**Signaling and Risk-mitigating Effects on Sequential Conversion Behavior of Convertible Bonds: A Recurrent Survival Approach**

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 **Abstract**

The main purposes of this paper are (i) to review convertible bond literature in detail and establish nine propositions to be tested empirically and (ii) to use data from 73 convertible bonds to test the hypotheses in terms of a recurrent survival approach. In addition, we discuss the recurrent survival model in detail and theoretically show how the recurrent survival model can be estimated. The Rversion of the computer program will be shown in the appendix.

This study finds a higher spread of conversion-stock prices and a higher buy-backratio of stock repurchase provide CBs’ debt-like signals of Constantinides and Grundy(1989); while a lower risk-free rate, higher capital expenditures, higher non-managementinstitutional ownership, and a higher total asset value provide CBs’ equity-like signals ofStein (1992). While the equity-like signals might accelerate the rate of sequentialconversions and weaken CBs’risk-mitigating effect in the presence of risk-shiftingpotential, this study shows this can happen only in a financially healthy firm witha higher free cash flow. For financially distressed firms, CBs’risk-mitigating effect ismaintained.

***JEL classification***: G3, C1

***KeyWords***. Debt-like signal, Equity-like signal, Stock repurchase, Sequential Conversion,

Risk-mitigating effect, Risk-shifting.

1. **Introduction**

Convertible bonds (CBs) are often adopted as a financing vehicle for firms with higher levels of agency costs as well as adverse selection costs associated with information asymmetry (see e.g., Loncarski, Horst, andVeld, 2006; Dutordoir, Lewis, Seward, and Veld, 2014). By reducing the agency and the adverse selection costs, CBs can serve as a risk-mitigating instrument (see e.g., Green, 1984; Brennan and Kraus, 1987; Brennan andSchwartz, 1988;Constantinidesand Grundy, 1989; see e.g., FriermanandViswanath, 1994;Mayers, 1998, 2000; Isagawa, 2002; CornelliandYosha, 2003). In Green (1984), it was shown that the issue of CBs can reduce the agency costproblem between bondholders and shareholdersof Jensen and Meckling(1976)by curbing shareholders’ incentives for risk-shifting in a one-period setting when the investmentopportunity and CBs issue are contemporaneous, as evidenced by Lewis,Rogalski,and Seward(1998, 1999, 2003), Krishnaswami and Yaman(2008), [Dorion](http://www.sciencedirect.com/science/article/pii/S0929119913000989##), [François](http://www.sciencedirect.com/science/article/pii/S0929119913000989##),Grass, and Jeanneret(2014), and King and Mauer (2014). Likewise, Chang et al. (2004) and Korkeamakiand Moore (2004)argued thatCBs can serve as a sequential-financing tool to reduce the agency cost problem of overinvestment between managers and shareholders when the firmhas a sequence of investment opportunities (see e.g., Mayers, 1998, 2000; Cornelli&Yosha, 2003;Wang, 2009).

On the other hand, as evidenced by an in-depth interview with convertible bond issuers(see e.g., Dong et al. (2013),CBs can help in reducing the adverse selection costsassociated with information asymmetryon the perceived risks of the firmsbetween managers and shareholders (see e.g., Brennan & Kraus, 1987; Brennan & Schwartz, 1988; Cornelli&Yosha, 2003).Asa risk-mitigating instrument, CBs can serve asan alternative to straight debt (see e.g., Lewis, Rogalski,and Seward, 1999), which is evidenced by Billingsley and Smith’s (1996) survey analysis that most managers use CBs to obtain favorable couponrates on bond financing. In this respect, CBs should have debt-like features.Lee, Lee, and Yeo (2009)discuss howfirms with a higher agency cost problem of expropriation minority shareholders’wealth by controllingshareholders are more likely to issue debt-likeCBs. In Europe, CB issuers tend to be largerfirmsand havethe potential to issue debt-like securities, and theCB market is perceived as an extension to the bond marketby CB investors(see e.g., Dutordoir&Van de Gucht, 2009).

In contrast, other survey studies found that the majority of managers issue CBs in the hope of eventually converting them into equity(see e.g., Graham&Harvey, 2001; Brounen et al., 2006).This result supports Stein (1992), who states that CBs serve as back-door equity issuestohelp in reducing the adverse selection costs associated with information asymmetry about firm values (see e.g., Kim, 1990; Stein, 1992; Nyborg, 1995;Davidson, Glascock, and Schwarz, 1995; Brown et al., 2012;Lyandres and Zhdanov,2014).In this respect, CBs should have equity-like features. Lyandres and Zhdanov’s (2014) investment-based theoretical framework highlights the fact that the issuance of CBs alleviates Myers’ (1977) underinvestment problem

Be it debt-like or equity-like, in response to the adverse selection costproblem associated with information asymmetry, both Constantinides and Grundy (1989) and Stein (1992) emphasize the signaling role of CBsto the market so that efficient investments can be achieved. However, there is a significant difference between Constantinides and Grundy (1989) and Stein’s (1992)models. In Constantinides and Grundy (1989), the issue of CBs must be accompanied with stock repurchase and investment financing in order to have the desired signaling effect. In an empirical study that parallels the argument of Constantinides and Grundy (1989), the issue of CBs represents a positive signal if it enables the firm to make profitable investments and to optimize its financial structure(see e.g., Gillet and de La Bruslerie, 2010).Spattand Sterbenz (1988) further argue that if the firm uses the proceeds from CB issues to repurchase common stocks and/or to increase the firm’s capital/investment expenditures, a gain from hoarding can arise and, as a result, the rate of conversions slowsdown. This is paralleled to the prediction of the warrant exerciserate by Constantinides (1984).

In contrast to Constantinides and Grundy (1989), Stein (1992) argued that firms issue CBsas back-door equity with the expectation to have more equity to eventually attaining a less-levered capital structure. It is less likely the proceeds of CB issues are to be used for stock repurchase (see e.g., Loncarski et al., 2006), and a shorter time to conversion facilitates the firm to acquire the equity financing it needs. Under this rationale, the stronger the signaling effect of the CB issues, the shorter time for CBs to become attractive enough to be converted (see e.g., Davidson et al.,1995). This is in line with Nyborg (1995), who states the benefits of CBs as back-door equity issues are preserved only if conversion is voluntarybefore their optimal conversion time.On the other hand, CBs′ risk-mitigating effect can be weakened if CBs are converted voluntarilyas the risk-shifting opportunities arise before their optimal conversion time. This is due to the possibilities that CB bondholders who convert their bonds before the optimal conversion time can rally the initial shareholders to restore incentives towards risk at the expense of the straightbondholders (see e.g., Francois et al., 2011).It can be expected that the higher the firm’s risk-shifting potential, the weaker the risk-mitigating effect of the equity-likeCB issues.

In literature, the optimal conversion time occurs at the maturity or at the call in a perfect market with no dividend payments and constant conversion terms (see e.g., Ingersoll 1977), or immediately prior to a dividend payment dateif there are dividend payments (see e.g., Brennan and Schwartz 1977, 1980). Given the optimal conversion time, it is implicitly assumed that conversions should occur in a block,i.e. all CBs are converted completely or not at all. However, as CB issuesare divisible and held by competitive bondholders, the conversions of CBs take place sequentially instead of in a block over aCB’s lifecycle (see e.g., Buhler and Koziol,2004). These observed sequential conversions might be an outcome of Nash equilibrium in a non-cooperative game played by competitive bondholders (see e.g., Emanuel, 1983; Constantinides, 1984; Constantinides & Rosenthal, 1984; Francois et al. 2011). The potential advantageofsequential conversionsover a block conversion has been addressed under the conditions: (a) the presence of straight debt vs. no straight debt and (b) thedistribution of CBs′ ownerships. When there is no straight debt in the capital structure, a sequential conversion strategy is optimal for a monopolist bondholder even without regular dividend payments (see e.g., Emanuel, 1983; Constantinides, 1984; Constantinides and Rosenthal, 1984). When there are straight debts in the capital structure, Bühler and Koziol (2004) showed that the values of CBs held by competitive bondholders adopting sequential conversions are greater than those under a block conversion. This is due to the fact that a conversion impacts the default probability of the straight bonds, which results in the wealth transfer between stockholders and straight debtholders.

Compared to previous studies that emphasize the potential advantageofsequential conversions from bondholders’ views, this paper aims to identify theparameters that determine CBs′ signaling role and risk-mitigating effects. In view ofthe aforementioned discussion, we differentiate thesignaling role of CBs as equity-like versus debt-like based on the rate of sequential conversion of bondholders: equity-like signals increase the rate of sequential conversions, while debt-like signals decrease the rate of sequential conversions. The rate of sequential conversions is estimated based on the recurrent survival approach, in which sequential conversions are considered as recurrent events over a CB’s lifecycle (see e.g., Andersen & Gill, 1982; Prentice, Williams, & Peterson 1981). A dataset that contains longitudinal records of the timings of sequential conversions over the lifespan of a set of 73 convertible bonds listed on the Taiwan Stock Exchange from January 2004 to December 2009 is used. To our knowledge, this is the first study to use the recurrent survival analysis technique in studying bondholders’ sequential conversion behaviors in a dynamic setting.

The paper is organized as follows: Section 2 gives the development of the empirical model and hypotheses, Section 3 describes the data and method of the empirical study, Section 4 gives the results of the empirical study, and Section 5 concludes.

**2. Hypotheses**

 In the following, nine propositions are proposed in regards to the relationships between the rate of sequential conversion and the signaling and risk-mitigating effects of CBs.

**2.1 Debt-like vs. equity-like signals and sequential conversions**

In Constantinides (1984), a higher dividend yield or a lower risk-free rate will result in a lower conversion rate. This is similar to Spatt and Sterbenz (1988) in that if a firm pays an extraordinary dividend to equity holders with the proceeds from the exercise of warrants, than the optimal exercise strategy is to hold thewarrants until maturity.Therefore,dividend yield and risk-free rate represent debt-like and equity-like signals, respectively. In addition, Spatt and Sterbenz (1988) argued that if the firm uses the proceeds from CB issues to repurchase common stocks and/or to increase the firm’s capital/investment expenditures, a gain from hoarding can arise and, as a result, the rate of conversions is slowed down. This is paralleled to the prediction of the rate of warrant exercises by Constantinides (1984). Following this rationale, as the issue of CBs must be accompanied with stock repurchase and investment financing to have the desired signaling effect (see e.g., Constantinides and Grundy, 1989), the buy-back ratio of stock repurchase and the firm’s capital/investment expenditures are considered debt-like signals.

Davidson, Glascock, and Schwarz (1995) used the expected time for CBs to become attractive to be converted as the measure of the signal provided by CBs. Following Davidson, Glascock, and Schwarz(1995), a higher difference between the conversion price and the current stock price signals the shorter expected time for CBs to become attractive andbe converted.Kim (1990) states that this is due to Kim (1990)ahigher conversion price, which signals higher expected future earnings.In this case, when investors would want to convert sooner, the issue of CBs is an equity-like issuance. For this reason, the spread between the conversion price and the current stock is modeled as an equity-like signal.

In Lee, Lee, and Yeo (2009), it was argued that firms with higher shareholder rights, characterized with a greater extent of non-management institutional ownership, tend to issue more equity-like CBs with a higher conversion probability at maturity (see e.g., Lewis, Rogalski, &Seward, 1999, 2003), which is given by *N*(*d*2), where

 (1)

*N*(•) is the cumulative probability of a standard normal distribution*, S* is the stock price, *H* is the conversion price, *r* is the risk-free rate at the time of bond’s issue, *div* is the firm’s dividend yield the year before the bond’s issue date, *T* is the time to maturity, and *σ* is the standard deviation of the equity return. It can be expected that a greater extent of non-management institutional ownership has a larger sequential conversion rate.

In addition to the aforementioned explanatory variables, this study includes the firm size as an explanatory variable for the equity-like signaling effect of CBs, sinceinformation asymmetries would not be the same with firms of different sizes, so does the use of CBs as a signal to acquire equity. In summary, eight explanatory variables are considered: the risk-free rate, the dividend, the capital and/or investment expenditures, the buyback ratio of stock repurchase, the percentage spread between the conversion price and the current stock price, the non-management institutional ownership, and the total asset. Seven propositions are considered.

**Proposition 1.** Risk-free rate is an equity-like signal in that a lower risk-free rate has a negative effect on sequential conversion rate.

**Proposition 2.** Dividend is a debt-like signal in that a higher dividend has a negative effect on sequential conversion rate.

**Proposition 3.** Capital/investment expenditure is a debt-like signal in that higher capital/investment expenditure has a negative effect on the sequential conversion rate.

**Proposition 4.** Buyback ratio of stock repurchase is a debt-like signal in that a higher buyback ratio has a negative effect on the sequential conversion rate.

**Proposition 5.** Percentage spread between the conversion price and the current stock price is an equity-like signal in that higher spread has a positive effect on the sequential conversion rate.

**Proposition 6.**Non-management institutional ownership is an equity-like signal in that a higher non-management institutional ownership has a positive effect on the sequential conversion rate.

**Proposition 7.**Total asset value is an equity-like signal in that a higher total asset value has a positive effect on the sequential conversion rate.

**2.2 Risk-shiftingpotential and sequential conversion**

Jensen and Meckling (1976) and Green (1984) haveshown that an appropriately designed convertible debt issue can reduce the agency conflict problem in which shareholders transfer wealth from bondholders by increasing asset risk. This is due to the fact that whenbondholders choose to convert, the existing shareholders are forced to share the firm’s upside potential with the new shareholders. As a result, the incentives for existing shareholders to engage in risky projects are diminished. However, this risk-mitigating effect is weakened if bondholders act to convert CBs as the risk-shifting opportunities arise prior to the bond’s maturity so they can rally the initial shareholders to restore the incentives for risk (see e.g., Francois et al. 2011). It can be inferred that higher risk-shifting potential results in a higher sequential conversion rate, which weakens the risk-mitigating effect ofCBs. Most firms, except for financially distressed firms,care little about the agency conflict between shareholders and bondholders;this negative relationship between risk-shifting potential and risk-mitigating effect is significant only for financially distressedfirms(see e.g., De Jong & Van Dijk2007; Eisdorfer2008).

In this study, a firm’s risk-shifting potential is proxided by (*i*) the ratio of discretionary assets, which is one minus the ratio of fixed asset vs. total asset and (*ii*) the ratio of free cash flow, which is defined as the ratio of the operating free cash flow vs. book value of total assets (Jensen 1986; Banko and Zhao 2010). Financially distressed firms are characterized by lower bond ratings (Kish and Livingston 1992). In summary, we have the following two propositions.

**Proposition 8.** A higher ratio of discretionary assets has a positive effect on the sequential conversion rate and reduces CB′s risk-mitigating effect, which is strengthened for financially distressed firms.

**Proposition 9.** Ahigher ratio of free cash flow has a positive effect on the sequential conversion rate and reducing the CB′s risk-mitigating effect, which is strengthened for financially distressed firms.

**3. Data and Method**

**3.1 Data**

By excluding companies in the financial sector due to the comparability of financial indicators, a total of 170 five-year convertible bonds on the list of Taiwan Stock Exchange that were outstanding from January2004 and terminated before December 2009 are eligible for the study. If the same company issued two or more convertible bonds, then only one of them was randomly selected.The bond issues that were traded with more shares for call or put were excluded from our sample. After removing them, there were 130bonds traded mainly for conversion. Among the 130 eligible convertible bonds, 73 of them have complete historical records of the 16 covariates including *X*1=the risk-free rate, *X*2=dividend, *X*3=the ratio of capital expenditure vs. total asset, *X*4=the ratio of investment expenditure vs. total asset, *X*5=buy-back ratio of repurchase, *X*6=the percentage spread between the conversion price vs. current stock price, *X*7=the ratio of non-management institutional ownership, *X*8=total asset, *X*9= the ratio of discretionary asset vs. total asset, and *X*10=the ratio of free cash flow over total asset. Finally, *X*11~*X*16 are the dummies of credit rating categories 4-9.

The conversion history of the 73 convertible bonds during January 2004 to December 2009 was collected from Taiwan Gre-Tai Securities Market for a total of 2453 records. A total of1534 conversion eventsareobservedout of the2453records. Among the 1534 conversion events, 862 of them have more than 10 shares of converted CBs.Each record contains one or more of the following events: (1) A conversion by the bondholder, (2) the buy-back of the bonds by the firm, (3) the exercise of put options by the bondholders, and/or (4) the maturity of the bond. The history of these covariates was collected from *Taiwan Economic Journal* (TEJ). To facilitate the analysis, each record is in the counting process format (see e.g., TherneauandGrambsch, 2000; Allison, 2010) given by (*id*, *τ*1,*τ*2, status, *X*1, *X*2, *X*3, …, *X*16),where *id* was the identity number of the bond;*τ*1and*τ*2denoted the starting and endtime points (monthly) of the record; status indicated whether conversion occurred at the interval specified by (*τ*1, *τ*2], and *X*1, …, *X*16 denote the 16 covariates.

The following is an example showing how the counting process is done. In our dataset, only monthly conversion data were provided by Taiwan Gre-Tai Securities Market, and the convertible bonds being analyzed were issued be converted within 60 months. Assume a bond was converted in months 1, 3, and 6. Using this data from the first six months, six monthly records were created as follows:
startstop Status
 0      1    1
 1      2    0
 2      3    1
 3      4    0
 4      5    0
 5      6    1

The dependent variable is time or months since issuance. That variable, which is1, 2, 3, 4, 5, and 6 in the above example, is expressed in terms of an interval, and together with an extra variable,Status, indicates whether conversion occurred at the interval.
 The independent variables are the variables that will affect the rate the event occurs, which is the conversion of convertible bonds.

**3.2 Method: Recurrent survival analysis technique**

The traditional Cox proportional hazard model (see e.g., Cox, 1972) failed to cope with the problem to estimate the instantaneous hazard rate of conversions that occur recurrently over time. Instead, the recurrent survival analysis technique, an extension of the Cox proportional hazard model, can resolve the problem(see e.g., Prentice et al. 1981; Andersen and Gill, 1982; Duchateau et. al. 2003; Box-Steffensmeier et al. 2006). In Appendix A, we present the difference between Cox’s regression model and Andersen and Gill’s recurrent regression model. In this appendix, the estimation procedure in the Rcomputer program is also presented.

Depending on events’ categories and dependency within an individual, two types of recurrent survival analysis were developed, namely, the Andersen-Gill (AG) recurrent survival model (see e.g., AndersenandGill, 1982) and the conditional risk set model (PWP) by Prentice, Williams, and Peterson (1981). The AG model is based on a non-homogeneous Poisson counting process, in which all the recurrent events within an individual are treated as identical and independent. The advantage of the AG model lies in its efficiency and precision to give the most reliable estimates of covariate effects (see e.g., Therneau&Grambsch, 2000). In the AG model, the instantaneous rate *λik*(*t*) of experiencing the *k*thconversion at time *t* for the *i*th bond, 1≤*i*≤*N*, is a semi-parametric function *λik*(*t*) in the form

*λik*(*t*)=*λ*0(*t*)*exp*{*β*1*X*1*i*(*t*)+…+*βqXqi*(*t*)} (2)

where *λ*0(．) is the baseline nonparametric hazard function, *q* is the number of covariates. The corresponding partial likelihood can be expressed as

 (3)

where ***β***′=(*β*1, *β*2, …, *βq*) are the coefficients of the *q* covariates in (2), *Tik* is the time of the *k*th conversion of the *i*th bond, 1≤*k*≤*Ki*, while *Ki* is the number of conversions for the*i*th bond and is the termination time of the *i*th bond, 1≤*i*≤*N*. The indicator *I*(•) is one if *Tik*∈(•) and zero otherwise**.**

In contrast to the AG recurrent survival model, if the recurrent events within the same individual involve different types of categories and the ordering of the events is important, than the conditional risk set (PWP) model (see e.g., Prentice, Williams, andPeterson, 1981) should be adopted. In the PWP recurrent survival model, the risk set of the *k*th recurrent event isrestricted to the individuals who have experienced the first *k*-1 recurrent events. In our case, under PWP model, a bond is not at risk for the *k*th conversion until its (*k*-1)th conversion has occurred, and then only the bonds whose (*k*-1)th and *k*th conversionoccurs prior to and after *Tik*, respectively, are considered to be at risk, where *Tik* is the *k*th conversion time for the *i*th bond, where 1≤*i*≤*N*, 1≤*k*≤*C*. Here *C* is the maximum number of conversions defined by *C*=*max*{*K*1,…,*KN*}, where *Ki* is the number of conversions for the*ith*bond, 1≤*i*≤*N*. Besides the risk sets being constructed differently from rsik setsunder the AG model, the baseline hazard functions are allowed to vary with *k* for 1≤*k*≤*C*. Specifically, the instantaneous rate of experiencing the *k*thconversion at time *t* for the *i*th bond, 1≤*i*≤*N*, is a semi-parametric function in the form



where*λ*0*k*(*t*)is the baseline nonparametric hazard at time *t* for the *k*thstratum, 1≤*k*≤*C*. The corresponding partial likelihood can be formulated as



while the indicatorδ*ik* equals to 1 if bond i is converted at *Tik* and zero otherwise.

A recent simulation study reveals that both AG model and PWP model are preferable for recurrent events with low correlation (see e.g., Villegas et al., 2013). On the other hand, when the general effect is of interest then it is recommended to use common baseline hazard and unrestricted risk set (see e.g., Lipschutz&Snapinn, 1997). Moreover, the AG model gives the most reliable estimates of covariate effects (see e.g., Therneau&Grambsch, 2000), the nine propositions are tested based on the AG model in (2).The following are the results of the analysis.

**4. Empirical Model and Results**

The recurrent model deals with longitudinal data, which integrates time series and cross-sectional data. However, the regression part is to describe the hazard function, which describes the instantaneous failure (here is the conversion) rate.

Table 1 gives a summary of statistics of the 73 bonds. The lifespan since issuance to the termination of the bond ranges between 10 and 60 months with an average of 35.15 months. The sizes of the bonds range between 5×104shares to 1×106 shares at issuance. The credit ratings of the 73 bonds range from 4 to 9 with an average of 5.87. A rating category of “4” indicates the lowest risk and “9” indicates the highest risk. In Figure 1, the distribution of the ratings at the time of the issuance of the bonds is depicted. For each bond, conversions with more than 10 shares of converted CBs are counted. Among the 73 bonds, the average number of times of conversions is 11.21, with a range of 2 to 25 times. In Figure 2, the distribution of the conversion frequencies isdepicted. The earliest time of the first conversion takes place one month after the issuance date of the bond, while earliest time of the last conversion takes place 24months after the issuance date of the bond. On average, the first conversion takes place 6.1 months after the issuance date, while the last conversion takes place 31.78 months after the issuance date. Figures 3 and 4 illustrate the Kaplan-Meier estimate (with 95% confidence bounds) of the first and last conversion, respectively. For the 73 bonds, the probability of conversion (1) at maturity ranges between 0.6005and 1.000, with an average of 0.9216.

In Table 2, the summary statistics of the ten covariates *X*1~*X*10, based on the 2453 records are given. From Table 2, the average risk-free rate is 1.9789%, while the average dividend yield is 0.432%. The ratio of capital expenditure over total asset ranges between -0.3941% and 0.5532%, and the ratio of investment expenditure over total asset ranges between -0.1351% and 0.8009%.The percentage spread between the conversion price and current stock price ranges between -6.6944% and 0.7418%, with an average of -0.5508%. The ratio of non-management institutional ownership ranges between 0.0000% and 78.5500%, with an average of 12.0984%. The size of the firms ranges between 3.63×109 to 10.13×109and hasan average of 6.30×109. The ratio of discretionary assets ranges between 0.0795% and 0.9986%, with an average of 0.7634%. The ratio of free cash flow ranges from -0.7337% to 0.2277%, with an average of -0.0329%.

For the *N*=73 bonds, the instantaneous rate *λik*(*t*) of experiencing the *k*thconversion at time *t* for the *i*th bond, 1≤*i*≤*N*, is modeled by anAndersen-Gill (AG) recurrent survival model. In Propositions 1 to 7, the concern is related to a single covariate, while two covariates are considered in Proposition 3. Therefore, eight models, each with one covariate, are to be tested. For the *i*th bond, consider the AG modelwith covariate *Xji*as

=exp{*βjXji*(*t*)} (4)

where 1≤*j*≤8,(．) is the baseline hazard functioncorresponding to *j*thmodel*i*, *X*1*i*, …, *X*8*i* are therisk-free rate, dividend yields, the ratio of capital expenditure over total asset, the ratio of investment expenditure over total asset, percentage spread between the conversion and the current stock price, buy-back ratio of repurchase, the ratio of non-management institutional ownership, andtotal asset, respectively.

Now, we specify Equation 4 in terms of a regression model.

Table 3 illustrates the eight estimated AG models in (4) for 1≤*j*≤8. For the risk-free rate (*X*1*i*), the coefficient *β*1 is significant and positive with a *p*-value of 0.0000. Thus, Proposition 1 is validated. This higher risk-free rate, which gives rise to a higher conversion rate, is in line with Constantinides (1984). On the other hand, the coefficient *β*2 of the dividend (*X*2*i*) is insignificant with a *p*-value of 0.3600, and Proposition 2 is not validated. This might be due to the contradictory conclusions drawn from Asquith and Mullins (1991) and Constantinides (1984), respectively. According to Asquith and Mullins (1991), due to the cash flow advantage principle, when the conversion value is sufficiently high, and/or the converted dividends are greater than the bond's coupon, the incentives to convert before the maturity increase. In contrast to Asquith and Mullins (1991), by deriving the path of sequential conversion, Constantinides (1984) showed that a higher dividend yield results in a lower conversion rate.

For the ratios of capital and investment expenditures (*X*3*i* and *X*4*i*), the coefficients *β*3 and *β*4 are significant and positive (*p*-values are 0.0000). Thus,Proposition 3 is not validated. This suggests that the issue of CBs accompanied with investment financing does not have the desired debt-like signaling effect as suggested by Constantinides and Grundy (1989). To the contrary, as both *β*3 and *β*4 are positive, CBs issues accompanied with investment financing provides the equity-like signaling effect.

The coefficient *β*5 of the buyback ratio of stock repurchase (*X*5*i*) is significant and negative with a *p*-value of 0.0003. This validatesProposition 4, which is in line with Spatt and Sterbenz (1988), and thus the debt-likesignaling role of the buyback ratioof stock repurchaseby Constantinides and Grundy (1989). For the percentage spread between conversion price and the stock price (*X*6*i*), the coefficient *β*6 is significant and negative with a *p*-value of 0.0029. Thus,Proposition 5 is not validated. This result contradicts the CB′s equity-like signaling role of Davidson, Glascock, and Schwarz (1995) and Kim (1990), but, on the other hand, is in line with the cash flow advantage principle by Asquith and Mullins (1991), which states that higher conversion value engenders conversions by bondholders.

Finally, Propositions 6 and 7 are accepted as the coefficients *β*7 and *β*8 of the ratio of non-management institutional ownership (*X*7) and total asset value (*X*8) are both significant and positive with *p*-values of 0.0334 and 0.0012, respectively. This impliesthe issue of CBs is an equity-like signal for firms with a higher ratio of non-management institutional ownership. This is line with Lee et al. (2009), which states firms with higher shareholder rights issue equity-like CBs. In addition, the higher the total asset value and the less information asymmetry, the more likely the issue of CBs is an equity-like signal.

For Propositions 8-9, consider the AG model

=exp{*βj,*1*Xji*(*t*)\**X*11*,i*(*t*)+…+*βj,*7*Xji*(*t*)\**X*16*,i*(*t*)} (5)

where 9≤*j*≤10, *X*9*i* is the ratio of discretionary asset over total asset, *X*10,*i* is the ratio of free cash flow over total asset, and *X*11,*i*, …, *X*16,*i* are the dummies of the bond rating categories 4-9.

Table 4 illustrates the two estimatedAG models (5) for 9≤*j*≤10.For *j*=9, the coefficients *β*9,11,…, *β*9,14 of the terms (*X*9*i*\**X*11,*i*) ,…, (*X*9*i*\**X*15,*i*) are not significant, while *β*9,15 and *β*9,16 of the terms (*X*9*i*\**X*15,*i*) and (*X*9*i*\**X*16,*i*) are significant and negative with *p*-values of 0.0005 and 0.0000, respectively. That is, a higher ratio of discretionary asset decreases the conversion rate only for bonds with ratings 8 and 9 (*X*15,*i*=1 and *X*16,*i*=1). This suggests that as a risky firm’s risk-shifting potential is high, the rate of CBs’sequential conversions do not speed up as expected, butactuallyslows down so the risk-mitigating effect is maintained. Therefore, Proposition 8 is not validated.

For *j*=10, the coefficients *β*10,11, *β*10,12 of the terms (*X*10,*i*\**X*11,*i*), (*X*10,*i*\**X*11,*i*) are not significant. On the other hand, the coefficients *β*10,13,…, *β*10,16 of the covariates (*X*10*i*\**X*13,*i*) ,…, (*X*10,*i*\**X*16,*i*) are significant and negative with *p*-values 0.001, 0.000, 0.003, and 0.000, respectively. This suggests a higher ratio of free cash flow decreases the sequential conversion rate anddoes not weaken the risk-mitigating effect of CBs for risky firms of credit ratings 6-9 (*X*13,*i*=…=*X*16,*i* =1). Therefore, Proposition 9 is not validated and concludes that the issues of CBs cannot deteriorate the agency cost problem of asset substitution.

**5. Conclusion**

Through the estimation of CBs′ sequential conversion rate, this study develops a recurrent survival model to explore the signaling and risk-mitigating effects of CBs. The empirical analysis shows that a higher risk-free rate, higher ratios of capital and investment expenditures, a higher ratio of non-management institutional ownership, and higher total asset value provide equity-like signal for CB issues. On the other hand, a higher spread between the conversion price and the current stock price is a debt-like signal for CB issues. This result supports the cash flow advantage principle by Asquith and Mullins (1991) that higher conversion value engenders conversions by bondholders, and rejects the hypothesis that higher conversion price signals higher expected future earnings and serves as an equity-like signal by Kim (1990) and Davidson, Glascock, and Schwarz (1995). In addition, the higher buyback ratio of stock repurchaseis a debt-like signal for CB issues. This result supports the signaling equilibrium model of Constantinides and Grundy (1989), whichprovides a debt rationing rationaleas opposed to the equity rationing rationale for CBs issues in combination with stock repurchase that is linked to arbitrage-related short selling by convertible arbitrageurs (see e.g., de Jong et al., 2011; see e.g., Brown et al., 2012).

Finally, firms’ risk-shifting potential cannot deteriorate CBs’ risk-mitigating effect. For riskier firms, higher risk-shifting potential, either higher ratio of discretionary asset or higher free cash flow, decreases the sequential conversion rate. This implies that aCBs’risk-mitigating effectthat reduces the agency cost problem between shareholders and bondholders is maintained for riskier firms.

**Table 1**

**Summary Statistics of 73 Convertible Bonds**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | N | Min | Max | Average  | St Dev.  |
| Lifespan (months) | 73 | 10 | 60 | 35.043 | 16.9067 |
| Number of shares issued | 73 | 1.00×105 | 1×106 | 6.66×105 | 12.30×105 |
| Credit rating at issuance1 | 73 | 3 | 9 | 5.87 | 1.020 |
| Total Asset at issuance | 73 | 3.96×109 | 9.98×109 | 5.99×109 | 1.058×109 |
| Conversion frequency2 | 73 | 2 | 25 | 11.214 | 5.0040 |
| 1st conversion time (months) | 73 | 1 | 24 | 7.157 | 5.3504 |
| last conversion time (months) | 73 | 8 | 60 | 31.786 | 14.2797 |
| Conversion probability3 | 73 | 0.6005 | 1.0000 | 0.9216 | 0.1137 |

Note. 1 The credit rating is rated at issuance by TCRI. 2The conversion frequency is counted

as the number of shares converted exceeds 10. 3The conversion probability at maturity by

Lewis, Rogalski, and Seward (2003) given in (1).

**Table 2**

**Summary Statistics of Covariates *X***1**~*X***10

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | N | Min | Max | Average  | St Dev.  |
| *X*1 | 2453 | 0.9695  | 2.6488  | 1.9789  | 0.2864 |
| *X*2 | 2453 | 0.0000  | 7.4937  | 0.4320  | 0.9700 |
| *X*3 | 2453 | -0.3941  | 0.5532  | 0.0747  | 0.1235 |
| *X*4 | 2453 | -0.1351  | 0.8009  | 0.0927  | 0.1260 |
| *X*6 | 2453 | -6.6944  | 0.7418  | -0.5508  | 0.6311 |
| *X*7 | 2453 | 0.0000  | 78.5500  | 12.0984  | 14.8151 |
| *X*8 | 2453 | 3.6326  | 10.1355  | 6.3044  | 1.1485 |
| *X*9 | 2453 | 0.0795  | 0.9986  | 0.7634  | 0.1831 |
| *X*10 | 2453 | -0.7337  | 0.2277  | -0.0329  | 0.1556 |

Note. *X*1 (%) denotes the risk-free rate; *X*2(%) denotes the dividend yield; *X*3(%) and

*X*4 (%) denote the ratios of “capital expenditure” and “investment expenditure”;

*X*5(%) denotes the buy-back ratio; *X*6(%) denotes the percentage spread between

Theconversion and current stock prices; *X*7(%) denotes the “institutional ownership”,

 i.e., the ratio of shares held by non-management; *X*8 (×109) denotes “total asset”;

*X*9(%) denotes the ratio of “discretionary asset”, i.e., one minus the ratio of fixed

asset over total asset; *X*10(%) denotes the ratio of “free cash flow”.

**Table 3**

**Fitted AG Recurrent Survival Models of Signaling Effect**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Covariate | *βj* | exp(*βj*)2 | SE(*βj*)3 | *z* | *p*-value4 | *L*5 | *p*-value5 |
| A.Proposition 1: The risk-free rate has a positive effect on *λ*1(*t*) 1 |
| *X*1 | 0.770 | 2.160 | 0.138 | 5.580 | 0.0000 | 32.40 | 0.0000 |
| B. Proposition 2: The dividendhas a positive effect on *λ*2(*t*) 1 |
| *X*2 | -0.044 | 0.957 | 0.034 | -0.919 | 0.3600 | 1.71 | 0.1920 |
| C. Proposition 3a: Thecapital expenditurehas a negative effect on *λ*3(*t*)1 |
| *X*3 | 1.520 | 4.580 | 0.310 | 4.903 | 0.0000 | 22.20 | 0.0000 |
| D. Proposition 3b: The investment expenditure has a negative positive effect on *λ*4(*t*)1 |
| *X*4 | 1.270 | 3.580 | 0.275 | 4.618 | 0.0000 | 18.90 | 0.0000 |
| E. Proposition 4: The buy-back ratiohas a negative effect on *λ*5(*t*) 1 |
| *X*5 | -0.003 | 0.997 | 0.001 | -3.414 | 0.0003 | 13.20 | 0.0003 |
| F. Proposition 5: Thepercentage spreadhas a positive effect on *λ*6(*t*)1 |
| *X*6 | -0.114 | 0.893 | 0.038 | -2.969 | 0.0029 |  7.83 | 0.0051 |
| G. Proposition 6: The institutional ownership has a positive effect on *λ*7(*t*) 1 |
| *X*7 | 0.005 | 1.010 | 0.002 | 2.127 | 0.0334 | 4.28 | 0.0386 |
| H. Proposition 7: The total asset has a positive effect on *λ*8(*t*)1 |
| *X*8 | 0.100 | 1.110 | 0.031 | 3.226 | 0.0012 | 10.20 | 0.0014 |

Note. 1*λj*(*t*)=*λ*0(*t*)*exp*{*βjXj*(*t*)} is the instantaneous conversion rate, 1≤*j*≤8, where *X*1 denotes

the risk-free rate; *X*2 denotes the dividend yield; *X*3 and *X*4 denote “capital expenditure” and “investment expenditure” standardized by total assets at the end of last year; *X*5 denotes the buy-back ratio; *X*6 denotes the percentage spread between the conversion and current stock prices; *X*7 denotes the “institutional ownership”, i.e., the ratio of shares held by non-management; *X*8 denotes “total asset”; 2 exp(*βj*) denotes the marginal contribution of *Xj*to the instantaneous rateof conversion *λj*(*t*); 3 SE denotes the standard deviation of the estimator of *βj* based on AG model. 4 The *p*-value of the hypothesis H0: *β j*=0 vs. H1: *βj* ≠0; 5 *L* and *p*-value are the likelihood ratio statistics and *p* value of the hypothesis H0: *λj*(*t*)=*λ*0(*t*)*exp*{*βjXj*(*t*)}.

**Table 4**

**Fitted AG Recurrent Survival Models of Risk-mitigating Effect**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Covariate | *βj* | exp(*βj*)2 | SE(*βj*)3 | *z* | *p*-value4 | *L*5 | *p*-value5 |
| A. Proposition 8: The ratio of discretionary asset has a positive effect on *λj*(*t*)1 |
| *X*9\* *X*11 | -0.624 | 0.536 | 0.213 | -1.792 | 0.073 | 47.5 | 0.0000 |
| *X*9\* *X*12 | -0.678 | 0.508 | 0.361 | -1.881 | 0.059 |
| *X*9\* *X*13 | -0.463 | 0.630 | 0.193 | -1.545 | 0.120 |
| *X*9\* *X*14 | -0.384 | 0.681 | 0.214 | -1.380 | 0.170 |
| *X*9\* *X*15 | -1.518 | 0.219 | 0.335 | -2.788 | 0.005 |
| *X*9\* *X*16 | -2.341 | 0.096 | 0.622 | -4.177 | 0.000 |
| B. Proposition 9: Thefree cash flowhas a positive effect on *λj*(*t*)1  |
| *X*10\* *X*11 | -2.283 | 0.102 | 2.211 | -1.033 | 0.300 | 52.5 | 0.0000 |
| *X*10\* *X*12 | -0.771 | 0.462 | 0.469 | -1.644 | 0.100 |
| *X*10\* *X*13 | -1.842 | 0.159 | 0.561 | -3.286 | 0.001 |
| *X*10\* *X*14 | -1.728 | 0.178 | 0.364 | -4.742 | 0.000 |
| *X*10\* *X*15 | -0.425 | 0.653 | 0.192 | 2.210 | 0.030 |
| *X*10\* *X*16 | -151.14 | 0.000 | 25.556 | -5.914 | 0.000 |

Note. 1*λj*(*t*)=*λ*0(*t*)*exp*{*βjXj*(*t*)+*β*11*X*11(*t*)+…+*β*16*X*16(*t*)} is the instantaneous conversion rate, 9≤*j*≤10, where *X*9 denotes the “discretionary asset”, i.e., one minus the ratio of fixed asset over total asset; *X*10denotes the “free cash flow”; *X*11,…, *X*16 are the 6 dummy variables of ratings categories 4~9; 2 exp(*βj*) denotes the marginal contribution of *Xj*to the instantaneous rateof conversion *λj*(*t*); 3 SE denotes the standard deviation of the estimator of *βj* based on AG model. 4 The *p*-value of the hypothesis H0: *β j*=0 vs. H1: *βj* ≠0; 5 *L* and *p*-value are the likelihood ratio statistics and *p* value of the hypothesis H0: *λj*(*t*)=*λ*0(*t*)*exp*{*βjXj*(*t*)+*β*11*X*11(*t*)+…+*β*16*X*16(*t*)}.

Note. The distribution of the credit ratings of the 73 CBs at the time of issuance,

ranged from 4 to 9 with a rating category of “4” indicating the lowest riskand “9”

indicating the highest risk.

Note. For each of the 73 bonds, conversions with more than 10 shares of converted CBs

are counted. Among the 73 bonds, the minimum conversion frequency is twotimes and the maximum conversion frequencyis 25 times.



Note. The earliest first conversion takes place one month after the issuance date of the bond; the latest first conversion takes place 24 months after the issuance date of the bond. On average, the first conversion takes place 6.1 months after the issuance date.



Note. The earliest last conversion takes place 8 months after the issuance date of the bond, while latest last conversion takes place 60 months after the issuance date of the bond. On average, the last conversion takes place 31.78 months after the issuance date.

**References**

Allison, P.D. (2010) *Survival Analysis Using the SAS System: A Practical Guide.* Second Edition. Cary, NC: SAS Institute.

Andersen, P. K., &Gill, R. D.(1982) Cox’s regression model for counting processes: a large sample study, *Annals of Statistics*, 10, 1100-1120.

Asquith, P., &Mullins, D.W.(1991) Convertible debt: Corporate call policy and voluntary conversion, *Journal of Finance* 46, 1273-1289.

Banko, JC. & Zhou, L. (2010) Callable bonds revisited, Financial Management, 39, 2,613-641.

Billingsley, R.S., & Smith, D.M. (1996). Why do firms issue convertible debt? *Financial Management*, 25, 93–99.

Brennan, M. J., &Schwartz, E.S. (1977) Convertible bonds: Valuation and optimal strategies for call and conversion, *Journal of Finance*32, 1699-1716.

Brennan, M. J., &Schwartz, E.S. (1980) Analyzing convertible bonds, *Journal of Financial andQuantitative Analysis*, 15, 4, 907–929.

Brennan, M.J., &Schwartz, E.S. (1988). The case for convertibles. *J. Appl. Corp. Finance* 1, 55–64.

Brennan, M. J., & Kraus, A. (1987) Efficient financing under asymmetric information. *Journal of Finance*, 42, 1225-1243.

Box-Steffensmeir, J. M., &Boef, S.D. (2006) Repeated events survival models: The conditional frailty model, *Statistics in Medicine*, 25, 3518-3533.

Brown, S.J., B.D.Grundy, C.M.Lewis, &Verwijmeren, P. (2012) Convertibles and hedge funds as distributors of equity exposure. *Rev. Financ. Stud*., 25, 3077–3112.

Brounen, D., de Jong, A. &Koedijk, K.(2006) Capital structure policies in Europe: survey evidence. *J. Bank. Finance* 30, 1409–1442.

Bühler, W., &C. Koziol (2004) Valuation of convertible bonds with sequential conversion, *Schmalenbach Business Review*, 54, 302-334.

Chang, S.C., S.S.Chen, &Y. Liu (2004)Why firms use convertibles: a further test of the sequential-financing hypothesis. *J. Bank. Finance* 28, 1163–1183.

Constantinides**,** G.M. (1984) Warrant exercises and bond conversion in competitive markets, *Journal of Financial Economics*, 13(1), 371-397.

Constantinides, G. & Rosenthal, R. (1984) Strategic analysis of the competitive exercise of certain financial options, *Journal of Economic Theory* 32, 128-138.

Constantinides, G., & BD. Grundy (1989) Optimal investment with stock repurchase and Financing as signals. *Review of Financial Studies* 2:445–65.

[Cornelli](http://restud.oxfordjournals.org/search?author1=Francesca+Cornelli&sortspec=date&submit=Submit), F. &Yosha, O. (2003) Stage financing and the role of convertible securities, [*Review of Economic Studies*](http://restud.oxfordjournals.org/) 70,1, 1-32.

Cox, DR. (1972) Regression models and life-tables (with discussion), *Journal of the Royal Statistical Society*, Series B, 34, 187-222.

Cox, J., & Rubinstein, M. (1985) Options Markets*.* Englewood Cliffs, NJ: Prentice-Hall.

Davidson, WN.,Glascock, JL. & Schwarz,TV. (1995) Signaling with convertible debt, *Journal of Financial and Quantitative Studies*, 30(3): 425-440.

de Jong, A., &Van Dijk, R. (2007).Determinants of leverage and agency problems: A regression approach with survey data. *The European Journal of Finance* 13, 565–593.

de Jong, A., Dutordoir, M. &Verwijmeren, P. (2011). Why do convertible issuers simultaneously repurchase stock? An arbitrage-based explanation.*Journal of Financial Economics* 100, 113–129.

Dong, M., M. Dutordoir,& C. Veld, C. (2013) Why do firms issue convertible bonds? Evidence from the field. Working paper. York University, University of Manchester, and University of Glasgow.

[Dorion](http://www.sciencedirect.com/science/article/pii/S0929119913000989##), C., [François](http://www.sciencedirect.com/science/article/pii/S0929119913000989##), P., Grass, G., &Jeanneret, A. (2010)Convertible debt and shareholder incentives, *Journal of Corporate Finance* 24, 38-56.

Duchateau L, Janssen, P.,Kezic, I.,&Fortpied, C. (2003) Evolution of recurrent asthma event rate over time in frailty models. *Applied Statistics*, 52, 355-363.

Dutordoir, M., Lewis, C., Seward, J., & Veld, C. (2014).What we do and do not know about convertible bond financing, *Journal of Corporate Finance* 24, 38-56.

Emanuel, D. C*.* (1983), Warrant Valuation and Exercise Strategy, *Journal of Financial Economics*, 12, 211-235.

Eisdorfer, A., (2008). Empirical evidence of risk-shifting in financially distressed firms.*Journal of Finance* 63, 609–638.

François, P., G. Hübner, &Papageorgiou, N. (2011) Strategic analysis of risk-shifting incentives with convertible debt.*Quarterly Journal of Finance*, 1, 293–321.

Frierman, M., &*Viswanath, P.* (1994).Agency problems of debt, convertible securities, and deviations from absolute priority in bankruptcy.*Journal of Law and Economics* 37, 455–476.

Gillet, R. &de La Bruslerie, H (2010). The consequences of issuing convertible bonds:dilution and/or financial restructuring? European Financial Management 16, 4, 552-584.

Graham, J.R., &Harvey, C.R.(2001). The theory and practice of corporate finance: evidence from the field. Journal of Financial Economics, 60, 187–243.

Green, R. (1984) Investment incentives, debt, and warrants, Journal of Financial Economics, 13, 115-136.

Ingersoll, J. E. (1977) A contingent-claims valuation of convertible securities, Journal of Financial Economics,4, 289-321.

Isagawa, N. (2002) Callable convertible debt under managerial entrenchment.*Journal of Corporate Finance* 8, 255–270.

Jensen, M. C. (1986)Agency costs of free cash flow, corporate finance and takeovers. *American Economic Review*, 76, 323-329.

Jensen, M.C. &Meckling,W.H. (1976) Theory of the Firm: managerial behavior, agency costs and capital structure, *Journal of Financial Economics*, 3305-360.

Kim, YO. (1990) Informative conversion ratios: A signaling approach, *Journal of Financial and Quantitative Analysis*, 25(2): 229-243.

King, T.H.D.,&Mauer, D.C., (2014) Determinants of corporate call policy for convertible bonds. *Journal of Corporate Finance* 24, 112–134.

Kish, RJ.,& Livingston, M. (1992) Determinants of the call option on corporate bonds, *Journal of Banking and Finance*, 16,4, 687-703.

Korkeamaki, T.P., &Moore, WT.(2004) Convertible bond design and capital investment: The role of call provisions. *Journal of Finance*59, 391–405.

Krishnaswami, S., &Yaman, Y. (2008) The role of convertible bonds in alleviating contracting costs. *Quarterly Review of Economics and Finance* 48, 792–816.

Koziol, C. (2006) Optimal exercise strategies for corporate warrant, *Quantitative Finance* 6, 37-54.

Lee, C.F., Lee,K.W., &G. Yeo (2009) Investor protection and convertible debt design, *Journal of Banking & Finance* 33, 985–995.

Lewis, C., Rogalski, R.,& Seward, J. (1998) Agency problems, information asymmetries, and convertible debt security design, *Journal of Financial Intermediation* 7, 32-59.

Lewis, C., Rogalski, R.,& Seward, J. (1999) Is convertible debt a substitute for straight debt or for common equity? *Financial Management* 28, 5-27.

Lewis, C., R. Rogalski, &J. Seward (2003) Industry conditions, growth opportunities and market reactions to convertible debt financing decisions. *Journal of Banking and Finance* 27, 153–181.

Lipschutz, K.H., &Snapinn, S. M. (1997). Discussion of paper by Wei and Glidden.*Statistics in Medicine*, 16, 841-51.

Loncarski, I., Ter Horst, J.,&Veld, C.(2006). Why do firms issue convertible bonds? A review of theory and empirical evidence. In: Renneboog, L.D.R. (Ed.), *Advances in Corporate Finance and Asset Pricing. Elsevier*, Amsterdam.

Lyandres, E., & Zhdanov, A.(2014). Convertible debt and investment timing.*Journal of Corporate Finance* 24, 21-37.

Mayers, D. (1998) Why firms issue convertible bonds: the matching of financial and real investment options, *Journal of Financial Economics*47, 83-102.

Mayers, D.(2000) Convertible bonds: Matching financial and real options, *Journal of Applied Corporate Finance* 13, 8-21.

Myers, S.C. (1977)The determinants of corporate borrowing. *Journal of Financial Economics*5, 147–175.

Nyborg, K.G. (1995) Convertible debt as delayed equity: Forced versus voluntary conversion and the information role of call policy. *J. Financ. Intermed*. 4, 358–395.

Prentice, R. L., Williams, B. J., &Peterson, A.V.(1981) On the regression analysis of multivariate failure time data, *Biometrika*, 68, 373-379.

Villegas, R., Julia, O., & J. Ocana (2013) Empirical study of correlated survival times for recurrent events with proportional hazards margins and the effect of correlation and censoring. *BMC Medical Research Methodology*, 13:95.

Spatt, C.S., &Sterbenz, F.P. (1988) Warrant exercise, dividends, and reinvestment policy, *Journal of Finance* 43, 494-506.

Stein, JC. (1992) Convertible bonds as backdoor equity financing, *Journal of Financial Economics*, 32(1): 3-22.

Therneau, T. M., &Grambsch, P.M. (2000) Modeling Survival Data: Extending the Cox Model. Statistics for Biology and Health New York: Springer–Verlag.

Wang, S. (2009).Convertibles in sequential financing.*Eur. Finan. Rev*. 13, 727–760.

**Appendix A: Cox’s Regression Model vs. Andersen and Gill’s Recurrent Regression Model**

In this appendix, we will discuss Cox’s regression model and AG’s model in detail. In addition, the relationship between these two models will also be explored. The estimation procedure of AG’s model and the Rcomputer program will also be presented.

**(I) Cox’s Regression Model**

In survival analysiswith covariates, the dependent variable is time to event. Without specifying the distribution for time to event, the proportional hazard regression model proposed byCox (1972) relates the hazard, *i.e.* the instantaneous failure rate, at time *t*to *q*covariates by the following equation



whereis the baseline hazard function usually unspecified and thus nonparametric.Since the form of  is unknown, it is impossible to write down the regression equation for the dependent variableby a function of the covariates and their coefficients as in the linear regression model.Given the covariates and utilizing the relationship between hazard function andsurvivalfunction, the probability of surviving (*i.e.* not experiencing the event) beyond time *t* is

,

whereis the baseline survival probability, and the probability density function is

.

One feature thatmakes survival analysis special is that some subjects haven’t experienced the eventupon their departuresandare right-censored. To accommodate the right-censored situation, the full likelihood for *N*subjects is

,

whereif subject*i* fails, if subject*i* is right-censored, and () are the covariates for subject *i*.

Since the main concern is to make an inference on , (*t*)can betreated as a nuisance. Cox(1972) proposed to construct a partial likelihood function of by ignoring .For easy illustration, assume *k*subjects fail; their failure times are untiedand denoted as . Let be the covariates for the subject failingat, and  denote the risk set at ,which containsthose neither failed nor censored prior to , then Cox’s partial likelihood is

.

Therefore, even though  is unspecified, the *q*regression coefficients can still be estimated from the partial likelihood, which considers probabilities for failed subjects, while censored subjects contribute to the corresponding risk sets. The maximum likelihood estimator of ,,is the solutions of*q* partial likelihood equations,

, for *m*=1,…,*q.*

The observed information matrix is the negative of the matrix of the second derivative of the log partial likelihood, denoted as , with the (*m*,*r*)th element being

.

Since these *q* equations are nonlinear in , the estimates are found numerically by Newton-Raphson method or some other numerical method iteratively. For large samples, approximates to a *q*-variate normal distribution with mean vector , and variance matrix .

**(II) Andersen and Gill’s Recurrent Survivor Model**

When time since entry makes the time scale, and covariates may be time-varying, then the hazard for the *i*thsubject under Cox model is



whereis the covariate vector at time *t*for subject *i*, and, withbeing the failure or censoring time for subject *i*.

For recurrentdata, let  be the total number of the event occurring, and be the time for the *k*th recurrence, *k*=1, …,, and  denote the termination time of subject *i*.Andersen andGill (1982)extendedthe Cox model by assuming that recurrent events occur randomly and the numbers of events occurring in non-overlappingintervals are independent. Thehazard for subject *i*experiencing the *k*th recurrence under Andersen and Gill’s(AG)model is



whereis the covariate vector at the *k*th recurrence for subject *i* at time *t*, and, *i.e.* remains to be 1 as long as the last event occurs at or after *t*.At time , the risk set  contains those not experiencing their last event or censoring prior to . Since covariates may be changed over time, their hazards are evaluated accordingly. Hence, the partial likelihood under the AG model is



Notice that the above function doesn’t depend on the baseline hazard function, which becomes

.

Clearly, the distinction between the Cox model and the AG model is that once experiencing the first event or censoring, the subject is excluded from all subsequent risk sets under the Cox model, while the subject is contained in all risk sets up to the one corresponding to the last occurrence under the AG model. Therefore, it is not appropriate to analyzerecurrent survival data via fitting the classic Cox model.

The log partial likelihood function of the AG model is

.

The maximum likelihood estimates () are obtained by simultaneously solvingthe following *q* partial derivative equations:

=0,

for*m*=1,…,*q.*

The observed information matrix is the negative of the matrix of the second derivative of the log partial likelihood, denoted as , with the (*m*,*r*)th element being







Just like Cox’s model, estimates of are found numerically by Newton-Raphson’smethod or some other numerical method. For large samples, approximates to a *q*-variate normal distribution with mean vector , and variance matrix.

To illustrate the formations of the partial likelihood under the AG model and Cox’s model, assume there was one covariate.Bond A fully converted atmonth 1, bond B converted at months2 and 5, and bond C converted at months3 and 6. The counting process format allowing recurrent conversions is given below.

Bond monthstartend Status*X*1
A 1 0 1 1 1

B 1 0 1 0 1

B 2 1 2 1 2

B 3 2 3 02

B 4 3 4 0 3

B 5 4 5 1 3

C 1 0102

C 2 1 20 1
C 3 2 3 1 2
C 4 340 2
C 5 450 4
C 6      5     6    1 3

Although there are six intervals, (0,1], (1,2], (2,3], (3,4], (4,5], (5,6], the ordered conversion times are 1, 2,3,5, and6.Under the AG model, at *t*=1,all bonds are at risk; at *t*= 2, 3, and 5, bonds B and C are at risk; at *t*=6, only bond C is at risk.Moreover, *K*1=1, *T*11=1; *K*2=2, *T*21=2, *T*22=5;*K*3=2, *T*31=3, *T*32=6. Based on thesethreebonds, the partial likelihood under the AG model is



.

Note that under Cox’s model, analyses focus only on months to the first conversion.Here, times to the first conversion are 1 for bond A, 2 for bond B, and 3 for bond C, respectively. (*i.e.t*1=1, *t*2=2, and *t3*=3.) Therefore,besides the only record of bond A, the first two records of bond B and the first three records of bond Ccontribute to the partial likelihood, which is

.

The AG model has been available in R, which is freely available under the [GNU General Public License](https://en.wikipedia.org/wiki/GNU_General_Public_License). To fit the AG models in R, the statements for fitting the single covariate corresponding to Propositions 1 to 7, respectively, are given below.

AG1<-coxph(Surv(start, stop, Status) ~ X1+cluster(id), data=cb)

AG2<-coxph(Surv(start, stop, Status) ~ X2+cluster(id), data=cb)

AG3<-coxph(Surv(start, stop, Status) ~ X3+cluster(id), data=cb)

AG4<-coxph(Surv(start, stop, Status) ~ X4+cluster(id), data=cb)

AG5<-coxph(Surv(start, stop, Status) ~ X5+cluster(id), data=cb)

AG6<-coxph(Surv(start, stop, Status) ~ X6+cluster(id), data=cb)

AG7<-coxph(Surv(start, stop, Status) ~ X7+cluster(id), data=cb)

AG8<-coxph(Surv(start, stop, Status) ~ X8+cluster(id), data=cb)

For Proposition 8 and 9, the statement is as follows:

AGj<- coxph(Surv(start, stop, Status) ~ Xj(X11) + Xj(X12) + Xj(X13) + Xj(X14) + Xj(X15)+ Xj(X16)+cluster(id), data=lastsave)

for j=9,10.