  | $7.60 | $7.92 | $8.25 | $8.59 | $8.95 |
| CFu |  | $8.19 | $8.53 | $8.89 | $9.26 | $9.64 |
| Value | $50.00 | $52.08 | $54.25 | $56.51 | $58.87 | $61.32 |
| Investment |  | $7.08 | $7.38 | $7.69 | $8.01 | $8.34 |
| RE |  | $1.56 | $1.63 | $1.70 | $1.77 | $1.84 |
| ROE |  | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 |
| (1-b) |  | 0.833333 | 0.833333 | 0.833333 | 0.833333 | 0.833333 |
| g  |  | 0.041667 | 0.041667 | 0.041667 | 0.041667 | 0.041667 |

Note that the intent of the Higgins’ sustainable growth rate allows only internal source and external debt financing. Chen et al. (2013) incorporate Higgins (1977) and Lee et al. (2011) frameworks, allowing company use both external debt and equity, and derive a generalized sustainable growth rate as

 , (46.17)

where

;

;

; and

.

Comparing Eq. (46.17), the generalized sustainable growth rate has an additional positive term, , when the new equity issue is taken into account. Therefore, Chen et al. (2013) show that Higgins’ (1977) sustainable growth rate is underestimated because of the omission of the source of the growth related to new equity issue.

* 1. **Statistical Methods**

Instead of relying on financial ratios to estimate firm’s growth rates, one may use statistical methods to determine firm’s growth rates. A simple growth rate can be estimated by calculating the percentage change in earnings over a time period, and taking the arithmetic average. For instance, the growth rate in earnings over one period can be expressed as:

 . (46.18)

The arithmetic average is given by

 . (46.19)

A more accurate estimate can be obtained by solving for the compounded growth rate:

 , (46.20)

or

 , (46.21)

where

 measure in the current period(measure can be sales, earnings, or dividends); and

 measure in period *t*.

This method is called the *discrete compound sum* method of growth-rate estimation. For this approach to be consistent with the dividend growth model, the duration of each period (e.g., quarterly or yearly) must be consistent with the compounding period used in the dividend growth model.

Another method of estimating the growth rate uses the continuous compounding process. The concept of continuous compounding process can be expressed mathematically as

 . (46.22)

Eq. (46.21) describes a discrete compounding process and Eq. (46.22) describes a continuous compounding process. The relationship between Equations (46.21) and (46.22) can be illustrated by using an intermediate expression such as:

 , (46.23)

where *m* is the frequency of compounding in each year. If *m* = 4, Eq. (46.23) implies a quarterly compounding process; if *m* = 365, it describes a daily process; and if *m* approaches infinity, itdescribes a continuous compounding process. Thus Eq. (46.22) can be derived from Eq. (46.23) based upon the definition

 . (46.24)

Then the continuous analog for Eq. (46.20) can be rewritten as

 . (46.25)

Therefore, the growth rate estimated by continuous compound-sum method can be expressed by

 . (46.26)

If you estimate the growth rate via Eq. (46.26), you are implicitly assuming the dividends are growing continuously and therefore the dividend growth model. In this case, according to Gordon and Shapiro’s (1956) model, .

To use all the information available to the security analysts, two regression equations can be employed. These equations can be derived from Equations (46.20) and (46.22) by taking the logarithm (*ln*) on both sides of equation:

 . (46.27)

If Eq. (46.27) can be used to estimate the growth rate, then the antilog of the regression slope estimate would equal the growth rate. For the continuous compounding process,

 . (46.28)

Both Equations (46.27) and (46.28) indicate that  is linearly related to *t*; and the growth rate can be estimated by the ordinary least square (OLS) regression. For example, growth rates for *EPS* and *DPS* can be obtained from an OLS regression by using

 , and (46.29)

 , (46.30)

where *EPSt* and *DPSt* are earnings per share and dividends per share, respectively, in period *t*, and *T* is the time indicators (i.e., *T* = 1, 2, ..., n). We denote  and  as the estimated coefficients for Equations (46.29) and (46.30). The estimated growth rates for *EPS* and *DPS*, therefore, are  and  in terms of discrete compounding process and  and  in terms of continuous compounding process.[[6]](#footnote-6)

Table 46.4 provides dividends per share of Pepsico and Wal-Mart during the period from 1981 to 2010. Using the data in Table 46.4 for companies Pepsico and Wal-Mart, we can estimate the growth rates for their respective dividend streams. Table 46.5 presents the estimated the growth rates for Pepsico and Wal-Mart by arithmetic average method, geometric average method, compound-sum method, and the regression method in terms of discrete and continuous compounding processes. Graphs of the regression equations for Pepsico and Wal-Mart are shown in Figure 46.1.

**Table 46.4 Dividend Behavior of Firms Pepsico and Wal-Mart in Dividends per Share (*DPS*)**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Year | T | PEP | WMT |  | Year | T | PEP | WMT |
| 1981 | 1 | 3.61 | 1.73 |  | 1996 | 16 | 0.72 | 1.33 |
| 1982 | 2 | 2.4 | 2.5 |  | 1997 | 17 | 0.98 | 1.56 |
| 1983 | 3 | 3.01 | 1.82 |  | 1998 | 18 | 1.35 | 1.98 |
| 1984 | 4 | 2.19 | 1.4 |  | 1999 | 19 | 1.4 | 1.25 |
| 1985 | 5 | 4.51 | 1.91 |  | 2000 | 20 | 1.51 | 1.41 |
| 1986 | 6 | 1.75 | 1.16 |  | 2001 | 21 | 1.51 | 1.49 |
| 1987 | 7 | 2.30 | 1.59 |  | 2002 | 22 | 1.89 | 1.81 |
| 1988 | 8 | 2.90 | 1.11 |  | 2003 | 23 | 2.07 | 2.03 |
| 1989 | 9 | 3.40 | 1.48 |  | 2004 | 24 | 2.45 | 2.41 |
| 1990 | 10 | 1.37 | 1.9 |  | 2005 | 25 | 2.43 | 2.68 |
| 1991 | 11 | 1.35 | 1.14 |  | 2006 | 26 | 3.42 | 2.92 |
| 1992 | 12 | 1.61 | 1.4 |  | 2007 | 27 | 3.48 | 3.17 |
| 1993 | 13 | 1.96 | 1.74 |  | 2008 | 28 | 3.26 | 3.36 |
| 1994 | 14 | 2.22 | 1.02 |  | 2009 | 29 | 3.81 | 3.73 |
| 1995 | 15 | 2.00 | 1.17 |  | 2010 | 30 | 3.97 | 4.2 |

**Table 46.5 Estimated Dividend Growth Rates for Pepsico and Wal-Mart**

|  |  |  |
| --- | --- | --- |
|  | Pepsico | Wal-Mart |
| Arithmetic average | 4.64% | 8.99% |
| Geometric average | 0.99% | 5.45% |
| Compound-sum method | 0.99% | 5.30% |
| Regression method (continuous) | 0.56% | 7.04% |
| Regression method (discrete) | 0.56% | 7.29% |

The slope of the regression for Pepsico shows an estimated coefficient for the intercept is 0.56. The estimated intercept for Wal-Mart is 7.04. The estimated growth rates for Pepsico and Wal-Mart, therefore, are 0.56% and 7.29% in terms of discrete compounding process. Figure 1 also shows the true DPS and predicted DPS for Pepsico and Wal-Mart. We find that the regression method, to some extent, can estimate the growth rate for Wal-Mart more precisely than for Pepsico. Comparing to the geometric average method, the regression method yields a similar value of the estimated growth rate for Wal-Mart, while not for Pepsico .

**Figure 1. Regression Models for Pepsico and Wal-Mart**

 

 

There are some complications to be aware of when employing the arithmetic average, the geometric average, and regression model in estimating the growth rate. The arithmetic average is quite sensitive to extreme values. The arithmetic average, therefore, has an upward bias that increases directly with the variability of the data. Consider the following situation. Dividends in years 1, 2 and 3 are $2, $4 and $2. The arithmetic average of growth rate is 25% but the true growth rate is 0%. The difference in the two average techniques will be greater when the variability of the data is larger. Therefore, it is not surprising that we find differences in the estimated growth rates using arithmetic average and geometric average methods for Pepsico and Wal-Mart in Table 46.5.

The regression method uses more available information than the geometric average, discrete compounding and continuous compounding methods in that it takes into account the observed growth rates between the first and last period of the sample. A null hypothesis test can be used to determine whether the growth rate obtained from the regression method is statistically significantly different from zero or not. However, logarithms cannot be taken with zero or negative numbers. Under this circumstance the arithmetic average will be a better alternative.

We further randomly select 50 companies from S&P 500 index firms, which paid dividends during 1981 to 2010, to estimate their dividend growth rates by arithmetic average method, geometric average method, compound-sum method, and the regression method in terms of discrete and continuous compounding processes. Table 46.6 shows averages of estimated dividend growth rates for 50 random companies by different methods. As we discussed before, the arithmetic average is sensitive to extreme values and has an upward bias. We, therefore, find a larger average of the estimated dividend growth rate using the arithmetic average method. We also find that on average, the geometric, and compound sum methods yield relatively smaller growth rate estimates as compared to the estimates obtained using the regression methods to estimate growth rate. However, it appears that estimates obtained using the geometric, compound sum and regression methods are very similar.

**Table 46.6 Estimated Dividend Growth Rates for 50 Randomly Selected Companies**

|  |  |  |  |
| --- | --- | --- | --- |
|  | 50 Firms | Firms with Positive Growth(35 Firms) | Firms with Negative Growth(15 Firms) |
| Arithmetic average | 4.95% | 7.27% | -0.47% |
| Geometric average | 0.93% | 3.00% | -3.88% |
| Compound-sum method | 0.83% | 2.91% | -4.02% |
| Regression method (continuous) | 0.66% | 2.32% | -3.22% |
| Regression method (discrete) | 0.71% | 2.37% | -3.15% |

Finally, Gordon and Gordon (1997) suggest that one can infer the growth rate using the dividend growth model. In particular, the practitioner can use regression analysis to calculate the beta of the stock and use the *CAPM* to estimate the cost of equity. Since

  (46.31)

and the price of the stock is given by the market, the cost of equity is obtained using the *CAPM*, and and the current dividend is known, one can infer the growth rate using Eq. (46.31). If the inferred growth rate is less than the practitioner’s estimate, then the recommendation will be to buy the stock. On the other hand, if the inferred growth is greater than the practitioner’s estimate, the recommendation will be to sell the stock. However, it should be noted that the explanatory power of the *CAPM* to explain the relationship between stock returns and risk has been extensively questioned in the literature. See for example, Fama and French (1992).

* 1. **Conclusion**

The most common valuation model is the dividend growth model. The growth rate is found by taking the product of the retention rate and the return on equity. What is less well understood are the basic assumptions of this model. In this paper, we demonstrate that the model makes strong assumptions regarding the financing mix of the firm. In addition, we discuss several methods suggested in the literature on estimating growth rates and analyze whether these approaches are consistent with the use of using a constant discount rate to evaluate the firm’s assets and equity. In particular, we demonstrate that the underlying assumptions of the internal growth rate model (whereby no external funds are used to finance growth) are incompatible with the constant discount rate model of valuation. The literature has also suggested estimating growth rate by using the average percentage change method, compound-sum method, and/or regression methods. We demonstrate that the average percentage change is very sensitive to extreme observations. Moreover, on average, the regression method yields similar but somewhat smaller estimates of the growth rate compared to the compound-sum method. We also discussed the inferred method suggested by Gordon and Gordon (1997) to estimate the growth rate. Advantages, disadvantages, and the interrelationship among these estimation methods are also discussed in detail. Choosing an appropriate method to estimate firm’s growth rate can yield a more precise estimation and be helpful for the security analysis and valuation. However, all of these methods use historical information to obtain growth estimates. To the extent that the future may differ from the past, will ultimately determine the efficacy of any of these methods.

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1. See Brick and Weaver (1984 and 1997) concerning the magnitude of error in the valuation using a constant discount rate when the firm does not maintain a constant market based leverage ratio. [↑](#footnote-ref-1)
2. Gordon and Shapiro’s (1956) model assume that dividends were paid continuously and hence . [↑](#footnote-ref-2)
3. Earnings in this model are defined using the cash-basis of accounting and not on an accrual basis. [↑](#footnote-ref-3)
4. Generally, practioners define ROE as the ratio of the Net Income to the end of year Stockholders Equity. Here we are defining ROE as the ratio of the Net Income to the beginning of the year Stockholders Equity. Brick, Palmon and Venezia (2012) demonstrate that the practitioner’s definition is one of the sources for the Bowman Paradox reported in the Organization Management literature. [↑](#footnote-ref-4)
5. Increased in Assets is the net increase in assets. The total investment should also include the depreciation expense as can be seen in our examples delineated in Tables 46.1 and 46.2. But depreciation expense is also a source of funding. Hence, it is netted out in the relationship between increases in assets and retained earnings and new borrowings. [↑](#footnote-ref-5)
6. If the earnings (or dividend) process follows Eq. (46.27), we can get same results from the non-restricted model as Equations (46.29) and (46.30). [↑](#footnote-ref-6)