Financial institutions in crisis: Modeling the endogeneity between credit risk and capital requirements*

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ABSTRACT

This manuscript presents a credit-risk-based model for the establishment of minimum bank capital requirements. The structural model determines the minimum level of equity required to yield a maximum acceptable cumulative probability of default given a bank's existing liability structure. It is based on a modified version of the Geske (1977) structural model which assumes that new equity is issued to pay down maturing debt; this is particularly appropriate for financial institutions in times of distress. The model is traditionally used to accurately measure the term structure of a firm's default risk, given its existing capital structure and its associated market value of equity. This manuscript demonstrates how Geske can be adapted to use a default probability threshold as an input and to solve for the corresponding level of equity that yields this default probability. The model overcomes one of the major pitfalls of current market-risk-based Basel capital requirements: the lack of inclusion of the firm's liability structure. The Geske model, unlike other structural credit risk models which simplify the liability structure by imposing an arbitrary default boundary, is particularly well-suited for financial institutions because of its ability to accommodate the complicated bank capital structures that are observed in reality. The manuscript examines the case of Lehman Brothers, to demonstrate how the model might be used by regulators as a dynamic tool to accurately measure default risk, assess bank solvency, and, in turn, to set appropriate credit-risk-based capital requirements.

1 Introduction

The recent financial crisis that has rocked the global financial system has demonstrated the need for new tools to measure and manage the risks and capital requirements in financial institutions. Marked by the dramatic demise of major Wall Street icons and a massive bailout of financial services companies, the financial system stands at a critical juncture. The risk based capital requirements established by Basel II have proved inadequate in the face of rapidly changing credit risks and market conditions. Central banks and regulators are grappling with the need to anticipate default and the capital infusion required to reduce default probability to acceptable levels. Critical to this task is the quick and accurate measure of default risk reflecting rapidly changing market conditions. In this paper, we propose a new approach to determine the level of default risk in financial institutions and show how regulators can use our measure of default risk to estimate the amount of capital infusion that a financial institution needs to raise to reduce default probability to acceptable levels.

The financial accords collectively termed as Basel II provide the current framework for auditing and monitoring bank operations. The First Pillar of these accords, which specifies the minimum capital requirements for credit, market, and operational risks, is *static* and is based on *historical analysis* of the bank's balance sheet and operations. The capital ratio is calculated using the definition of regulatory capital and risk-weighted assets. Specifically, the requirements mandate that the total capital ratio must be no lower than 8%. The regulations also specify that Tier 2 capital is limited to 100% of Tier 1 capital, where Tier 1 and Tier 2 refer to the amount of capital required against specific instruments held as assets on the bank's balance sheet. The primary goal of frameworks such as Basel II is to ensure that banks have a "capital cushion" to absorb losses and reduce the probability of financial distress and default. The larger the amount of capital held on the bank's balance sheet, the lower the probability of default. The recent financial crisis, and indeed other debacles such as the S&L crisis in the 1980s, the fall of Long Term Capital Management in the 1990s, have shown that static, asset-based regulatory requirements are ineffective. Well-known papers such as Koehn and Santomero (1980) and Kim and Santomero

(1988) have questioned the use of such static asset-based regulatory ratios which are incapable of reacting to a fast moving credit environment and changing market risk.

In this paper, we propose a dynamic approach to measuring default risk and develop regulatory tools to specify capital requirements that keep the probability of default within an acceptable range. Our approach is based on the Geske (1977) model and estimates default risk using a contingent claims framework in which equity is a call option on the underlying assets of the firm. Our approach is along the lines proposed by Merton (1974) and other structural models, which use the probability that the call option expires out-of-the-money to measure default probability. Unlike other structural models, our approach is applicable to firms characterized by large and complex liability structures and is applicable to estimating default risk in financial institutions.

Our model provides predictive power of default probability in the sense that it takes future cash flow obligations and the current market condition as inputs to calculate the probability of bankruptcy over different horizons. Regulators can identify not only high default risk cases, but also the horizon over which the financial institution faces the most problems.

Our model also provides the regulator with a prescriptive tool. The financial institution has several degrees of freedom in controlling default risk. One way is that the institution can sell assets on the balance sheet and retire debt. Reducing debt will automatically reduce default risk. However, selling assets may not be desirable or feasible in illiquid market conditions. The financial institution can also restructure its debt, i.e. retire debt by issuing new debt in a targeted fashion. The financial institution can relieve financial constraints caused by a particular cash flow by issuing new debt with cash flows during periods in which it has financial slack. A better designed liability structure could therefore ease the cash flow constraints and bring default probability to acceptable levels. Finally, the financial institution can raise capital to meet cash flow needs. Of course, the financial institution need not restrict itself to only one of the above three strategies but can incorporate all the three methods in a comprehensive approach to manage default risk. Our model can be used to evaluate the impact of any and all these mechanisms and gives regulators critical insight on whether fresh infusions of capital will (or will not) improve default risk in any specific case. Our model thus represents a way of merging the monitoring ability of market participants and the regulator as discussed by Flannery (1998).

We apply our model to understand the implied default risk and capital requirements for the case of Lehman Brothers during the Financial Crisis of 2008. We show that the one year bankruptcy probability reached 50% in April 2008, well before Lehman Brothers actually failed. We find that volatility plays a crucial role and that in addition to the market valuation, the level of market uncertainty is critical in determining default risk. We estimate our model for Lehman Brothers before and after the capital infusion of $25 Billion and are able to show that this capital was not sufficient enough to lower its default risk to acceptable levels. Our model thus shows that capital infusion alone cannot dramatically reduce default risk and Lehman Brothers required substantial restructuring of its debts and assets in order to reduce bankruptcy risk.

Our work extends the literature along several dimensions. Our model represents a liability driven estimation of default risk for a financial institution in contrast to a static asset driven procedure. The insight that equity markets can be used to infer default risk parallels the concepts discussed by Black and Scholes (1973) and Merton (1974) in a single period European option framework. For example, Crosbie and Bohn (2001) present the KMV model that represents a practical implementation of the original Black-Scholes-Merton (BSM) approach. This approach to estimating default risk, however, assumes an exogenously specified default boundary by reducing the entire liability structure of a firm to a weighted average of two points — short-term and long-term debt — in order to adhere to the single-period European option framework. On the other hand, the Geske (1977) approach that underlies our model endogenously solves for the default boundary over time as a function of the entire liability structure and the corresponding future cash flows. Financial institutions have very complex liability structures; over-simplifying the liabilities and using an exogenously specified default boundary can dramatically distort estimates of credit risk. Our implementation of the Geske (1977) model goes a long way towards realizing the potential of structural models to generate a practical measure of default risk and capital requirements to measure and manage the default risk in financial institutions.

Our work also represents a practical implementation of the Geske (1977) model and incorporates several computational improvements. The computational algorithm proposed by Geske (1977) requires that the default barrier to be solved for recursively and is difficult to implement. We present an algorithm that does not require a recursive solution. In our model, the default barrier is instead a direct output of a binomial model. It has also been shown in the literature that even the simplest term structure model will highly complicate the solution of the Geske (1977) model. In our binomial framework, this generalization is straightforward and adds only reasonable computation time.

Structural models require assumptions regarding how firms deal with intermediate cash-flow obligations. In our model the firm replaces debt with equity. We note that alternate models do exist; e.g. the Leland (1994) and Leland and Toft (1996) models assume that the firm retires debt by issuing more debt. Our model, and the original Geske (1977) model, are more conservative in assuming that maturing debt is replaced with equity. This avoids the problem that default risk estimates represent the impact of future new debt rather than the firm's current maturity structure. Further, assuming that debt is replaced with equity ensures that we do not overestimate default risk and imposes an additional regulatory constraint on the financial institution.

Our model allows a capital structure solution to managing default risk and preserving the integrity of the financial institution. There are two important issues in reducing liabilities. First, which debt does the financial institution retire? Preferably the financial institution should retire the debt that most constrains the firm. Second, from where does the financial institution raise the funds to pay down debt? Our approach can identify the specific debt issues that cause default risk to be high and the reduction in default risk by replacing the debt with equity.

The rest of the paper is as follows. Section 2 reviews the relevant literature. Section 3 discusses the dataset and methodology employed. Section 4 provides an overview of the Geske (1977) model framework and a numerical example. Section 5 presents the liability structure and time-line of events for the case of Lehman Brothers. Section 6 develops estimates of default risk and clearly shows that our model can accurately predict the higher default risks. Section 7 presents our conclusions and the policy implications of our work.

2 Related Literature

Several papers have begun to document, examine, and explain the events that led up to and resulted in the financial crisis. Crouhy, Jarrow, and Turnbull (2008) provide an in-depth look at the flawed mechanisms driving the "Subprime Crisis of 2007" from securitization, to capital requirements, and even the role of ratings agencies. Gorton (2009) explains how the events went from a relatively localized debacle involving derivatives on subprime mortgages to a systemic crisis that engulfed the global financial system. He attributes the credit crisis to opaque markets, asymmetric information, and interconnectedness between financial intermediaries with their collective exposure to housing prices magnified many times over through the web of complex contracts. One of the first high-profile victims of the crisis occurred in the UK, with a run on Northern Rock Bank in September of 2007. This case is profiled by Shin (2009) where a new type of bank run is described; one that is not only due to liquidity issues and maturity mismatch but further exacerbated by high leverage. For an excellent discussion of liquidity and the credit crisis please see Brunnermeier (2009). Central to the aforementioned papers is the topic of capital adequacy standards. Sadly, the existing market value risk-based bank capital requirements delineated in Basel II were not capable of immunizing banks from the perils of credit risk.

There is a rich body of literature that examines the role of bank capital and relevancy of minimum bank capital adequacy standards for the mitigation of bank risk.[1] Diamond and Rajan (2000) offer an explanation for the role of bank capital. They argue that bank capital makes banks safer and helps avoid distress, which in turn leads to an increased survival probability: bank capital is important because "deposits are fragile and prone to runs." The flip side, however, is that bank capital reduces liquidity, and one of the primary functions of a bank is its ability to create liquidity. Since bank capital is crucial to a bank's survival, Santomero and Watson (1977) ask the fundamental question of whether or not bank capital levels should be regulated? Is there an optimal level of bank capital? They find that the optimal level of bank capital is a trade-off between the cost of bank failures (as a result of undercapitalization) and the cost of over-capitalization (in terms of the constraints placed on the bank).

This research on optimal bank capital levels leads to a discussion of bank capital adequacy and minimum capital requirements. Kareken and Wallace (1978) present an equilibrium model of banking under different regulatory schemes, two of which include deposit insurance and capital requirements. [2] They find that capital requirements, in and of themselves, do not do much to mitigate bankruptcy risk. Sharpe (1978) examines the role of bank capital where the bank's liabilities consist only of deposits which are insured by a third party, effectively making them risk-free. He defines capital adequacy in terms of minimizing the insurer's liability so that it is "no larger than an insurance premium." Buser, Chen, and Kane (1981) argue that capital requirements can serve as an implicit risk-based deposit insurance premium. Therefore, using their line of reasoning, regulating a bank's capital levels may be used in conjunction with a flat-rate deposit insurance scheme to achieve the same results as one that sets insurance premiums according to risk.

Although Sharpe (1978) indirectly demonstrates that capital adequacy standards should be risk-based, Koehn and Santomero (1980) formally and directly argue the

[1]For a comprehensive review of the bank capital regulation literature see Santos (2001).

[2]The literature frequently cites risk-based deposit insurance and minimum capital requirements as two methods for controlling financial institution risk. Our paper deals with the latter and as such we do not discuss the abundant literature on risk-based deposit insurance. For a good review of this literature the reader is referred to Allen and Saunders (1993). Also, Santos (2001) does a good job of expounding the link between deposit insurance and capital requirements as regulatory tools.

point. They examine how minimum capital requirements affect banks' behavior. Their idea is that a regulator wants to ensure that the bank sets aside enough capital to keep the probability of failure low (within "acceptable levels"). They demonstrate that placing restrictions on bank capital might actually have counter-intuitive results: namely, banks with too much risk may actually become riskier as a result of stricter capital requirements. Kim and Santomero (1988) extend Koehn and Santomero (1980) to include capital requirements that account for the riskiness of the bank's portfolio. Hendricks and Hirtle (1997) provide an overview of risk-based capital requirements and present a case in favor of banks using internal models to determine regulatory capital, citing that they should "conform more closely to banks' true risk exposures".

The Basel Accords, both I and II, created minimum capital adequacy standards for banks. The standards were based on market risk; specifically, the riskiness in the market value of the assets.[3] One of the major differences lies in the definition of how to determine the risks. There are many critiques of the capital adequacy standards set forth in Basel and with capital adequacy standards in general. We include only the subset most relevant to our paper here. Thakor (1996) argues that that risk-based capital requirements can potentially lead to credit rationing. Hellmann, Murdock, and Stiglitz (2000) also find that capital requirements, by themselves are insufficient to mitigate bank bankruptcy risk, and that instead they can lead to the perverse result where banks gamble. Estrella, Park, and Peristiani (2000) claim that simple capital ratios (specifically, the leverage ratio and gross revenue ratio) are as good or better at predicting bank failure than risk-weighted Basel-type metrics. Altman and Saunders (2001) examine two aspects of Basel II risk-based capital regulation, but find that using agency ratings for the risk-weighting could lead to "cyclically lagging capital requirements" thereby possibly making the financial system less stable over time. Furthermore, they argue that the current risk-bucketing lacks granularity. As a substitute, they propose a revised weighting system that fits historical default loss statistics rather closely. Similarly, Krainer (2002) examines alternatives

[3]For background on Basel II please see Basel Committee on Banking Supervision (2001)

to risk-based capital requirements (i.e. full-reserve banking). He proposes a corporate governance system that reduces the agency problem between creditors/depositors and stockholders. The optimal contract allows creditors/ depositors to "offset" risk shifting decisions that may benefit the bank's shareholders at the creditors' expense.

Herring (2004) takes a different approach. The paper claims that Basel II standards and credit risk management best practices cannot be aligned, and offers an alternative to Basel II: the mandatory issuance of subordinated debt. Kashyap and Stein (2004) examine the Basel II Internal Ratings Based (IRB) capital requirements and the effect that it may have on cyclicality in the financial system. They find that the Basel capital requirements may exacerbate business cycle effects.[4]

Jarrow (2007) offers an even stronger critique of the Basel II capital requirements; one that is based on Basel's lack of concern for credit risk. He argues that Value at Risk does not account for credit risk appropriately. Kashyap, Rajan, and Stein (2008) call for a complete overhauling and even rethinking of bank capital regulation, and propose alternatives including "capital insurance".

In light of the recent credit crisis and failure of many banking institutions, perhaps a rethinking of how regulators might set minimum capital standards, especially ones that take into consideration credit risk, is in order. The first to consider such a problem is Ronn and Verma (1989). Using an option pricing model, they focus on deposit insurance, and solve for capital infusion that will lower deposit insurance premium to a specified level. Our approach differs from Ronn and Verma (1989) in that we endogenously solve for the market value of debt (they use book value), our model is dynamic, multi-period with sequential cash flows (theirs is a single period, static model), and our model uses the bank's default probability to find the risk-based capital requirement (theirs uses deposit insurance).

[4]Interestingly, Pennacchi (2005) argues that risk-based capital requirements are inclined to yield greater procyclical effects than risk-based deposit insurance. He, therefore, suggests that Basel II incorporate some elements of risk-based deposit insurance to help moderate the procyclical nature of the existing framework.

2.1 Credit Risk Models: Theory and Applications

Since we employ a credit-risk-based model of capital adequacy our paper is clearly related to the credit risk literature. Generally speaking, there are two classes of credit risk models: reduced-form and structural models. The former define default purely in statistical terms where it is modeled as a jump process with a given intensity, or hazard rate. Reduced-form models are calibrated to market data and typically do not take any firm-specific information into account. In this framework default is *unpredictable* and the default time is said to be "inaccessible". Popular reduced-form models include Jarrow and Turnbull (1995), Duffie and Singleton (1999), Jarrow, Lando, and Turnbull (1997), and Duffie and Lando (2001).

Our model falls into the structural class. Structural credit risk models view default as an economic event where firm value declines to a level that is too low to justify servicing outstanding debt obligations. Structural models originated with Black and Scholes (1973) where, in their seminal option pricing paper, they noted that when a firm has debt in its capital structure equity is like a call option on the firm's unlevered assets. This idea was later formalized by Merton (1974). In the original BSM framework default can only occur at maturity. If the value of the firm's assets is not greater than the face value of the debt then shareholders choose to let the call option expire (i.e. default) and bondholders do not receive their promised payment but rather take ownership of the assets. If, however, the value of the firm's assets is greater than the face value of the debt then shareholders liquidate the assets and use the proceeds to pay bondholders with the retaining the residual amount for themselves. The probability that the call option is not exercised at maturity can be thought of as the default probability.

Due to the oversimplifying assumptions that underlie the original BSM framework, the decades that followed would generate many extensions looking to incorporate more realistic features of debt. One such class of extensions, commonly referred to as "barrier" structural models, was pioneered by Black and Cox (1976). In their paper Black and Cox (1976) develop an analytical model for valuing corporate debt with certain indenture agreements that can result in default occurring before maturity.

Specifically, they examine "safety covenants" – contractual provisions that stipulate the conditions that force restructuring. In the model there is a lower boundary on firm value, below which the firm is considered "bankrupt"; once the asset value hits this barrier debtholders take control of the firm's assets. Longstaff and Schwartz (1995) develop a model that is a direct extension of Black and Cox (1976). The Longstaff and Schwartz (1995) model includes stochastic interest rates with a flat, exogenous barrier. Collin-Dufresne and Goldstein (2001) incorporate mean-reverting leverage ratios into an exogenous barrier model. Leland (1994) derives a barrier structural model with *endogenous default*. The model solves for both the optimal capital structure and the price of risky debt in the presence of taxes and bankruptcy costs. Leland and Toft (1996) extend the Leland (1994) model to take debt maturity into account as well.

Distinct from the barrier models is the compound option model of Geske (1977).[5] In the Geske model, shareholders own a *compound* option on the firm's unlevered assets. Every time a cashflow is due to bondholders the shareholders must decide whether to make the payment, effectively exercising the compound option, or default giving the bondholders (who are the writers of the compound call option) ownership of the firm's assets. Default is determined endogenously as a function of the liability structure and the associated promised cashflows. The Geske model is described in much greater detail below (see Section 4.1).

Structural models are commonly used for predicting default as they naturally provide quantitative measures of default probabilities. In the barrier structural models, default probabilities are given by the first passage time density. In the compound option model, default probabilities are found by calculating the probability that the compound option is not going to be exercised at a particular future cash flow time. This allows us to calculate both the conditional and unconditional default probabilities and actually results in a term structure of default probabilities.

[5]The Geske compound option model is also derived in a no-arbitrage setting in Geske (1979) where a "leverage effect" is shown to result in nonconstant volatilities. The original Geske (1977) model was also corrected by Geske and Johnson (1984) to properly account for the seniority structure of debt.

The KMV model, as described by Crosbie and Bohn (2001), is a modified version of the original BSM model that is very popular in practice. Using market data as well as information from a firm's balance sheet the KMV model calculates expected default frequencies and the distance to default. Leland (2004) looks at default probabilities calculated using two structural models – the *endogenous barrier* model of Leland and Toft (1996) and the *exogenous barrier* model of Longstaff and Schwartz (1995) – and observes how well the predictions fit historical default frequencies. Delianedis and Geske (2003) compute risk-neutral default probabilities (RNDP) using the original BSM model and the Geske model. They draw two interesting conclusions that have very important implications for our model: first, they show that RNDPs serve as an upper bound to risk-adjusted default probabilities (for both models) and, second, the Geske model is able to provide important information that other structural models cannot; specifically, it provides a full term structure of default probabilities. Bharath and Shumway (2008) compare the distance to default from the BSM/KMV model with forecasts obtained using a hazard model. Structural credit risk models can also be used to analyze distress in the financial sector. Mason (2005) uses a real options approach to model bank failure as the trustee's optimal decision to liquidate.

3 Methodology

Our approach works as follows. First, we explicitly document the time and dollar amount of the liability cash flows. This often requires the complete identification of over a thousand different bonds with their associated coupon and redemption payments over time. We note that while we had to hand collect the data, the efficiency of this step can be greatly improved by regulators by requiring that financial institutions provide a detailed breakdown of their liabilities on a weekly, or in the case of a financial crisis daily, basis to the regulator. This should not be particularly onerous for the Financial Institutions given that they need to collect the data for internal risk management purposes.

Second, we estimate the market value and volatility of the firm's equity. In this paper, we use the most recent price of the financial institutions traded stock and multiply by the outstanding number of shares to calculate the market capitalization of the firm as our estimate of market value of equity. For an estimate of equity volatility, we use the historical standard deviation based on the most recent 30, 60, and 90 days of traded stock prices. The latest price and the most recent estimate of volatility allows for the use of measures that incorporate the most relevant information to estimate the financial health of the financial institution. In dramatically changing circumstances such as those that exist during a financial crisis, measures that portray a more stable environment can be misleading.

Third, we construct the default boundary and the term structure of default probabilities. As in the original Geske (1977) model, default is triggered when the value of the financial institution's assets fall sufficiently low so as to prevent it from raising new equity capital. The default boundary is therefore defined as the breakeven point, or the asset value that sets the current market value of the firm's equity equal to the next cash flow obligation. The default probabilities, in conjunction with the endogenous default boundary, provide an accurate and complete picture of the survivability of the financial institution over both the short-term and the longer-term.

We begin our analysis in January 2008 when Lehman Brothers was in little danger of default and then evaluate the firm monthly until September when Lehman Brothers declared bankruptcy. We collect data on the outstanding debt of Lehman Brothers from the FactSet database. The data in FactSet includes the CUSIP number, the total face value of the issue, the issue date, the coupon rate, and the maturity date. Using the Fixed Income Explorer in Factset, we collect all of these data items for each and every bond as well as the seniority, redemption options, and credit ratings when available. We identify several thousand bonds issued by Lehman Brothers and carefully document the outstanding amounts, the coupon rates and maturity of these bonds taking into account the possibility that the bond has been retired. We supplement the data on Lehman Brothers debt structure with data from the firms financial statements (annual income statements from 1998 to 2007, quarterly balance sheets from 3Q.2005 to 4Q.2007, and quarterly cash flows from 3Q.2005 to 4Q.2007). These data sources allow us to specify in detail the outstanding cash flow obligations of Lehman Brothers at different horizons. We also estimate the market capitalization of Lehman Brothers at the end of each month and the historical 30-day and 60-day volatilities.

We note that we do not include private debt and off-balance sheet obligations of Lehman Brothers because of a lack of data on these obligations. We offer two arguments on this front. First, our estimates are necessarily a lower bound on the true level of default risk. In order to estimate the impact of ignored cash-flow obligations, we re-estimate our model using ad-hoc assumptions on margin requirements on Lehman's off-balance sheet activities and the level of private debt. As expected, the level of default risk rises substantially when additional cash-flow obligations are incorporated into the model. Second, we note that regulators can demand access to these data and can easily incorporate the cash-flow liabilities in any real-world implementation of our model. Indeed, one advantage of our model is the ease with which additional cash-flows can be incorporated into the analysis when such cash-flow obligations are deemed to be relevant by the regulators.

4 Model Framework

In this section we discuss our dynamic lattice based structural model for estimating default risk in financial institutions. Structural models are especially well-suited for managing and monitoring credit risk, either internally or externally, as they use the most recent inputs from financial statements and market data. Our approach is based on the compound option pricing model as developed by Geske (1977) (henceforth Geske). We first review the Geske model and the related Geske and Johnson (1984) model. We then present our lattice implementation of the Geske model.

4.1 Geske model and Endogenous Default

Structural option pricing methods have been used extensively in the banking literature to quantify and price deposit insurance (see e.g. Ronn and Verma (1989)) and in the credit risk literature to estimate default risk (see e.g. Crosbie and Bohn (2001)). Black and Scholes (1973) and Merton (1974) pioneered the notion that when a firm has risky debt outstanding, the equity is very much like a call option where shareholders are faced with the decision to exercise when payment is due to debtholders. Upon maturity of the debt shareholders can choose to make the payment, effectively exercising the call option or default by filing for bankruptcy and letting the call option expire unexercised. In the Merton (1974) approach, the liabilities are modeled as a fixed point barrier, hence the name barrier structural model, and there is only one future date in which the exercise decision is made. The firm has the option to default only at one point - on the final maturity date of the debt. The overly simplistic and exogenous specification of the default boundary has limited the application of the Merton approach to estimating default risk, especially for financial institutions.

The Geske (1977) model incorporates multiple cash flows and allows for default at each time that a cash flow, either a coupon or a face value payout, is due. The Geske approach thus fundamentally differs in the way it models the default boundary and the conditions under which the call option is exercised by the shareholders. First, the Geske approach relaxes the fixed point barrier rendering of the default boundary to allow for multiple sequential cash flows. Exercising the option to pay a cash flow due to bondholders delivers to the shareholders a sequence of options corresponding to the multiple cash flows over the maturity structure of the various components of debt. Second, the compound option specification evaluates the decision to default at every cash flow period. Consequently, with the compound option structural model each and every cash flow is important and the endogenous default barrier is explicitly a function of these cash flows. Geske (1979) and Geske and Johnson (1984) extended the basic framework to incorporate a "leverage effect" that causes volatility to change with leverage and account for the seniority structure of multiple debt claimants. The model is thus especially suited for analyzing default risk in financial institutions and for estimating bank capital.

Our model is a discrete time implementation of the Geske approach. The endogenous default boundary does make obtaining an analytical solution difficult; we, however, develop a lattice approach to estimating the default boundary and asset values. Our discrete time implementation directly solves for the endogenous boundary without the need for iterative calculations.

The Geske (1977) model and its extensions make several assumptions regarding the conditions that trigger default, the sequence in which various options are exercised, and how the equity holders raise the funds needed to exercise the call option and pay the debtholders. First, the model assumes that the ability to make a cash payment and the requirement that the market value of the assets is greater than the market value of future liabilities is important in order to exercise the call option. The default boundary in these models is therefore *endogenous* and depends on the market value of debt and the market value of assets. Second, the model assumes that earlier payments are more senior to later payments. This naturally leads to the assumption that bonds with shorter maturities have priority over long maturity bonds, which is consistent with short-term debt being more important in triggering credit constraints in practice. Finally, the model assumes that all payments to bondholders are financed by issuing new equity. This implies that the firm will ultimately become all equity. As a result the default risk estimated by the model is conservative and is contingent on the firm being able to raise equity in order to remain solvent. This approach arguably is the correct approach in the context of financial institution regulation. The focus is on identifying firms that have an unacceptably high theoretical default risk and require these firms to restructure by issuing more equity or restructuring its debt in order to reduce leverage and the associated default risk.

4.2 n-Period Geske Model

Let the firms assets evolve according to a diffusion process with dynamics described by the Stochastic Differential Equation:

$$\frac{dA_t}{A_t} = \mu_t dt + \sigma_t dW_t \tag{1}$$

Let K_k denote the current value of a cash flow paid at time T_k. We note that the value of the cash flow at T_k is also a function of all the cash flows paid prior to time T_k, that is $K_1 \cdots K_{t-1}$. Let A represents the value of the firm's assets, D represents the market value of the firms debt, S represents the value of the firm's equity, r represents the risk free rate, and σ represents the volatility of the firm's assets.

$$\begin{aligned} D(0,T_k) = \textstyle\sum_{i=1}^{k} e^{-rT_i} K_i[N_i(h_1^-(\bar{A}_{1k}), \cdots, h_i^-(\bar{A}_{ik})) - N_i(h_1^-(\bar{A}_{1k-1}), \cdots, h_i^-(\bar{A}_{ik-1}))] \\ +A_0[N_{k-1}(h_1^+(\bar{A}_{1k}), \cdots, h_{k-1}^+(\bar{A}_{k-1k-1})) - N_k(h_1^+(\bar{A}_{1k}), \cdots, h_k^+(\bar{A}_{kk}))] \end{aligned} \tag{2}$$

where,

$$h_i^{\pm}(\bar{A}_{ij}) = \frac{\ln A(0) - \ln \bar{A}_{ij} + (r \pm 0.5\sigma^2 T_i)}{\sigma\sqrt{T_i}} \tag{3}$$

$\bar{A}_{ij}$ for $i < j$ is the internal solution for $A(T_i)$ to the following equation:

$$A(T_i) = \sum_{k=i}^{j} D(T_i, T_k) \tag{4}$$

Note that $D(T_i, T_i) = K_i = \bar{A}_{ii}$ and that $N_0(.) = 1$ and $N_1(.) = 1$ are i-dimensional Gaussian probability functions.

We need to solve for $\bar{A}_{ij}$ sequentially in a process similar to bootstrapping. We start the second to last period, $k-1$. There is one critical value to solve: $\bar{A}_{(t-1)k}$ which is the solution to:

$$A_{k-1} = K_{k-1} + D(T_{k-1}, T_k) \tag{5}$$

where,

$$D(k-1,k) = A_{k-1}\left(1 - N_1\left(\frac{\ln A_{k-1} - \ln K_k + (r+0.5\sigma^2)\Delta t}{\sigma\sqrt{\Delta t}}\right)\right) + e^{-r\Delta t} K_k N_k\left(\frac{\ln A_{k-1} - \ln K_k + (r+0.5\sigma^2)\Delta t}{\sigma\sqrt{\Delta t}}\right) \quad (6)$$

This gives the solution to $A_{(k-1)k}$ as a function of K_{k-1} and K_k and other parameters such as risk free rate and volatility. Then we move backwards one period to $k-2$. In this period, we need to solve for $A_{(k-2)(k-1)}$ and $A_{(k-2)k}$. The former $(A_{(k-2)(k-1)})$ can be solved the same way as $\bar{A}_{(k-1)k}$ since it is a one period bond as a function of K_{k-2} and K_{k-1}. The latter $(\bar{A}_{(k-2)k})$ is more complex. It is a solution to:

$$A_{k-2} = K_{k-2} + D(T_{k-2}, T_{k-1}) + D(T_{k-2}, T_k) \quad (7)$$

where,

$$D(T_{k-2}, T_k) = e^{-2r\Delta t}\int_{\bar{A}_{k-1k}}^{\infty} E[\min\{A_k, K_k\}] + e^{-r\Delta t}\int_{K_{k-1}}^{\bar{A}_{k-1k}} E[A_{k-1} - K_{k-1}] \quad (8)$$

and hence $A_{(\bar{k}-2)k}$ is a function of K_{k-2}, K_{k-1}, and K_k as well as $A_{(\bar{k}-1)k}$ which must be solved first. As we work backwords in the lattice, all the critical values $\bar{A}_{ij}$ for $i < j$ are solved.

The total value of debt is:

$$\sum_{k=1}^{n} D(0, T_k) = A(0)[1 - N_n(h_1^+(X_{1n}), \cdots, h_n^+(X_{nn}))] + \sum_{i=1}^{n} e^{-rT_i} K_k N_n(h_1^+(X_{1n}), \cdots, h_n^+(X_{nn})) \quad (9)$$

The quasi-closed form solution for the n-period Geske model involves n-dimensional cumulative normal distribution functions. Since this cannot be computed analytically we have to use a numerical methods. We use a binomial lattice method to implement the Geske model along the lines of Eom, Helwege, and Huang (2004).

4.3 The Lattice Model

The Geske (1977) model involves an endogenous default boundary and we develop a lattice approach to implement the model. Consider a 6-period binomial lattice model for the value of the firm's assets as shown in Figure 1. The shareholders of the firm have three exercise decisions to make, i.e. there are three cash flows K_1, K_2, and K_3 that they have to make to debt holders. K_3 is the debt face value at the final node and K_1 and K_2 are debt cash flows at the intermediate nodes. As shown in the figure, there is an intermediate node In between any two of the cash-flows.

The option to default can be exercised at the nodes a cash flow payment is due. That is, we model bankruptcy as the option shareholders have to not make a payment to the debt holders and continue the firm for at least the time interval to the next cash flow. The decision to exercise the option to default takes into account the relative value of the assets and the debt of the firm and, as discussed above, is equivalent to a call option. If the shareholders choose not to make a payment, the firm is in default and the *default boundary* refers to the asset value for which the shareholders will exercise the option to default at different points in time in the lattice. Equivalently, the default boundary represents the "cut off" asset values below which the firm is in default at each time t. We estimate the default boundary, debt value, and equity value by backward recursion in the tree.

An important assumption we make is the source of the funds shareholders use to meet the cash flow obligation and keep the firm alive until the next cash flow period. Following Geske (1977) we assume that the shareholders issue new equity at the current market price to raise the required capital. This assumption simplifies the calculations and has several implications which we believe makes it especially suitable to the setting of highly levered financial institutions. First, the transaction reduces the amount of debt and increases the amount of equity as time evolves, thereby decreasing the default risk over time. The largest default risk, therefore, arises from meeting the next cash flow payment. This is typically true of financial institutions that rely heavily on trust and reputation in the marketplace; failing to make a near-term cash flow obligation has disastrous consequences. Second, the model will be conservative in estimating default probabilities and unlikely to imply onerous capital requirements for financial institutions with longer-term cash flow obligations rising from debt issues, which can usually be refinanced.[6]

Consider the final nodes at the end of the lattice. On the date of the final contractual cash flow, i.e. terminal time T, the firm liquidates. Since equity is a call option on the assets, the value to equity holders and the optimal exercise decision is given by,

$$E_T = \max\{A_T - K_3, 0\} \tag{10}$$

where E_T is the equity value at the terminal time T , A_T is the asset value at the terminal time T and K_2 is the final redemption value of debt. In the above diagram, the solid dots represent economic states where the equity has positive value and those hollow dots represent those states where the equity has no value.

Moving backwards along the lattice, the equity value is computed as the risk neutral expectation of the values in the next period, that is $E_t = \mathbf{E}_t[e^{-r\Delta t}E_{t+1}]$ for any t in between any two cash flows.

Consider the second-to-last cash flow K_2. At this time, the firm decides if it should pay down the debt with equity. If the equity value is more than the cash obligation, then it is rational for the firm to pay down the debt and continue to survive; otherwise, the debt holders will seize and liquidate the firm, and the equity becomes worthless. That is: $E_t = \max\{\mathbf{E}_t[e^{-r\Delta t}E_{t+1}] - K_2, 0\}$

There are two important quantities we compute in the lattice. One is the default boundary and the other is the survival (default) probability. The default boundary is the critical value above which the asset value must stay in order for the firm to remain alive. In the Geske (1977) notation, it is $A_{i,i+1}$, which is the highest critical value at any given time i. If the asset price at time i, A_i, falls below this value the equity would become worthless. Note that at the critical value the total value of

[6] We can easily modify the assumption and have the firm issue debt to finance the cash flow payment. This is the assumption underlying the Leland and Toft (1996) model. The model essentially implies a flat default boundary and does not take into account the potential to reduce default risk by raising capital when needed.

outstanding debts is equal to the asset value. Above this critical value, the firm can issue new equity to pay down the debt due at time i. The above definition of default boundary is effective for all i.

The other quantity of great importance is the survival probability. In the original Geske (1977) model, as well as in our model, the survival probability is the probability that the asset value stays above the default boundary. That is,

$$Q(0,i) = \Pr(A_1 > \bar{A}_{12} \cap A_2 > \bar{A}_{23}, \cdots, A_i > \bar{A}_{ii+1}) \tag{11}$$

This joint probability is easy to compute within the lattice. We simply trace each path in the lattice and count only those that survive. Note that the default probability between any two cash flow periods is:

$$Q(0,i-1) - Q(0,i) \tag{12}$$

This means that to compute the i^{th} default probability, the firm must survive until year $i-1$ and then default in year i. The associated recovery values are rather difficult to obtain in an integration format in the original Geske and yet is rather easy to obtain in the lattice. Similar to the computation of the survival probability, we trace defaults along the lattice. We then compute the expected value of the assets, equity, and debt.

4.4 Numerical Example

To demonstrate with a numerical example, consider a firm with annual cash flows 10, 20, and 75 at $t = 1, 2, 3$ respectively. There is one intermediate node between cash flows, i.e. $\Delta t = 0.5$,. Let $A_0 = 300$, $\sigma = 10\%$, $r = 3\%$, . Using the standard binomial model, we have: $u = e^{\sigma\sqrt{\delta t}} = 1.0733$ and $d = e^{-\sigma\sqrt{\delta t}} = 0.9317$. The probabilities are $p = \frac{e^{r\Delta t} - d}{u-d} = 0.5891$ and $1 - p = 0.4109$.

Figure 2 shows the asset value binomial tree. Figure 3A, 3B, and 3C respectively shows the three time periods, working backwards from the last time step, representing the equity value taking into account the optimal decision to not make the debt payment and declaring bankruptcy (shown as zero values in the lattice).

From Figure 3, the value of equity at $t = 0$ is equal to 27.40. Since asset value is \$300, the resulting debt value is 272.60. To compute the survival probability, we trace survival through the lattice, i.e. wherever the value of equity is greater than 0. For the first cash flow, survival occurs at the asset values of 57.57 and 17.06. The probability is equal to $\pi^2 + 2\pi(1-\pi)$ and is equal to 83.12%. Similarly, in the second year, the survival probability is 74.94%, and 70.00% in the third year. The default probabilities are therefore 16.88%, 8.18%, and 4.94% in Years 1, 2, and 3 respectively.

The default boundary is also obtained by tracing the lattice. For example, in Year 1, default occurs in the bottom state, i.e. the state with the lowest asset value. Hence, the default boundary must in between 300 and 260.44. We note that this seems to be a rather large range, but as the time period (and step size) shrinks the gap will narrow and converge. We assume that the firm defaults at the average of the asset values where the firm just survives and the firm just defaults, e.g. the bottom two nodes in Year 1. In Year 2, the bottom two states default and top three states survive. Hence, the default boundary falls in between 300 and 260.44. In Year 3, the default boundary falls between 300 and 260.44, but this time we know for sure it is 275, the last coupon. The default boundary curve is therefore: 280.22, 280.22, and 275 at Year 1, 2, and 3 respectively.

To compute the expected recovery value of the i^{th} bond (cash flow), we need to compute $\mathbf{E}_0\left[e^{-ri}A_i I_{A_1>\bar{A}_{12}\cap A_2>\bar{A}_{23}\cdots A_{i-1}>\bar{A}_{i-1i}\cap A_i<\bar{A}_{ii+1}}\right]$. For the three years, we obtain the following values from the lattice: 42.67 that is composed of 1.64 of the first bond (out of face value 10), 3.18 of the second bond (out of face value 20), and 37.85 (out of face value 275). Note that the survival probability of the first year is 83.12%. Hence, for the first bond, its coupon value is: $\$10 \times 0.8312 \times \exp(-5\%) = \$8.07$. Combining this with its recovery value gives a total value of: $\$8.07 + 1.64 = \$9.71$, which is effectively a risk free bond. Similarly, the second bond has a coupon value of 14.12 and a recovery value of 4.72. Combining these together gives a value of 18.84, which is also a risk free bond. We also note that for the second bond the recovery value of

4.72 comes from 3.18 in the first year and 1.54 in the second year, if the shareholders default. Finally, for the last bond (face value 275), he first year has a recovery of 37.85, the second year has a recovery of 18.51, and third year has a recovery of 11.77, for a total recovery value of 68.13. The coupon value is 175.92. Therefore, the total bond value is 244.05. In terms of yield, the third year bond has a yield-to-maturity of 3.98% yield, showing a spread of 98 basis points.

5 Lehman Brothers: An Anatomy of a Financial Crisis

We implement our approach to analyze the default risk for Lehman Brothers leading up to the firm's failure. Lehman Brothers was a major financial institution that experienced severe financial problems disastrous enough that the Wall Street powerhouse was forced to file bankruptcy. Figure 4 shows a time line of events at Lehman Brothers. In this section, we show that our model accurately predicts the substantial increase in default risk for Lehman Brothers over the first few months of 2008 and we examine whether capital infusions alone could have prevented bankruptcy.

5.1 The Liabilities

We compile a comprehensive data set representing the liabilities of Lehman Brothers using the FactSet database. FactSet aggregates financial data from various sources which allowed us to analyze the firm's capital structure at extremely detailed levels, specifically looking into the debt profile of Lehman Brothers. In particular, FactSet allows us to collect data on every single debt issuance at any given point in time. The dataset includes basic information of the bonds in FactSet such as the CUSIP number, the total face value of the issue, the issue date, the coupon rate, and the maturity date. In addition, it also includes other detailed information such as the "Status" (matured/redeemed/active), redemption options (callable/putable/convertible), the redemption date (where applicable), the ratings as per Moodys and S&P, the coupon type (fixed/floating/etc.), and the seniority from the Fixed Income Explorer. This specific data regarding the debt structure was supplemented with data from the firms financial statements (annual income statements from 1998 to 2007, quarterly balance sheets from 3Q.2005 to 4Q.2007, and quarterly cash flows from 3Q.2005 to 4Q.2007). This permits us to study Lehman Brothers through September 2008, right before it filed for bankruptcy.

We collate the debt of Lehman by calendar year and estimate the dollar value of Lehman's debt in each calendar year. Table 3 details the notional amount of the liabilities maturing in each calendar year as of January 2008. As seen in the table, Lehman has substantial amount of debt maturing in the short term in 2008 and 2009. The table also shows other details of the debt outstanding.

Figure 5 displays the debt maturity structure as of each month in 2008. The figure shows the notional debt value for each month. As the figure shows, short-term debt maturing in one to three years and long-term debt with maturities greater than or equal to 20 years dominate in Lehman's liability structure. This is typical for a financial institutions that use short-term debt for working capital needs and finance their core activities using long-term debt. Figure 5 also shows that the short-term debt increased dramatically in March of 2008, around the time that the market realized that Lehman was in financial trouble and did not have enough assets to meet its financial liabilities.

Figure 5 presents the level of debt at the beginning of each month in 2008. Both the time at which the liability structure is determined and the time period over which different tranches of debt are collated can be changed. For example we can calculate the debt profile every week or every fortnight and collate the liabilities over six-month windows. Our choice here represents a compromise between the level of detail and clarity of presentation and does not affect any of our results.

5.2 Market Value Inputs

Our model uses two market-determined inputs for calculating the default probability of Lehman Brothers. The first is the market value of Lehman's equity and the second is the volatility of stock returns on Lehman's.

Figure 6 shows the book value and the market value of Lehman Brothers. The market value of equity is calculated as the product of the closing stock price on the estimation day and the number of shares outstanding on that day. The book value of equity is as reported by COMPUSTAT. As the figure shows, the market value of equity had a precipitous decline over 2008. In February the equity value dropped from \$35.1 billion to \$26.8 Billion, a drop of 23.6%. As Figure 4 shows, the financial crisis reflected in this dramatic drop in equity value led Lehman to raise additional funds in March 2008. Lehman obtained a \$2 billion 3 year credit line from a consortium of 40 banks including JPMorgan Chase and Citigroup. The infusion stabilized Lehman over the next two months; however a reported loss of \$2.8 Billion in May led to a further stock price decline of 27.9% in May as shown in the figure.

Figure 7 shows the historical 30-day stock return volatility for Lehman. We calculate the volatility as the annualized standard deviation of Lehman's daily stock returns over the prior 30-day period. We use the most recent period to focus on the most recently available market information of Lehman's stock returns. As Figure 7 shows, the volatility estimate spiked in March 2008 reflecting the increase in uncertainty in February.

5.3 Default Probability

We use the hand collected data on the liability structure, the equity market value, and the volatility estimates to determine the endogenous default boundary for Lehman Brothers at the beginning of each month for the period from January 2008 to September 2008.

Figure 8 shows the market value and book value of Lehman's debt. The book value of debt is the cumulative dollar value of Lehman's debt from FactSet. The market value of debt is as estimated by our model. As Figure 8 shows, the market value of debt is substantially lower than its book value indicating a high default risk premium for the debt.

Figure 9 shows our model estimates of the 2-year default probability. Default probability spiked in March 2008 to well over 50% after staying low in January and February of 2008. Thus, our model predicts the dramatic rise in default risk for Lehman Brothers, well before the September 2008 bankruptcy filing. The short-term funds obtained by Lehman did lower the default risk substantially in May and June. In fact, the default probability was lower than the default probability in April, possibly because the market anticipates a government intervention and a rescue package.

Figure 10 shows the cumulative default probability over time for Lehman Brothers at the beginning of each month in 2008. As seen in the figure, the cumulative default probability is upward sloping with a steep slope in the initial period and the default probability quickly reaches the highest value. The picture of cumulative probability is typical for the Geske model and the intuition behind the behavior of cumulative default is as follows. Financial institutions are characterized by having large amounts of liabilities, a substantial amount of which typically matures in the short term. The present value of debt is therefore high and in adverse market conditions a financial institution faces higher default risk in the short term. The initial rise in default probability captures the degree of financial distress faced by the financial institution. If the financial institution does not default in the initial period, the model assumes that the firm raises equity to meet the debt obligation. This lowers the default probability in the future periods and results in the cumulative default probability that levels off in the future periods. We note that the results in Figure 10 reinforce the results from Figure 9 in that the default risk is the highest for Lehman during March 2008 when the firm faced financial distress and there was a high degree of uncertainty regarding whether the government would play a role in supporting financial institutions.

6 Managing Default Risk

In this section, we discuss the implications of our model and present how our model can be used to manage default risk in financial institutions using the case of Lehman Brothers as an illustration.

The pattern of default probability over time show in Figure 10 has important implications for the management of default risk in financial institutions. As shown in the figure, default risk rises initially and levels off for longer term. The pattern of default probability is a function of the liability structure of financial institutions and the assumptions made by the model on capital structure changes when liabilities come due. We argue that these features are important in measuring and managing default risk in financial institutions. Financial institutions are saddled by a large amount of financial liabilities and a substantial fraction of these liabilities are due in the short-term. It is therefore not surprising that the per-period default is the highest in the initial period and is lower in the later periods. Further, the model's assumption that debt is retired using equity further mitigates default risk in later periods.

Our model indicates that the largest threat to the survival of a financial institution is in the short term. Regulators have to be especially vigilant when the short-term default risk is unusually high, as for example when the default risk for Lehman brothers rose to over 50% in April 2008.

We next examine how default risk changes for varying capital structure. Figure 11 graphs default risk as a function of volatility for two different levels of equity capital in April 2008. The top line shows default risk for Lehman as existed with a market value leverage ratio of 72.7%. The bottom line shows the default probability at a market value leverage ratio of 10.4%. As Figure 10 shows, given market conditions as of April 2008, Lehman would need to dramatically lower leverage in order to reduce default risk to acceptable levels. In other words, the elasticity of default probability with respect to market value leverage is fairly low, indicating that default risk can be lowered substantially only by large infusions of equity capital.

The implications for managing default risk in financial institutions are profound. Our model validates the Federal Reserve's focus on the short-term viability of financial institutions and provides tools that the Fed (and other regulators) can use to evaluate institutions in dramatically changing market conditions. However, the model also predicts that in adverse market conditions, tweaking the firm's capital to reach the magic 8% ratio required by the Basel II accord, would not result in substantially lowering default risk. Regulators and management at financial institutional have to fundamentally rethink the operating model when the firm is in a crisis.

7 Conclusions

In this paper we develop and present a new approach to estimating the default risk of Financial Institutions. Our model is based on the Geske (1977) framework for valuing compound options and allows for the accurate estimation of default risk, with an endogenous default boundary, and works for even complex portfolio of liabilities as is typical for financial institutions. Our model can be readily applied to estimate the default risk of financial institutions and gives regulators a powerful dynamic tool that reflects current market conditions in managing default risk of financial institutions.

While an options based approach using structural models has been advocated as a way to estimate default risk since the seminal work of Black and Scholes (1973) and Merton (1974), the application to financial institutions has several practical impediments. Traditionally, such structural models have used a single point equivalent of a firm's liabilities and have used exogenously imposed default barriers. Our model considers the entire liability structure and uses an endogenous default boundary to develop accurate estimates of default risk.

Our model uses the most recent market based inputs, specifically market value of equity and the volatility of stock returns to estimate default risk. We implement our model for the first few months of 2008 to the case of Lehman Brothers. We use hand collected data from FactSet to determine the firm's detailed liability structure and daily stock data to estimate market value of equity and stock return volatility. We show that default risk spiked to over 50% in April 2008, well before the bankruptcy filing by Lehman Brothers in September of 2008.

We show that financial institutions such as Lehman Brothers are characterized by rapidly rising default risk in the short term that then levels off over the longer term. The pattern of cumulative default probability has important implications for the regulation of financial institutions. Our model validates the Federal Reserve's focus on the short-term survival probability of a financial institution as the ability to meet the large short-term cash flows is the most critical. However we also show that in times of financial crisis when the short-term default risk is very high, a rather large level of capital infusion is required to increase survival probability. Small levels of capital infusions do not eliminate risk as illustrated by the case of Lehman Brothers: default risk remained high following the capital infusion in April 2008.

Our model represents a powerful predictive and diagnostic tool that will enable regulators to accurately estimate default risk and analyze the impact of any intervention. Moreover our model uses the most recent market data, as opposed to the static default risk measures that use book values. This is crucial when facing fast moving market conditions such as those that exist in the midst of a financial crisis.

References

Allen, Linda, and Anthony Saunders, 1993, Forbearance and valuation of deposit insurance as a callable put, *Journal of Banking and Finance* 17, 629–643.

Altman, Edward I., and Anthony Saunders, 2001, An analysis and critique of the bis proposal on capital adequacy and ratings, *Journal of Banking and Finance* 25, 25–46.

Basel Committee on Banking Supervision, 2001, *The New Basel Accord: an explanatory note* (Bank for International Settlements).

Bharath, Sreedhar T., and Tyler Shumway, 2008, Forecasting default with the merton distance to default model, *Review of Financial Studies* 21, 1339–1369.

Black, Fischer, and John C. Cox, 1976, Valuing corporate securities: Some effects of bond indenture provisions, *Journal of Finance* 31, 351.

Black, Fischer, and Myron Scholes, 1973, The pricing of options and corporate liabilities, *Journal of Political Economy* 81, 637.

Brunnermeier, Markus K., 2009, Deciphering the liquidity and credit crunch of 2007-2008, *Journal of Economic Perspectives* 23, 77–100.

Buser, Stephen A., Andrew H. Chen, and Edward J. Kane, 1981, Federal deposit insurance, regulatory policy, and optimal bank capital, *Journal of Finance* 36, 51–60.

Collin-Dufresne, Pierre, and Robert S. Goldstein, 2001, Do credit spreads reflect stationary leverage ratios?, *Journal of Finance* 56.

Crosbie, Peter J., and Jeffrey R. Bohn, 2001, Modeling default risk, (KMV, LLC, San Fransisco, CA).

Crouhy, Michel, Robert A. Jarrow, and Stuart M. Turnbull, 2008, The subprime credit crisis of 2007, *The Journal of Derivatives*.

Delianedis, Gordon, and Robert Geske, 2003, Credit risk and risk neutral default probabilities: Information about rating migrations and defaults, *EFA 2003 Annual Conference Paper No. 962*.

Diamond, Douglas W., and Raghuram G. Rajan, 2000, A theory of bank capital, *Journal of Finance* 55, 2431–2465.

Duffie, Darrell, and David Lando, 2001, Term structures of credit spreads with incomplete accounting information, *Econometrica* 69, 633–664.

Duffie, Darrell, and Kenneth J. Singleton, 1999, Modeling term structures of defaultable bonds, *Review of Financial Studies* 12.

Eom, Young Ho, Jean Helwege, and Jing Zhi Huang, 2004, Structural models of corporate bond pricing: An empirical analysis, *Review of Financial Studies* p. 499544.

Estrella, Arturo, Sangkyun Park, and Stavros Peristiani, 2000, Capital ratios as predictors of bank failure, *Economic Policy Review* 6.

Flannery, Mark J., 1998, Using market information in prudential bank supervision: A review of the u.s. empirical evidence, *Journal of Money, Credit and Banking* 30, 273–305.

Geske, Robert, 1977, The valuation of corporate liabilities as compound options, *Journal of Financial and Quantitative Analysis* 12, 541–552.

——— , 1979, The valuation of compound options, *Journal of Financial Economics* 7, 63–81.

——— , and H. E. Johnson, 1984, The valuation of corporate liabilities as compound options: A correction, *Journal of Financial and Quantitative Analysis* 19, 231.

Gorton, Gary, 2009, The subprime panic, *European Financial Management* 15, 10–46.

Hellmann, Thomas F., Kevin C. Murdock, and Joseph E. Stiglitz, 2000, Liberalization, moral hazard in banking, and prudential regulation: Are capital requirements enough?, *American Economic Review* 90, 147 – 165.

Hendricks, Darryll, and Beverly Hirtle, 1997, Bank capital requirements for market risk: The internal models approach, *Economic Policy Review* 3.

Herring, Richard J., 2004, The subordinated debt alternative to basel ii, *Journal of Financial Stability* 1, 137 – 155.

Jarrow, Robert A., 2007, A critique of revised basel ii, *Journal of Financial Services Research* 32, 1 – 16.

———, David Lando, and Stuart M. Turnbull, 1997, A markov model for the term structure of credit risk spreads, *The Review of Financial Studies* 10, 481–523.

Jarrow, Robert A., and Stuart M. Turnbull, 1995, Pricing derivatives on financial securities subject to credit risk, *The Journal of Finance* 50, 53–85.

Kareken, John H., and Neil Wallace, 1978, Deposit insurance and bank regulation: A partial-equilibrium exposition, *Journal of Business* 51, 413–438.

Kashyap, Anil K., Raghuram G. Rajan, and Jeremy C. Stein, 2008, Rethinking capital regulation, in *Maintaining Stability in a Changing Financial System* Jackson Hole, Wyoming. Federal Reserve Bank of Kansas City.

Kashyap, Anil K., and Jeremy C. Stein, 2004, Cyclical implications of the basel ii capital standards, *Economic Perspectives* 28, 18 – 31.

Kim, Daesik, and Anthony M. Santomero, 1988, Risk in banking and capital regulation, *Journal of Finance* 43, 1219–1233.

Koehn, Michael, and Anthony M. Santomero, 1980, Regulation of bank capital and portfolio risk, *Journal of Finance* 35, 1235–1244.

Krainer, Robert E., 2002, Banking in a theory of the business cycle: a model and critique of the basle accord on risk-based capital requirements for banks, *International Review of Law and Economics* 21, 413 – 433.

Leland, Hayne E., 1994, Corporate debt value, bond covenants, and optimal capital structure, *The Journal of Finance* 49, 1213–1252.

———, 2004, Predictions of default probabilities in structural models of debt, *Journal of Investment Management*.

———, and Klaus Bjerre Toft, 1996, Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads, *Journal of Finance* 51, 987–1019.

Longstaff, Francis A., and Eduardo S. Schwartz, 1995, A simple approach to valuing risky fixed and floating rate debt, *The Journal of Finance* 50, 789–819.

Mason, Joseph R., 2005, A real options approach to bankruptcy costs: Evidence from failed commercial banks during the 1990s, *Journal of Business* 78, 1523–1553.

Merton, Robert C., 1974, On the pricing of corporate debt: The risk structure of interest rates, *The Journal of Finance* 29, 449:470.

Pennacchi, George G., 2005, Risk-based capital standards, deposit insurance, and procyclicality, *Journal of Financial Intermediation* 14, 432–465.

Ronn, Ehud I., and Avinash K. Verma, 1989, Risk-based capital adequacy standards for a sample of 43 major banks, *Journal of Banking and Finance* 13, 21–29.

Santomero, Anthony M., and Ronald D. Watson, 1977, Determining an optimal capital standard for the banking industry, *Journal of Finance* 32, 1267–1282.

Santos, Jo A. C., 2001, Bank capital regulation in contemporary banking theory: A review of the literature, *Financial Markets, Institutions and Instruments* 10, 41.

Sharpe, William F., 1978, Bank capital adequacy, deposit insurance and security values, *Journal of Financial and Quantitative Analysis* 13, 701–718.

Shin, Hyun Song, 2009, Reflections on northern rock: The bank run that heralded the global financial crisis, *Journal of Economic Perspectives* 23, 101–119.

Thakor, Anjan V., 1996, Capital requirements, monetary policy, and aggregate bank lending: Theory and empirical evidence, *Journal of Finance* 51, 279–324.

Figure 1: Lattice model implementation of the Geske model

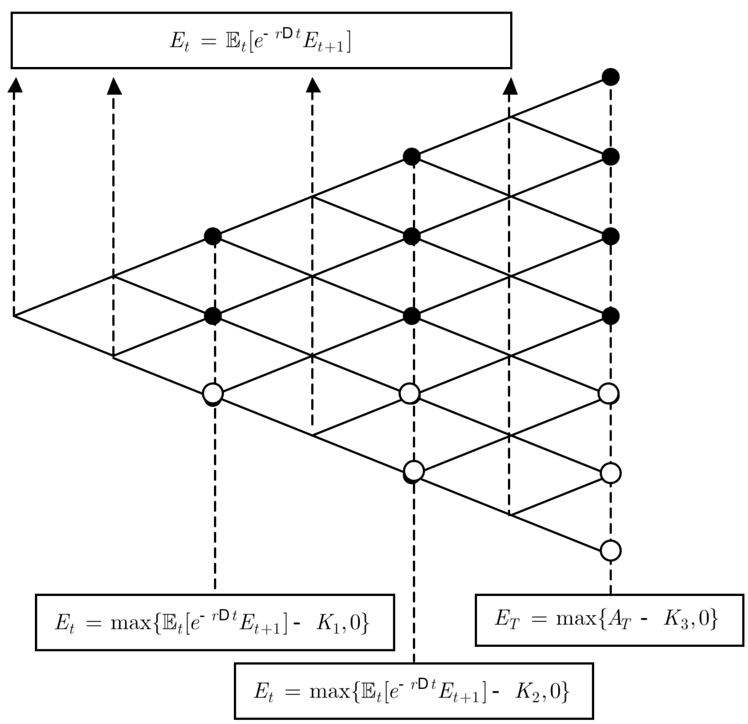

Figure 2: Numerical Example - 3-period Asset Binomial Tree

						458.54
					427.24	
				398.07		398.07
			370.89		370.89	
		345.57		345.57		345.57
	321.98		321.98		321.98	
300.00		300.00		300.00		300.00
	279.52		279.52		279.52	
		260.44		260.44		260.44
			242.66		242.66	
				226.09		226.09
					210.66	
						196.28

Figure 3: Numerical Example:
Equity Values in a Discrete 3-period Binomial Tree

Figure 3A

						183.54
					156.33	
				131.20		123.07
					99.99	
				78.70		70.57
					51.08	
				35.51		25.00
					14.51	
				8.42		0.00
					0.00	
				0.00		0.00
					0.00	
						0.00

Figure 3B

				111.20
			88.29	
		67.57		58.70
			40.35	
		27.06		15.51
			9.00	
		5.22		0.00
			0.00	
				0.00

Figure 3C

		57.57
	40.31	
27.40		17.06
	9.90	
		0.00

Figure 4: Lehman Brothers Time Line

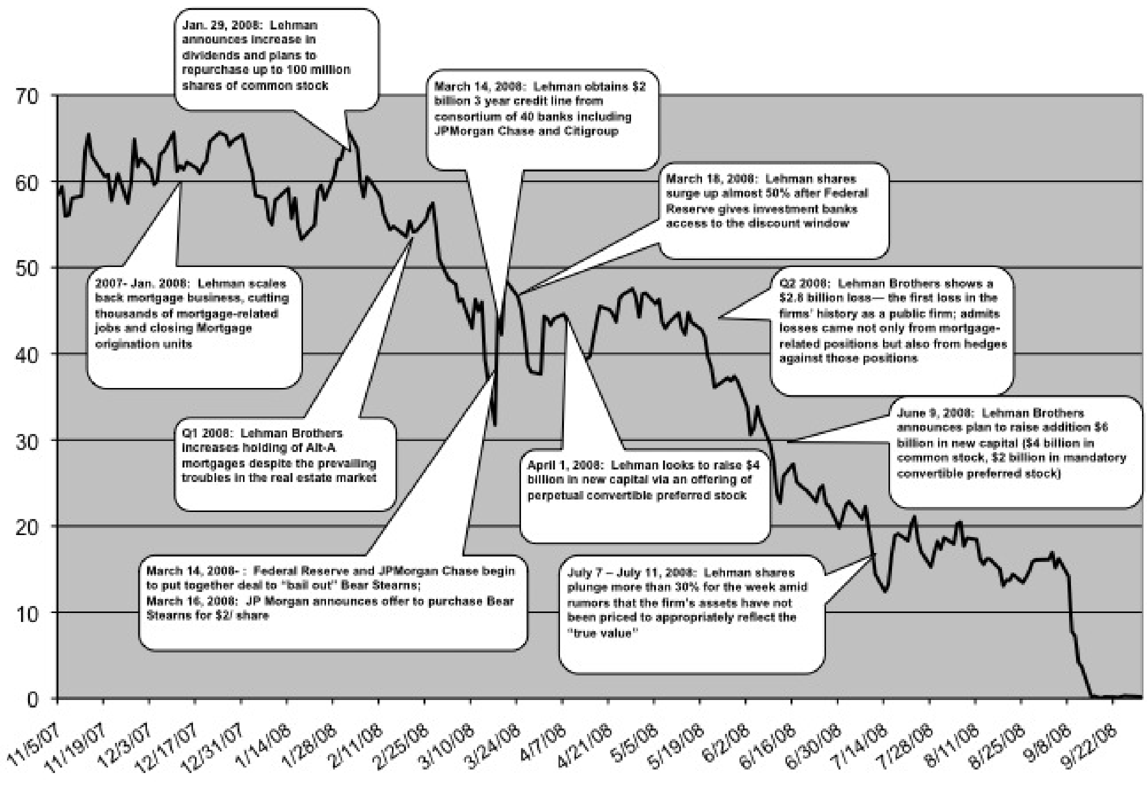

Figure 5: Lehman Brothers Debt Maturity

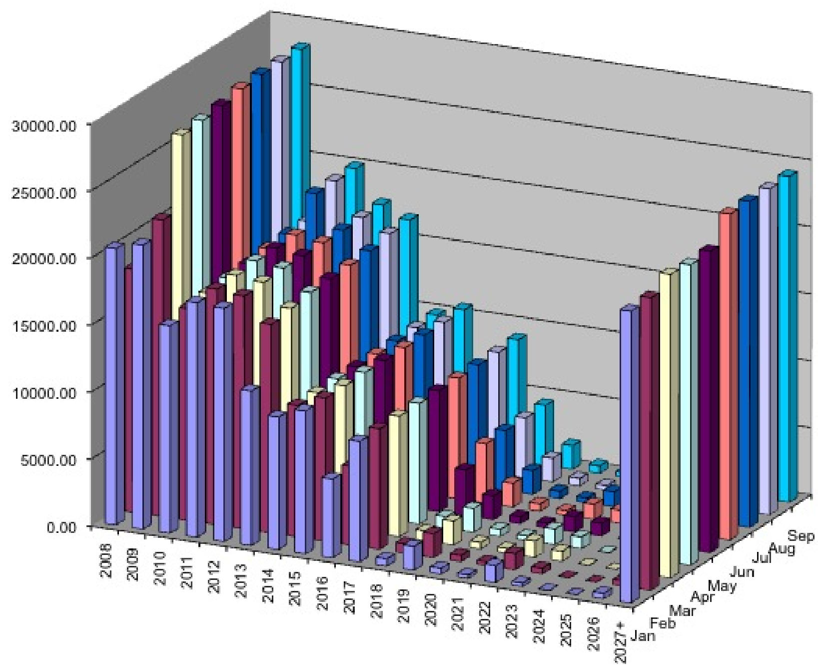

Figure 5 shows the dollar level of debt maturing in each year from 2008 to 2038 for Lehman Brothers. Data is shown monthly for the period from January 2009 to September of 2008.

Figure 6: Lehman Brothers Equity Value

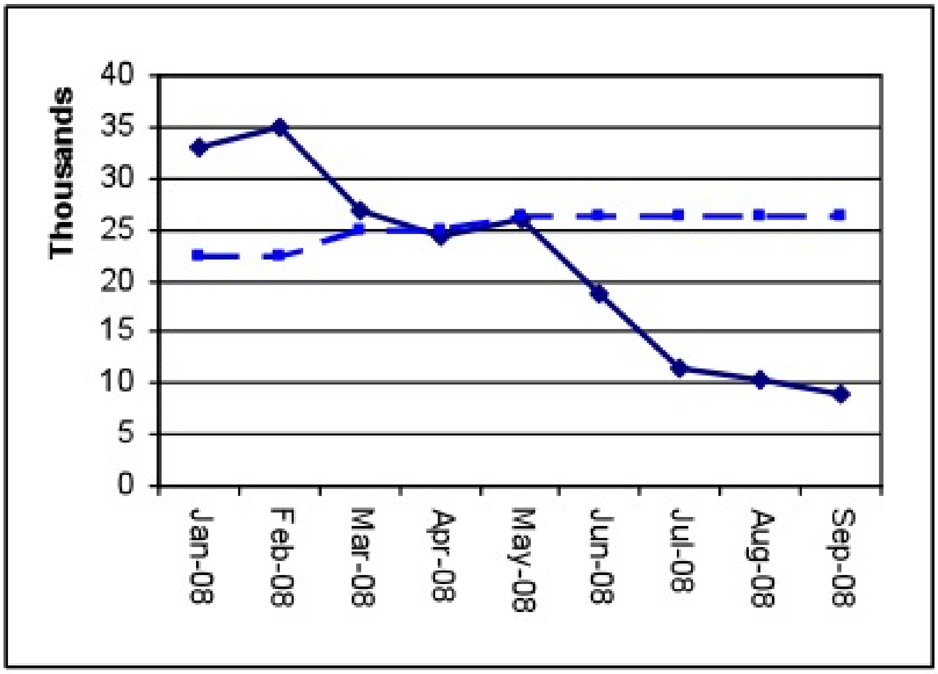

Figure 6 shows the book value and the market value of equity for Lehman Brothers at the beginning of each month from Jan 2008 to September 2008. Market value of equity is calculated as the product of the stock price multiplied by the number of shares outstanding and is represented by the solid line. Book value of equity is as reported in COMPUSTAT, The market value of equity is one of the market determined inputs for our model.

Figure 7: Lehman Brothers Volatility

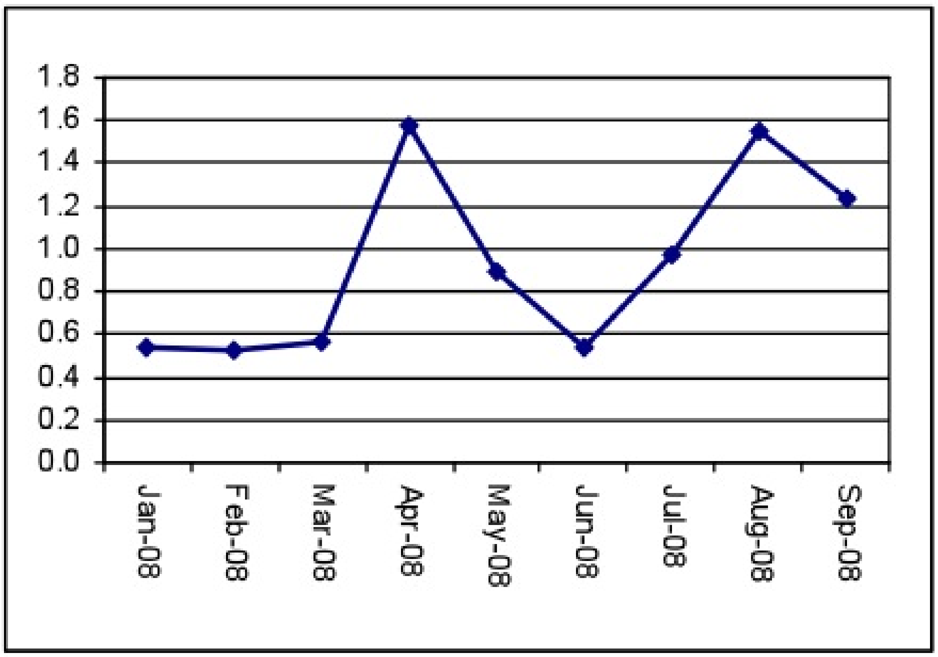

Figure 7 shows the historical 3-month volatility by at the beginning of each month from Jan 2008 to September 2008. Historical volatility is calculated by annualizing the standard deviation of daily returns over the prior 3-month period. The data on volatility is the second market determined input for our model.

Figure 8: Lehman Brothers Debt Value

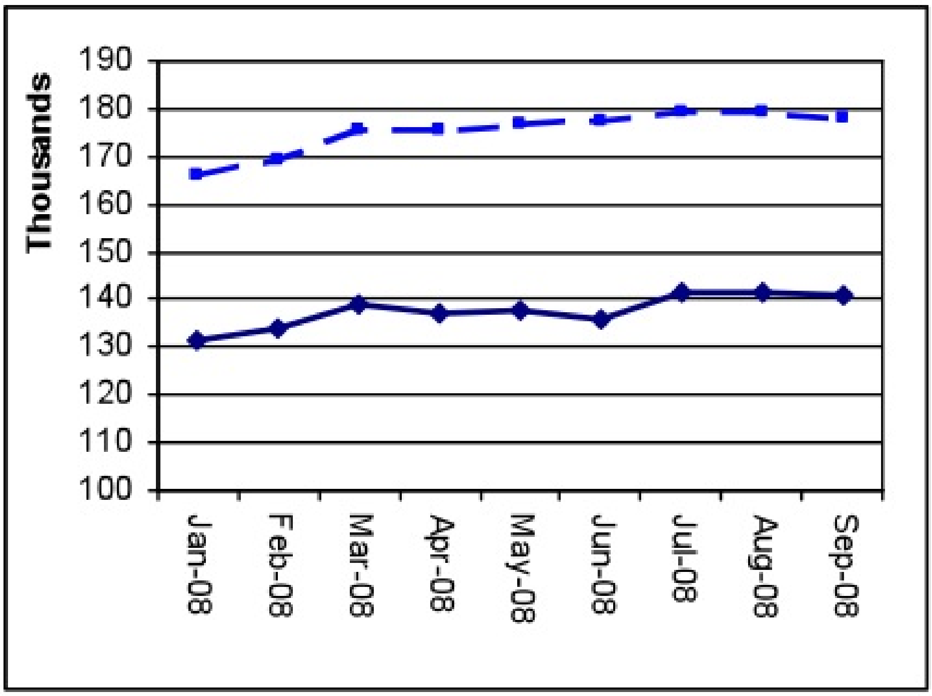

Figure 8 shows the book value and the market value of outstanding debt of Lehman Brothers at the beginning of each month from Jan 2008 to September 2008.e. Book value of debt represents the cumulative dollar total of debt outstanding for Lehman Brothers as reported by FactSet. Market value of debt is estimated by our model.

Figure 9: Lehman Brothers Default Probability

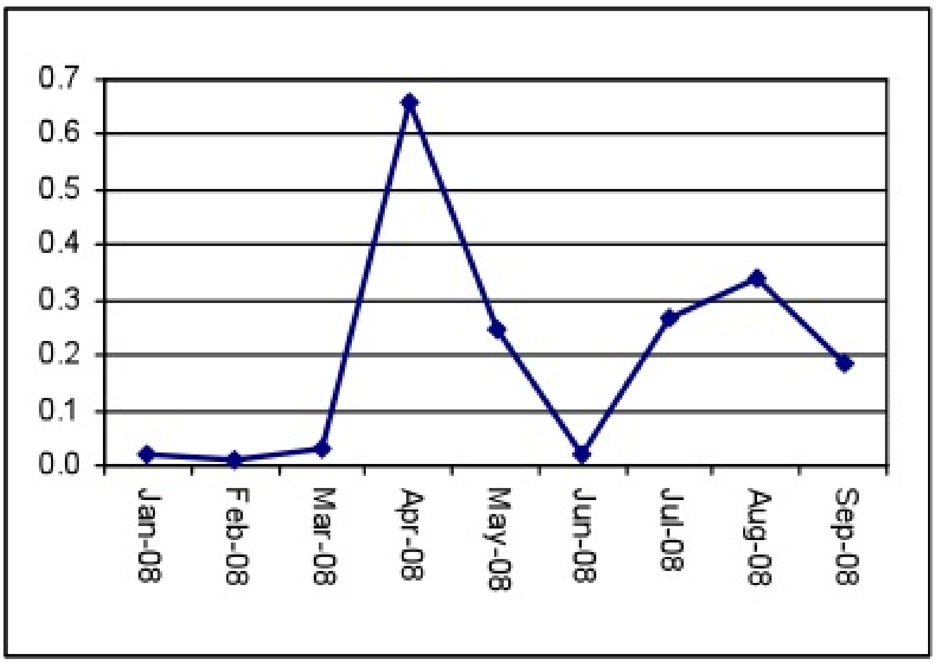

Figure 9 shows the cumulative 2-year default probability at the beginning of each month from Jan 2008 to September 2008 as estimated by our model.

Figure 10: Lehman Brothers Cumulative Default Probability

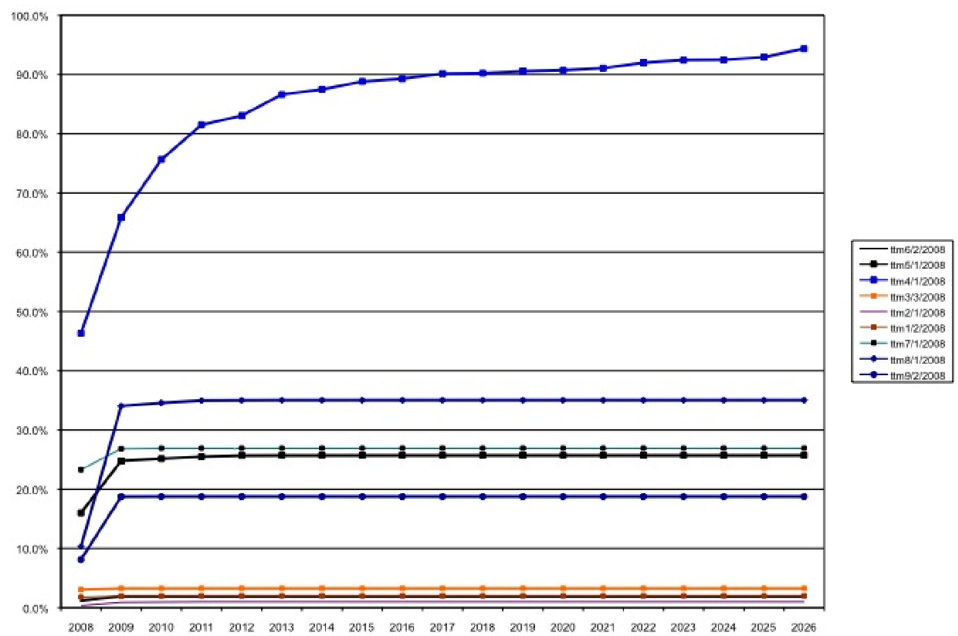

Figure 10 shows the cumulative probability of default through 2028 which is the longest date debt in our sample. The cumulative default probability is at the beginning of each month from Jan 2008 to September 2008 as estimated by our model.

Figure 11: Lehman Brothers One-Year Default Probability

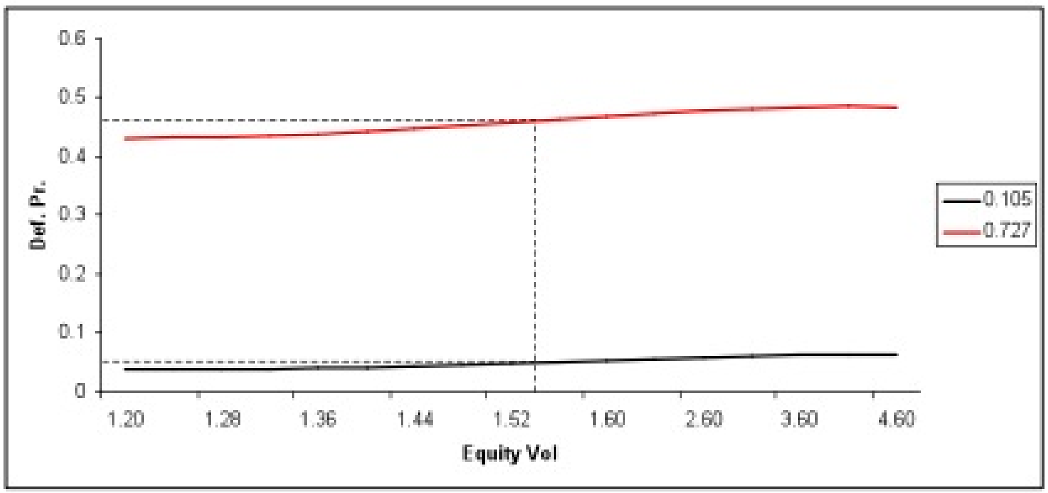

Figure 11 shows the default probability of Lehman Brothers in April 2008 as a function of equity volatility. In April, Lehman had a market cap of $24,448,000,000 and equity volatility of 157%. Our model indicates that the one-year default probability was 46.29% (and the two-year default probability was 36.46% (not shown in the figure). The market value leverage ratio (D/A) is 72.7%. The bottom line shows the impact of adding equity. As the graph shows, Lehman Brothers can reduce default probability to 5%, but this requires that Lehman substantially lower its leverage to 10.5%.

Table 1: Geske Model Vs KMV

Table 1 reports the values of the default probability for the Lattice model and for the KMV model for three different scanrios. In each scenario, Asset Value=300, $\sigma = 0.1$. Column One on each panel shows the time of the cash flow, Column Two shows the magnitude of the cash flow, Column 3 shows the default probability for the Lattice model, and Column Four shows the default probability for the KMV model.

Year	Cashflow	Geske	KMV
Panel A: Even Cashflows			
1	100	0.828	0.999
2	100	0.828	0.978
3	100	0.828	0.769
Panel B: $\bigcup$-shaped Cashflows			
1	100	0.905	0.999
2	50	0.905	0.978
3	150	0.905	0.769
Panel C: $\bigcap$-shaped Cashflows			
1	100	0.777	0.999
2	150	0.777	0.978
3	50	0.777	0.769

Table 2: Time Line of Events at Lehman Brothers

- 2007- Jan. 2008: Lehman scales back mortgage business, cutting thousands of mortgage-related jobs and closing Mortgage origination units
- Q4 2007: Lehman Brothers shows $886 million in quarterly earnings (flat compared to 3rd quarter); reported earnings of $4.192 billion for 2007 fiscal year (a 5% increase from the previous fiscal year)
- Jan. 29, 2008: Lehman announces increase in dividends and plans to repurchase up to 100 million shares of common stock
- Q1 2008: Lehman Brothers increases holding of Alt-A mortgages despite the prevailing troubles in the real-estate market
- March 14, 2008: Lehman obtains $2 billion 3 year credit line from consortium of 40 banks including JPMorgan Chase and Citigroup
- March 14, 2008: Federal Reserve and JPMorgan Chase begin to put together deal to bail out Bear Stearns
- March 16, 2008: JP Morgan announces offer to purchase Bear Stearns for $2/ share
- March 18, 2008: Lehman shares surged up almost 50% after Federal Reserve gives investment banks access to the discount window
- April 1, 2008: Lehman looks to raise $4 billion in new capital via an offering of perpetual convertible preferred stock
- Q2 2008: Lehman Brothers shows a $2.8 billion loss the first loss in the firms history as a public firm; admits losses came not only from mortgage-related positions but also from hedges against those positions
- June 9, 2008: Lehman Brothers announces plan to raise addition $6 billion in new capital ($4 billion in common stock, $2 billion in mandatory convertible preferred stock)

Table 2: Time Line of Events at Lehman Brothers (continued)

- July 7 July 11, 2008: Lehman shares plunge more than 30% for the week amid rumors that the firms assets have not been priced to appropriately reflect the true value
- September 9, 2008: Markets punish Lehman Brothers for not raising capital more aggressively; Lehmans share price falls 45% to $7.79 on fears that the firms capital levels not sufficient to support exposure to deteriorating real-estate investments
- September 10, 2008: Lehman CEO Dick Fuld reveals plans to spin off real-estate assets and sell a portion of the asset management division, insisting that the firm is solvent enough to survive
- September 11, 2008: Talks of a Lehman takeover permeate the markets as Lehman shares fall further, closing at $4.22
- September 12, 2008: Lehman approaches several potential buyers, including Bank of America and Barclays
- September 15, 2008: Lehman Brothers officially files bankruptcy after Treasury Secretary Paulson refuses to back any takeover; Shares close at $0.21
- September 16, 2008: Lehman Brothers dropped from the S&P 500
- September 18, 2008: Lehman shares close at $0.052 in OTC trading as effects of the biggest bankruptcy in history ripples through the financial markets
- September 22, 2008: Lehmans U.S. operations reopen for business under Barclays Capital after approval for the acquisition was granted by the federal bankruptcy court presid

Table 3: Lehman Brothers Debts as January 2008 (Million)

Maturity	Notional
2007	7541
2008	15931
2009	11931
2010	8564
2011	8853
2012	7403
2013	4805
2014	2217
2015	2185
2016	1234
2017	1861
2018	617
2019	59
2020	156
2021	428
2022	331
2023	172
2024	56
2025	120
2026	43
2027+	1336
Grand Total	75843

Coupon Type	Notional	Seniority	Notional
Combination	3262	Senior	38001
Combo - Fixed/Floating	706	Senior Secured	1061
Combo - Floating/Fixed	1533	Subordinate	1000
Deferred Interest	21	NONE	312
Fixed Listing	30084	N/A	35469
Floating Rate	39853	Grand Total	75843
Graduated Rate	280		
Step-Up/Down	84		
N/A	21		
Grand Total	75843		