Asset Pricing with Left-Skewed Long-Run Risk in Durable Consumption

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Abstract

I document that durable consumption growth is highly persistent and predicted by the price-dividend ratio. This provides strong and direct evidence for the existence of a highly persistent expected component. I also document robust evidence that durable consumption growth is left skewed and exhibits time-varying volatility. These properties motivate a model for durable consumption growth as driven by shocks with counter-cyclical volatility. I embed the durable consumption growth dynamics and i.i.d. nondurable consumption growth in non-separable Epstein-Zin preferences. The resulting model, together with dividend growth as containing a leverage on the expected component of durable consumption growth, can explain a number of asset pricing phenomena, including pro-cyclical price-dividend ratio, large and counter-cyclical equity premium and return volatility, low and smooth risk-free rate, and the predictability of stock returns. The model also generates the volatility feedback effect and an upward sloping term structure of real bond yields.
1 Introduction

Bansal and Yaron (2004) demonstrate that persistent predictable components and time-varying volatility in nondurable consumption and dividend growth, in conjunction with Epstein and Zin (1989) preferences, can explain major asset market phenomena. Their study has spawned a growing literature on long-run consumption risk. This literature, like a large part of the broader consumption based asset pricing literature, focuses on nondurable consumption.\footnote{See Bansal (2006) for a survey on the long-run consumption risk literature. See Campbell (2003) and Cochrane (2006) for general surveys of the consumption based asset pricing literature.}

Besides spending on nondurables, consumers also possess a large stock of durable goods. Service flows from durables arguably occupy a substantial fraction of aggregate consumption.\footnote{According to the Bureau of Economic Analysis, in 2007, U.S. consumers spent about 23 thousand dollars per person in nondurable goods and services, and held a net stock of durable goods of about 15 thousand dollars per person at the year end. Both are in chained year 2000 dollars. Major sub-categories of durable goods are motor vehicles and parts, furnishings and durable household equipment, recreational goods and vehicles, and others.} The traditional focus on nondurables is based on the underlying assumption of separability between different consumption goods. There is, however, considerable evidence for non-separability.\footnote{See, among others, Dunn and Singleton (1986), Eichenbaum and Hansen (1990), Ogaki and Reinhart (1998), Lustig and Nieuwerburgh (2005), Yogo (2006), Piazzesi, Schneider, and Tuzel (2007), and Pakos (2005).} This paper expands the scope of the long-run risk literature by investigating durable consumption and incorporating non-separability.

Empirically, I document that durable consumption growth is persistent and predicted by the price-dividend ratio. This provides strong and direct evidence for the existence of a highly persistent expected component — that is, the long-run risk component. I also document robust evidence that durable consumption growth is left skewed and exhibits time-varying volatility. These properties motivate a model for durable consumption growth as driven by shocks with counter-cyclical volatility. I incorporate non-separability in Epstein-Zin preferences and show that it allows the empirically motivated dynamics of durable consumption...
growth to generate interesting asset pricing implications, even though nondurable consumption growth is modeled as random walk and consumption shares are constant. Hence, this study puts the focus, and consequently the burden, almost entirely on durable consumption in explaining asset market phenomena.

To document the empirical properties of durable consumption growth, I choose the post-war sample period of 1952–2007, following Ogaki and Reinhart (1998) and Yogo (2006). I focus on the annual data, but also provide the results from the quarterly data that confirm the same empirical properties at even higher significance levels.

I report strong and direct evidence that durable consumption growth contains a highly persistent expected component. In the data, the first and the second order autocorrelations are 0.65 and 0.42, both highly significant. More importantly, I document strong evidence that durable consumption growth is predicted by the price-dividend ratio. At the 1-year horizon, the slope is 0.017 with a standard error of 0.005 and a $R^2$ of 0.10. At the 5-year horizon, the slope increases to 0.092 with a standard error of 0.032, and $R^2$ increases to 0.20. These findings motivate me to model durable consumption growth as containing a highly persistent expected component plus a random walk component – that is, a long-run risk component and a short-run risk component. Moreover, the predictability results provide suggestive evidence that the long-run risk component is linked with the business cycles.

I show that durable consumption growth is left skewed. The skewness in the annual data is -0.51, significantly negative at the 2% level, and this robust feature is exhibited in sub-samples. Notably, the skewness in the quarterly data is -0.46, indicating an increasing pattern with respect to time aggregation.

Durable consumption growth also exhibits slow time-varying volatility. I perform the variance ratio test on the realized volatility of innovations to durable consumption growth, and find that the variance ratio increases substantially over horizons from 2 to 10 years, indicating positive serial correlation in the realized volatility. The ratios at horizons beyond
4 years all reject the null hypothesis of an i.i.d. process at the 10% significant level, and the significance is at the 5% level for the 5-year horizon. Additionally, the realized volatility is predicted by and predicts the price-dividend ratio, both with negative slopes.

Together, left skewness and time-varying uncertainty motivate me to model the durable consumption growth dynamics as driven by shocks with conditional volatility that varies negatively with the long-run risk component. I show that this mechanism is critical in order for the model to match the empirical skewness estimates at both annual and quarterly frequencies. The model also matches the empirical variance ratios of the realized volatility. In the model, the unconditional distribution of the long-run risk component is negatively skewed. Hence, while the long-run risk component spends more time above 0, there is also an important fat left tail. This left-skewed long-run risk component is broadly consistent with historically long spans of expansions and short spans of contractions.

I also report that nondurable consumption growth exhibits a significant first order autocorrelation of 0.36, an insignificant skewness of -0.18, and slowly increasing variance ratios for the realized volatility of innovations. To highlight the role of durable consumption, I model nondurable consumption as a pure random walk. Lastly, following Bansal and Yaron (2004), I model dividend growth as a levered claim on the expected durable consumption growth, plus an independent random walk.

Following the convention in the literature, I calibrate the parameters of the growth rate models at the monthly frequency so that the model implied annual growth rates replicate the salient features of the observed data. In particular, given the high persistence of durable consumption growth and the predictability by the price-dividend ratio, I set the persistence parameter of the expected component at 0.99. The leverage parameter is set to be 0.5.

These growth rate dynamics are set into non-separable Epstein and Zin (1989) preferences. In each period the agent values durable and nondurable consumption with a Cobb-Douglas utility. This implies that the elasticity of substitution between the two goods is 1,
which is close to the estimates reported in empirical studies [for example, Ogaki and Rein-hart (1998), Yogo (2006), and Piazzesi, Schneider, and Tuzel (2007)]. The Cobb-Douglas utility also implies constant consumption shares. Following the estimate in Yogo (2006), the share of durable consumption is set to 0.8, indicating a large role of durable goods in total consumption. I then embed the intra-period utility in Epstein-Zin recursive preferences, which differentiate risk aversion and elasticity of intertemporal substitution. Similar to Bansal and Yaron (2004), the results of this paper are based on a reasonable risk aversion of 8, and a large elasticity of intertemporal substitution of 1.5 is needed to generate small and smooth risk-free rates. Taken together, since the elasticities between goods and across time are different, the resulting utility is non-separable between the two goods, which allows the durable consumption growth dynamics to generate important asset pricing implications.

I present approximate analytical solutions to illustrate the model intuition, and rely on numerical solutions and simulations to assess the model’s quantitative implications. The results show that the model is able to justify a broad set of asset pricing phenomena, including pro-cyclical price-dividend ratio, large and counter-cyclical equity premium and return volatility, low and smooth risk-free rate, and the predictability of stock returns. The model is also capable of generating the volatility feedback effect. In addition, it generates positive, counter-cyclical real bond risk premia, and an upward sloping term structure. Lastly, the model is capable of matching the empirically documented predictability of durable consumption growth, its realized volatility, and the price-dividend ratio.

The mechanism in this model in generating large equity premium and return volatility relies on the high persistence of the expected durable consumption growth. Due to high persistence, a shock to the expected component generates substantially amplified responses in both the pricing kernel and the returns to consumption and dividend claims — that is, the market price of this long-run risk is high, and the exposure of consumption and dividend claims to this long-run risk is also large. Taken together, they result in large equity premium
and high return volatility.

Time-varying equity premium and return volatility arise in my model as a result of the conditional volatility in durable consumption growth, which varies counter-cyclically with respect to the long-run risk component. Again, due to the high persistence of the expected growth, the counter-cyclical volatility is greatly amplified in the pricing kernel and the returns to consumption and dividend claims, which then result in substantial counter-cyclical variations in both equity premium and return volatility.

Overall, these model implications, together with the strong empirical evidence and the underlying fact that durable consumption is a large fraction of aggregate consumption, suggest long-run durable consumption risk as a plausible channel to justify key asset market phenomena. In particular, the direct evidence for the existence of the long-run durable consumption risk component corroborates the evidence documented in Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2007) for nondurable consumption.

This paper is also related to several recent studies that incorporate multiple consumption goods in the agent’s utility. In my model, the Cobb-Douglas utility implies that the shares of durable and nondurable consumption are constant. The implications of time-varying composition are emphasized in Piazzesi, Schneider, and Tuzel (2007), in which the utility is non-separable between nondurable consumption and housing. Also, my paper highlights the asset market implications of long-run durable consumption risk in a calibrated model economy. The objective and the approach of my study are complementary to the empirical studies of Yogo (2006) and Pakos (2005) that focus on the GMM estimation.

In the rest of the paper, I first discuss the data in Section 2. Section 3 focuses on the empirical properties of the durable consumption growth data. These properties motivate the model of durable consumption growth, which is presented in Section 4, followed by the calibration of the parameters. Section 5 specifies the pricing kernel, calibrates the preferences parameters, and discusses the model intuition using approximate analytical solutions. In
Section 6, I present the numerical solutions and the simulations to discuss the model’s quantitative implications. The concluding section addresses the limitations of the paper, and the appendices collect additional details of the model.

2 Data

Annual data on consumption are from the Bureau of Economic Analysis. Nondurable consumption is the sum of real personal consumption expenditures on nondurable goods and services. Following the convention, I assume the service flow of durable goods is proportional to the stock. Consequently, durable consumption is measured by the year-end real stock of consumer durable goods. Both durable and nondurable consumption series are divided by the population to obtain per capita values. While annual data are available from late 1920s, I use the sample period of 1952–2007. The Great Depression era experienced unusually negative consumption growth rates. The period after World War II but before 1952 is associated with unusually high growth in the stock of durables. Consequently, following Ogaki and Reinhart (1998) and Yogo (2006), I exclude these unusual periods to insure that the key data characteristics are stable, and in particular, not driven by outliers.

Quarterly data are also available for the period after WWII. However, they may involve seasonalities that confound the inference. In addition, stock of durable goods is only reported at year ends, and thus the quarterly series of the stock must be constructed from the annual series using quarterly real expenditures on durable goods and imputed depreciation rates.

4 Ogaki and Reinhart (1998) find that when their sample period begins in 1947, the cointegrating relation, with which they estimate the elasticity of substitution between durables and nondurables, is strongly rejected. Consequently, they exclude the unusual period of restocking of durables immediately following WWII.

5 I also conduct robustness checks for longer sample periods. For the period 1929–2007, annual nondurable consumption growth exhibits a first-order autocorrelation of 0.44 and a skewness of -1.61, while in 1932–2007 the autocorrelation is 0.13 and the skewness is essentially zero. In comparison, for 1929–2007, annual durable consumption growth shows a first-order autocorrelation of 0.76 and a skewness of -0.49, while in 1932–2007 the two estimates are 0.73 and -0.53, respectively. Hence, high persistence and left skewness appear to be robust features of the durable consumption data.
Yogo (2006). Consequently, I focus on the annual data following the convention in the literature. Still, I also construct the quarterly series of the stock of durables for the same period of 1952–2007, and find that the results obtained from the annual data are confirmed with even higher significance levels by the quarterly data. The relevant results from the quarterly series will be reported mostly in footnotes.

The stock market returns are value-weighted annual returns for NYSE, and the risk-free rates are 3-month T-bill rates, both adjusted for inflation. All data series are obtained from the CRSP. Year-end price-dividend ratios and annual real dividend growth rates are computed from the value-weighted annual returns for NYSE with and without distributions, and adjusted for inflation, following the procedure in Cochrane (2008).

3 Empirical properties of growth rates

Fig. 1 plots annual durable and nondurable consumption growth, with shaded regions marking the NBER dated economic contractions. On average, durable consumption grows at about 4% per year. This rate is about two times that of nondurable consumption. As a result, the ratio of durable consumption to nondurable consumption has tended to rise over time. As pointed out in Ogaki and Reinhart (1998) and Yogo (2006), this trend is consistent with the downward trend in the relative purchase price of durables with respect to nondurables.

Durable consumption growth is also more volatile. Its volatility of about 2% is about two times that of nondurable consumption growth. In time series, both growth rates tend to be low (high) during recessions (expansions). Durable consumption growth also tends to decrease consecutively during recessions, and rise consecutively during expansions. This suggests that durable consumption growth is persistent.
3.1 Durable consumption growth

Panel A of Table 1 reports that in the full sample, the first order autocorrelation of durable consumption growth is 0.65, which is highly significant. Moreover, the second order autocorrelation is also significant at 0.42. To insure that high persistence is an innate property rather than sample period specific, I split the full 1952–2007 sample period into halves, and report the estimates for 1952–1979 and 1980–2007 separately. In spite of shortened periods, both sub-samples exhibit significant first and second order autocorrelations. Taken together, there is compelling evidence that durable consumption growth is persistent.

More strikingly, as reported in Panel B of Table 1, durable consumption growth is predicted by the price-dividend ratio. Here, cumulative durable consumption growth from $t$ to $t+J$ is regressed on log price-dividend ratio of time $t$,

$$\Delta d_{t+1} + \ldots + \Delta d_{t+J} = a_J + b_J(p-d)_t + \eta_{t+J}. \quad (1)$$

The results indicate positive slope coefficients that are highly significant, and both the slope and $R^2$ increase with the horizon. At the 1-year horizon, the slope is 0.017 with a standard error of 0.005 and a $R^2$ of 0.10. At the 5-year horizon, the slope increases to 0.092 with a standard error of 0.032, and $R^2$ increases to 0.20. With first and second order autocorrelations of 0.91 and 0.81, the price-dividend ratio is highly persistent. Hence, these predictive regressions provide strong and direct evidence that durable consumption growth contains a highly persistent predictable component – that is, a long-run risk component. In addition, they also provide suggestive evidence that the long-run risk component is linked with the business cycles.

6For the data on quarterly durable consumption growth, the 4th and 8th order autocorrelations (standard errors) are 0.57 (0.08) and 0.34 (0.12), respectively.
Table 2 brings the focus to skewness. To assess the statistical significance of the point estimates, I use the bootstrap method to find the confidence intervals. More specifically, I generate 10000 bootstrap samples, measure skewness for each sample, and report the percentiles of these estimates. If, for example, the 95th percentile is still negative, then I conclude that the point estimate of skewness is significantly negative at the 5% level.

Table 2 reports that in the period 1952–2007, durable consumption growth exhibits a negative skewness of -0.51, and is significantly negative at the 2% level. To insure that negative skewness is a stable and consistent characteristic of the data, I again split the sample period into two halves. The estimates of skewness for both periods are close to -0.5. In spite of the shortened samples, skewness of the 1952–1979 period is significantly negative at the 10% level, while that for 1980–2007 is significantly negative at the 5% level. In addition, in the quarterly data for durable consumption growth, the skewness is -0.46, significantly negative at a confidence level much better than 1%. Taken together, negative skewness appears to be a robust property of the empirical durable consumption growth data.

Finally, I investigate time-varying volatility in durable consumption growth. As in Bansal and Yaron (2004), I perform the variance ratio test on the realized volatility of innovations. I first obtain residuals from the regression

$$\Delta d_t = a_0 + \sum_{i=1}^5 a_i \Delta d_{t-i} + \epsilon_{\Delta d,t}. \tag{2}$$

Using five lags is more than adequate to account for the autocorrelation in durable consump-

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7The skewness estimator is corrected for the small sample bias,

$$\sqrt{n(n-1)} \frac{\frac{1}{n} \sum (x_i - \bar{x})^3}{n - 2} \frac{1}{\left(\frac{1}{n} \sum (x_i - \bar{x})^2\right)^{3/2}}.$$

8In other words, here the resampling and the confidence intervals are obtained under the null hypothesis that skewness is equal to the empirical estimate (which is not zero).
tion growth. Then I compute the variance ratios for the absolute value of the residuals

$$\frac{\text{var} \left[ \sum_{j=0}^{J-1} |\epsilon_{\Delta d,t+j}| \right]}{J \cdot \text{var} \left[ |\epsilon_{\Delta d,t}| \right]}.$$

where $J$ is the horizon.

For statistical inference, I generate 10000 bootstrap samples and compute variance ratios for each sample. Here, the resampling treats each data point in the time series as independent. Hence, the bootstrap samples and consequently the percentiles are obtained under the null hypothesis of serial independence. This allows me to test the null by comparing the empirical estimates with the bootstrap percentiles under the null. To examine the dependence of the variance ratio on the horizon, I follow Poterba and Summers (1988) and adjust for the small-sample bias by dividing the empirical estimates by the averages of the bootstrap results.

Without time-varying volatility, the adjusted variance ratios would be flat with respect to the horizon, and stay close to 1. Panel A of Table 3 shows the opposite: the adjusted variance ratios are all above 1, and increase substantially with the horizon. Moreover, for all horizons beyond 4 years, the null of no serial correlation is rejected at the 10% level. At the 5-year horizon, the null is also rejected at the 5% level. The larger than 1 and increasing variance ratios suggest positive serial correlation in the realized volatility. 9

To provide additional supportive evidence for time-varying volatility, Panel B of Table 3 explores the predictive relations between the realized volatility and the price-dividend ratio. The results indicate that realized volatilities in the future are predicted by the log

9 For the quarterly data of durable consumption growth, I verify that the residuals from an AR(4) regression are not serially correlated. Variance ratio tests on the absolute value of the residuals indicate that the null of no serial correlation is rejected at the 5% level beyond horizons of 8 quarters, and at the 2% level beyond horizons of 12 quarters.
price-dividend ratio,

\[ |\epsilon_{d,t+J}| = a_J + b_J (p-d)_t + \eta_{t+J}, \]

(4)

with negative slopes. Conversely, log price-dividend ratios in the future are also predicted by the realized volatility,

\[ (p-d)_{t+J} = a_J + b_J |\epsilon_{d,t}| + \eta_{t+J}, \]

(5)

also with negative slopes. These results are similar to those reported in Bansal, Khatchatrian, and Yaron (2004) for nondurable consumption growth.

### 3.2 Nondurable consumption and dividend growth

Table 4 presents the empirical properties for non-durable consumption growth in the period of 1952–2007. Panel A reports a significant first order autocorrelation of 0.36, while the second order autocorrelation is insignificant. These estimates are close to those reported in Bansal and Yaron (2004) for the period 1929-1998. Panel B indicates that nondurable consumption growth exhibits a small negative skewness, which is not significantly different from zero at the 10% level. Panel C shows that for innovations to nondurable consumption growth, the variance ratio for the realized volatility increases slowly with respect to the horizon, and are all bracketed within the 10% confidence intervals. The adjusted variance ratios are close to 1. Hence, the null of no serial correlation cannot be rejected.

Overall, these results are consistent with the evidence in previous studies and are sug-

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10 As emphasized in Bansal and Yaron (2004), in finite samples, it is difficult to distinguish between a pure i.i.d. process and a process with small persistence [Shephard and Harvey (1990)].

11 The bootstrap mean and percentiles are almost identical in Panel A of Table 3 and Panel C of 4. These quantities are obtained under the same null of no serial independence. They appear to depend almost solely on the sample length, and very little on the specific data. The small differences between the two tables appear to be sampling variations.
gestive of a long-run predictable component in nondurable consumption growth [Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2007)]. To sharpen the focus of this paper and highlight the implications of the empirical properties of durable consumption, I subsequently model nondurable consumption growth as a pure random walk.

Lastly, for dividends, the average growth rate is about the same as that of nondurable consumption, while the volatility is high. The first order autocorrelation is about 0.29. The correlation between dividend and durable consumption growth rates is 0.11. As presented subsequently, dividend growth is modeled as a levered claim on the expected component in durable consumption growth. The small correlation suggests that the leverage parameter will be small. The empirical properties of dividend growth are presented later in Table 6, in which they will be compared with the simulation results implied by the calibrated model.

4 Models of growth rates

In this section I present the models for consumption and dividend growth rates, and then calibrate the parameters. Following the convention in the literature,\textsuperscript{12} the models are calibrated at the monthly interval, so that simulated monthly series, after time aggregation to the annual frequency, replicate the salient features of the empirical data. The calibrated parameters are listed in Table 5, while Table 6 compares the salient properties of the empirical data with those obtained from the model implied, 10000 simulated 56-year samples.

4.1 Durable consumption growth

The model for durable consumption growth is motivated by the robust empirical properties documented earlier. Specifically, durable consumption growth has a mean of $\mu_d$, and contains

\textsuperscript{12}See Kandel and Stambaugh (1991), Campbell and Cochrane (1999), and Bansal and Yaron (2004), among others.
two components as specified in

\[ \Delta d_{t+1} = \log \frac{D_{t+1}}{D_t} = \mu_d + x_t + \sigma_d \omega_t \varepsilon_{d,t+1}, \tag{6} \]

\[ x_{t+1} = \phi x_t + \sigma_x \omega_t \varepsilon_{x,t+1}, \tag{7} \]

\[ \omega_t = \sqrt{1 - \zeta x_t}, \tag{8} \]

\[ \varepsilon_{x,t+1}, \varepsilon_{d,t+1} \sim i.i.d. N(0, 1). \tag{9} \]

The expected component \( x_t \) is an AR(1) process of the persistence parameter \( 0 < \phi < 1 \), driven by shocks of time-varying volatility \( \sigma_x \omega_t \). The random walk component is independent shocks with time-varying volatility \( \sigma_d \omega_t \). The time-varying \( \omega_t \) is a function that varies negatively with \( x_t \), with a sensitivity parameter of \( \zeta \). The square root functional form is inspired by the sensitivity function in Campbell and Cochrane (1999). This ultimately contributes a term linear in \( x_t \) to the expected excess returns. The volatility approaches zero when \( x_t \to 1/\zeta \), and in the continuous time limit, \( x_t \) never exceeds \( 1/\zeta \).

In this specification, durable consumption growth contains two types of risk. The “short-run” risk comes from the random walk shocks, while the “long-run” risk comes from the shocks to the expected component. The persistence of \( x_t \) implies that its shocks will have a long lasting effect. The conditional volatility for the shocks varies counter-cyclically with respect to the long-run risk component. When \( x_t \) is high, durable consumption is expected to grow fast, and volatilities are low for both short-run and long-run shocks. When \( x_t \) is low, durable consumption is expected to grow slowly, and volatilities are high.

As documented earlier, durable consumption growth is predicted by the price-dividend ratio. With first and second order autocorrelations of 0.91 and 0.81, the price-dividend ratio is highly persistent. This provides direct evidence for the highly persistent \( x \) component in the model. Consequently, I set the persistence parameter \( \phi \) to 0.99.\(^\text{13}\) The two volatility

\(^{13}\text{This is consistent with } 0.91^{1/12} = 0.9922. \text{ Also, Bansal and Yaron (2004) calibrate a persistence of 0.979}\)
parameters $\sigma_x$ and $\sigma_d$ are calibrated to match the autocorrelations and the standard deviation of the annual data. The mean $\mu_d$ is chosen to match the average annual growth rate.

The counter-cyclical volatility following $\omega_t$ is motivated by, and allows the model to generate, both negative skewness and time-varying uncertainties documented above. In particular, negative skewness arises because the shocks have high (low) volatility when the expected growth is low (high). I set the parameter $\zeta$ to match the skewness of the annual data, which leads to a value of 278. The resulting average skewness of the simulated quarterly series is about -0.52, broadly consistent with the empirical estimate of -0.46 for the quarterly data. Hence, this mechanism is able to replicate skewness at both annual and quarterly frequencies. The variance ratios for the realized volatility of durable consumption growth are not included as targets of matching in the calibration. Still, Panel A of Table 6 shows that two variance ratios of the simulated data are largely consistent with the empirical estimates.

The top panel in Fig. 2 plots the counter-cyclical volatility function $\omega(x)$ with respect to $x$ scaled by $\sigma_x/\sqrt{1 - \phi^2}$, which would be the unconditional volatility of $x$ without the counter-cyclical $\omega$. By construction, $\omega = 1$ at $x = 0$. With the calibrated $\zeta = 278$, the volatility becomes zero at $x = 2.097 \sigma_x/\sqrt{1 - \phi^2}$. This is also the maximum of $x$. For negative $x$, $\omega$ rises slowly and reaches about 1.6 when $x/(\sigma_x/\sqrt{1 - \phi^2}) = -4$. The bottom panel of Fig. 2 plots the model implied unconditional density of $x$. The distribution, due to the conditional volatility, is negatively skewed. Hence, $x$ is more often positive, but there are also important, although less frequent, realizations in the fat left tail. This left-skewed long-run risk component is broadly consistent with historically long spans of expansions and short spans of contractions.\(^{14}\)

Additional simulation results suggest that the counter-cyclical volatility mechanism is for their expected nondurable consumption growth, and 0.987 for the volatility state variable. Campbell and Cochrane (1999) calibrate a persistence of $0.87^{1/12} = 0.9885$ for log surplus consumption ratio.\(^{14}\) The surplus consumption ratio in Campbell and Cochrane (1999) is heavily negatively skewed, and they interpret it as a business cycle indicator.
critical for the model to match the empirically observed skewness. Consider, for example, an alternative in which negative skewness is unrelated to conditional volatility, but arises solely because long-run and/or short-run shocks follow a skewed distribution. Simulation results indicate that this alternative mechanism is too weak — monthly *i.i.d.* shocks with a very negative skewness of -1.5 generate only a skewness about -0.1 in the time-aggregated annual series.

Furthermore, it is important that volatility varies with the expected component $x_t$ of durable consumption growth. Should $\omega_t$ depend on some additional state variable unrelated to $x_t$, the resulting growth rate series would exhibit zero skewness. In addition, it is also important that volatility is counter-cyclical not just for the short-run shocks, but also for the long-run shocks. With counter-cyclical volatility, both types of shocks can generate negative skewness, but with opposite behaviors with respect to time aggregation. Simulation results indicate that, with counter-cyclical volatility only in the long-run shocks, skewness becomes much more negative when quarterly series are aggregated to the annual frequency. In contrast, if counter-cyclical volatility appears only for the short-run shocks, annual series are much less negatively skewed than quarterly series. As reported earlier, the empirical skewness is -0.46 in the quarterly data and -0.51 in the annual data. This slightly increasing pattern suggests that both long-run and short-run components are at play. For parsimony, I specify the same counter-cyclical function $\omega_t$ for both components. As shown above, the model replicates the skewness estimates at both annual and quarterly frequencies.$^{15}$

Finally, in my model, time-varying uncertainties are modeled together with negative skewness by specifying the conditional volatility following $\omega_t$. The simulation results in Table 6 suggest that this parsimonious mechanism appears to be largely adequate in matching the empirical variance ratios of the realized volatility of durable consumption growth. However,

$^{15}$ An anonymous referee points out that, in a regime-switching framework with $x$ as the filtered growth rate, both positive and negative correlations between volatility and $x$ may arise. This is an interesting point that is worth pursuing in future work.
the resulting perfect negative correlation between the conditional volatility and \( x_t \) is a strong restriction. In particular, it implies that all asset pricing results vary with one single state variable \( x_t \), which is doomed to be over-simplifying vis-a-vis the empirical asset market data.

A potential extension of this specification is to introduce a separate state variable to represent a component of time-varying uncertainty unrelated to \( x_t \), in the spirit of Bansal and Yaron (2004). This additional variable enhances the flexibility of the model. It also introduces an additional source of risk, and will raise equity premium and generate additional time-varying implications. On the other hand, it does not change the model intuition that counter-cyclical variations in equity premium and return volatility are due to the negative dependence of \( \omega_t \) on \( x_t \). Hence, my specification of \( \omega_t \) maintains parsimony without sacrificing the model’s capability to match the empirical variance ratios and generate major asset market implications. Consequently, I focus on this parsimonious setup in my paper and defer the extension to future research.

4.2 Nondurable consumption and dividend growth

To highlight the implications of the durable consumption growth dynamics, I specify nondurable consumption growth as a pure random walk,

\[
\Delta c_{t+1} = \log \frac{C_{t+1}}{C_t} = \mu_c + \sigma_c \varepsilon_{c,t+1}, \quad \varepsilon_{c,t+1} \sim i.i.d. N(0,1). \tag{10}
\]

Here, \( \mu_c \) is the mean and \( \sigma_c \) is the volatility of the \( i.i.d. \) shocks. These parameters are calibrated to match the average and the standard deviation of the empirical nondurable consumption growth data, as confirmed in Panel B of Table 6.

Lastly, similar to Bansal and Yaron (2004), I specify dividend growth as

\[
\Delta y_{t+1} = \mu_y + \lambda x_t + \sigma_y \varepsilon_{y,t+1}, \quad \varepsilon_{y,t+1} \sim i.i.d. N(0,1). \tag{11}
\]
Here, $\mu_y$ is the mean, $\lambda$ is the leverage parameter characterizing the exposure of dividend growth to the expected component of durable consumption growth. The random walk component of dividend growth has a volatility of $\sigma_y$.

The correlation between dividend and durable consumption growth, reported in Panel C of Table 6, suggests a small leverage parameter $\lambda$, which is set to 0.5. The mean parameter is set to match the average dividend growth, and a large $\sigma_y$ is needed to replicate the empirical volatility. Panel C of Table 6 confirms the match, and in particular, shows that the model replicates the first-order autocorrelation of dividend growth as well as the correlation between durable consumption growth and dividend growth. With a small $\lambda$ and a large $\sigma_y$, the dividend growth process is close to a random walk. However, the high persistence of the expected durable consumption growth implies that even small exposure to long-run risk is able to deliver quantitatively large asset pricing implications.

For parsimony, all shocks — $\varepsilon_{x,t+1}$, $\varepsilon_{d,t+1}$, $\varepsilon_{c,t+1}$, and $\varepsilon_{y,t+1}$ — are mutually independent. This leaves the burden of the model performance entirely on the expected durable consumption growth $x_t$. The model could be enriched by allowing for correlations between the shocks. For example, in the empirical data, the correlation between annual durable and nondurable consumption growth rates is about 0.18. This can be replicated by correlated $\varepsilon_{d,t+1}$ and $\varepsilon_{c,t+1}$ shocks. Similarly, the correlation between annual dividend and nondurable consumption growth rates is about 0.35. This can be replicated by correlated $\varepsilon_{y,t+1}$ and $\varepsilon_{c,t+1}$ shocks. These enrichments only result in minor qualitative and quantitative changes in the model’s asset pricing implications, because the prices of risk are small for $\varepsilon_{d,t+1}$ and $\varepsilon_{c,t+1}$ shocks.\footnote{Bansal and Yaron (2004) focus on the long-run risk in nondurable consumption growth. The correlation between dividend and nondurable consumption growth, replicated by modeling dividend growth as containing a leverage on the expected component in nondurable consumption growth, yields major asset pricing results in their paper.}

Lastly, as modeled in my paper, both log durable consumption $d$ and log nondurable

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16 Bansal and Yaron (2004) focus on the long-run risk in nondurable consumption growth. The correlation between dividend and nondurable consumption growth, replicated by modeling dividend growth as containing a leverage on the expected component in nondurable consumption growth, yields major asset pricing results in their paper.
consumption \( c \) are difference stationary, and consequently, \( d - c \) is difference stationary with a drift. Ogaki and Reinhart (1998) make the same assumptions and confirm the empirical validity of these assumptions in their paper. Fig. 1 also suggests that these assumptions are empirically plausible.

5 Asset pricing

This section presents the details of the model for a representative agent with non-separable utility over both durable and nondurable consumption embedded in Epstein and Zin (1989) preferences. This is followed by the calibration of the model parameters and the discussions of the model intuition using approximate analytical solutions.

5.1 Utility

In each period, the representative agent consumes \( C_t \) units of the nondurable good, and derives the service flow proportional to the stock, \( D_t \), of the durable good. The agent has an intra-period Cobb-Douglas utility function

\[
V_t = C_t^{1-\alpha} D_t^\alpha,
\]

where \( 0 < \alpha < 1 \) is a constant.

The nondurable good is the numeraire. Since the durable good lasts for more than one period, the relative price of consuming durables, measured in terms of nondurables, is the user cost

\[
Q_t = \frac{\partial V_t / \partial D_t}{\partial V_t / \partial C_t} = \frac{\alpha C_t}{1 - \alpha D_t},
\]

(13)
rather than the purchase price of the durable good. Consequently, the total consumption is

$$G_t = C_t + Q_t D_t = \frac{1}{1 - \alpha} C_t. \quad (14)$$

Total consumption grows at the same rate as that of nondurable consumption. In addition, the consumption shares are constant regardless of the ratio $D/C$, as

$$\frac{C_t}{C_t + Q_t D_t} = 1 - \alpha, \quad \frac{Q_t D_t}{C_t + Q_t D_t} = \alpha. \quad (15)$$

This provides an interpretation of $\alpha$ as the share of durable consumption.

The empirical evidence presented earlier indicates that the ratio $D/C$ has risen considerably over time, which is consistent with the downward trend in the relative purchase price of durables. The Cobb-Douglas utility also implies that the relative user cost of durables $Q_t$ is proportional to $C_t/D_t$. Hence, like the purchase price, $Q_t$ also tends to decrease over time, and its decrease exactly cancels the increase in $D_t/C_t$. Consequently, with the Cobb-Douglas utility and when measured in terms of the nondurables numeraire, total consumption is proportional to nondurable consumption, and the consumption shares are constant.

The Cobb-Douglas utility is the special case of $\rho = 1$ for the constant elasticity of substitution utility function

$$\left( (1 - \alpha)C_t^{1 - \frac{1}{\rho}} + \alpha D_t^{1 - \frac{1}{\rho}} \right)^{\frac{1}{1 - \rho}}.$$

There are two motivations for the choice of $\rho = 1$ in this paper. First, empirical estimates of $\rho$ are not very different from 1. For example, Ogaki and Reinhart (1998) find $\rho$ to be about 1.2,\(^{17}\) Yogo (2006) estimates it at about 0.8, and Piazzesi, Schneider, and Tuzel (2007) also

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\(^{17}\) Specifically, Ogaki and Reinhart (1998) assume $c$ and $d$ are difference stationary, and then show that the log price of durables in terms of nondurables, $p$, is also difference stationary, and $[p, c-d]$ is cointegrated with a cointegrating vector $(1, -1/\rho)$. They confirm the empirical validity of these stationarity assumptions,
report estimates close to 1 for housing. Secondly, under a general \( \rho \), the durable consumption share is

\[
\frac{\alpha (D_t/C_t)^{1-\frac{1}{\rho}}}{1 - \alpha + \alpha (D_t/C_t)^{1-\frac{1}{\rho}}}.
\]

Hence, \( \rho \neq 1 \) introduces another state variable \( D_t/C_t \) to the model, and the share of durable consumption varies with time. This induces composition risk, similar to that in Piazzesi, Schneider, and Tuzel (2007). In the data the ratio of \( D/C \) has tended to rise over time. This implies an increasing (decreasing) share of durable consumption if \( \rho > 1( < 1) \), while \( \rho = 1 \) implies a constant share of durable consumption.\(^{18}\) It turns out that when \( \rho \) is not very different from 1, this only generates very small temporal variations in the quantities of interest.\(^{19}\) Overall, setting \( \rho = 1 \) maintains parsimony, sharpens the focus of the paper, and the implications — consumption shares are constant, and total consumption grows at the same rate as that of nondurable consumption — appear to be economically reasonable.

The intra-period utility is imbedded in Epstein and Zin (1989) recursive preferences

\[
U_t = \left( (1 - \delta)V_t^{1-\frac{1}{\psi}} + \delta \left( E_t[U_{t+1}^{1-g}] \right)^{1-\frac{1}{g}} \right)^{1-\frac{1}{\psi}}.
\]

(16)

Here, \( 0 < \delta < 1 \) is the time discount factor, \( \gamma \) is the risk aversion parameter, and \( \psi \) is the elasticity of intertemporal substitution. Following Bansal, Tallarini, and Yaron (2008) and utilizing the cointegration relation, they estimate \( \rho \) to be about 1.2. With \( \rho = 1 \), my model implicitly assumes that \( [p, c - d] \) is cointegrated with a vector \((1, -1)\).

\(^{18}\) For a general \( \rho \), the user cost is

\[
Q_t = \frac{\partial V_t/\partial D_t}{\partial V_t/\partial C_t} = \frac{\alpha (D_t/C_t)^{-\frac{1}{\rho}}}{1 - \alpha}.
\]

Hence, the user cost decreases when \( D/C \) increases as long as \( \rho > 0 \). When \( \rho > 1( < 1) \), the decrease in user cost under-compensates (over-compensates) the rise in \( D/C \), resulting in an increasing (decreasing) share of durable consumption.

\(^{19}\) The details of the model and the solutions under a general \( \rho \) are available upon request.
Yogo (2006), the pricing kernel is

\[ M_{t+1} = \delta^\theta \left( \frac{C_{t+1}}{C_t} \right)^{-\theta} \left( \frac{(D_{t+1}/C_{t+1})^\alpha}{(D_t/C_t)^\alpha} \right)^{\theta(1-\frac{1}{\psi})} R_{g,t+1}^{\theta-1}, \]  

where \( \theta = (1 - \gamma)/(1 - \frac{1}{\psi}) \). Here,

\[ R_{g,t+1} = \frac{W_{t+1}}{W_t - G_t} \]  

is the return to wealth, and the wealth \( W_t \) is the claim to the entire future stream of total consumption.

The parameters in the preferences, as presented in Table 5, are chosen to take into account the empirical evidence and economic considerations. I calibrate the durable consumption share \( \alpha = 0.8 \). This is the estimate in Yogo (2006), and it indicates that durable consumption has a significant role in total consumption. The parameters of Epstein-Zin preferences largely follow Bansal and Yaron (2004). The time discount factor \( \delta < 1 \). The risk aversion parameter \( \gamma = 8 \) is somewhat smaller than that in Bansal and Yaron (2004). The elasticity parameter is \( \psi = 1.5 \). As in Bansal and Yaron (2004), the model in this study relies on \( \psi > 1 \) to generate small and stable risk-free rates.\(^{20}\)

With \( \psi = 1.5 \) and consequently \( \psi \neq \rho = 1 \), the utility is non-separable between durable and nondurable consumption. This is a critical ingredient of the model that allows durable consumption growth, in particular, its long-run risk component, to generate important asset pricing implications.\(^{21}\)

\(^{20}\)Bansal and Yaron (2004) present more detailed discussions regarding the debate whether \( \psi \) is above or below 1.

\(^{21}\) For general \( \rho \), define

\[ \tilde{V}_t = \tilde{V}(D_t/C_t) = V_t/C_t = \left( 1 - \alpha + \alpha(D_t/C_t)^{1-\frac{1}{\rho}} \right)^{-\frac{1}{\rho}}. \]
5.2 Pricing kernel

The model has only one state variable \( x_t \), which fully characterizes the solutions. As shown in Appendix A, the approximate analytic solution for the log price-total consumption ratio is a linear function of \( x_t \),

\[
\log \frac{P_{g,t}}{G_t} = z_{g,t} \approx A_{g0} + A_{g1}x_t, \quad A_{g1} = \frac{(1 - \frac{1}{\psi})\alpha}{1 - \kappa_{g1}\phi}.
\]

(19)

Here, \( P_{g,t} \) is the ex-dividend price of the claim to the entire future stream of total consumption, \( \kappa_{g1} \) is a constant very close to 1, \( A_{g0} \) and \( A_{g1} \) are constants, and \( A_{g1} > 0 \) when \( \psi > 1 \). Therefore, a rise in the expected durable consumption growth increases the price of the total consumption claim. A higher persistence parameter \( \phi \) amplifies the response to a much greater extent. Since durable consumption makes up a fraction \( \alpha \) of total consumption, the magnitude of \( A_{g1} \) is proportional to \( \alpha \).

Following the price-total consumption ratio, the innovation to the pricing kernel is

\[
m_{t+1} - E_t[m_{t+1}] = -\gamma \sigma_c \varepsilon_c, t+1 - (\gamma - 1)\alpha(\sigma_d \omega_t \varepsilon_d, t+1 - \sigma_c \varepsilon_c, t+1) - (\gamma - \frac{1}{\psi})\frac{\kappa_{g1}\alpha}{1 - \kappa_{g1}\phi} \sigma_x \omega_t \varepsilon_x, t+1
\]

(20)

\[
= -\pi_c \sigma_c \varepsilon_c, t+1 - \pi_d(\sigma_d \omega_t \varepsilon_d, t+1 - \sigma_c \varepsilon_c, t+1) - \pi_x \sigma_x \omega_t \varepsilon_x, t+1.
\]

(21)

The first term is familiar from the studies based on power utility: the risk is the i.i.d. shocks to nondurable consumption growth, and its market price of risk is risk aversion \( \gamma \). In the second term, the risk is the random walk shocks to \( d_t - c_t \), or short-run shocks to the

The pricing kernel is [Bansal, Tallarini, and Yaron (2008) and Yogo (2006)]

\[
M_{t+1} = \delta^\theta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{\theta}{\gamma}} \left( \frac{\tilde{V}_{t+1}}{\tilde{V}_t} \right)^{\theta(\frac{1}{\rho} - \frac{1}{\psi})} R_{g,t+1}^{\theta-1}.
\]

If \( \rho = \psi \), then durable and nondurable consumption are separable, and durable consumption drops out of the pricing kernel.
ratio of durable to nondurable consumption. Its market price of risk is positive if $\gamma > 1$.\footnote{Under a general $\rho$, this is actually $\gamma > \frac{1}{\rho}$.}

The small volatilities of nondurable and durable consumption growth imply that, in order for these two terms to generate the observed level of equity premium, an extraordinarily high risk aversion is required.

In contrast, in the third term of the pricing kernel innovation, the risk is shocks to the expected durable consumption growth, and its market price of risk is positive if $\gamma > 1/\psi$, and can be very large with high persistence $\phi$. This term arises as a result of the long-run risk and Epstein-Zin preferences. It is contributed by the long-run risk component in durable consumption growth, and thus the market price of risk contains $\alpha$ and inherits the time-varying conditional volatility following $\omega_t$.

### 5.3 Real bond yields

Following the pricing kernel, the approximate analytical solution for the log risk-free rate can also be computed. The details are left to Appendix B. The risk-free rate is measured in terms of the numeraire nondurable good, and

$$r_{f,t} \approx -\log \delta + \frac{1}{\psi} \mu_c - (1 - \frac{1}{\psi}) \alpha (\mu_d + x_t - \mu_c) - \frac{\theta - 1}{\theta} E_t [r_{g,t+1} - r_{f,t}].$$

The equation highlights several determinants of the risk-free rate. In particular, the first three terms above are actually a special case of $\rho = 1$ for the general result of

$$- \log \delta + \frac{1}{\psi} \left( (1 - \alpha) \mu_c + \alpha (\mu_d + x_t) \right) - \frac{1}{\rho} \alpha (\mu_d + x_t - \mu_c).$$

Here, the first term is the discount effect. If $\delta$ is small, or future consumption is heavily discounted, the agent wants to borrow to increase consumption today, driving up the risk-
free rate. The second term reflects the effect of intertemporal substitution. If $\psi$ is small, the agent prefers to smooth over time. Hence, when both durable and nondurable consumption growth are high, the agent will want to borrow, driving up the risk-free rate. Lastly, the third term represents the effect of substitution between goods. If $\rho$ is small, the agent wants to smooth between durable and nondurable consumption. Consequently, when the expected durable consumption growth is higher than that for nondurable consumption, the agent will want to save so as to increase nondurable consumption in the future, driving down the risk-free rate.

The variations of the risk-free rate are practically determined by its dependence on $x_t$ shown above. In the subsequent calibration, $\psi = 1.5 > \rho = 1$, and thus the effect of goods smoothing dominates that of intertemporal smoothing. As a result, the risk-free rate varies counter-cyclically with $x_t$ and is high when $x_t$ is low (and conditional volatility, proportional to $\omega_t = \sqrt{1 - \zeta x_t}$, is high). The small unconditional volatility of $x_t$ also implies that the risk-free rate is rather smooth.

Appendix B also computes the log price for a $n$-period real zero-coupon bond

$$b_{n,t} \approx B_{n,0} + B_{n,1}x_t, \quad B_{n,1} = (1 - \frac{1}{\psi})\alpha \frac{1 - \phi^n}{1 - \phi}.$$ \hfill (23)

With $\psi > 1$, real bond yields, $-b_{n,t}/n$, also vary counter-cyclically. For the holding period return,

$$hpr_{n,t+1} = b_{n-1,t+1} - b_{n,t},$$ \hfill (24)

$$hpr_{n,t+1} - E_t[hpr_{n,t+1}] \approx B_{n-1,1}\sigma x_t \omega_t \varepsilon_{x,t+1}.$$ \hfill (25)
The positive loading implies a positive, counter-cyclical bond risk premium

\[ E_t[hpr_{n,t+1} - r_{f,t}] + \frac{1}{2} \text{var}_t[hpr_{n,t+1}] \approx - \text{cov}_t[m_{t+1}, hpr_{n,t+1}] \]
\[ = B_{n-1,1} \pi_x \sigma_y^2 \omega_t^2 = B_{n-1,1} \pi_x \sigma_y^2 (1 - \zeta x_t). \] (26)
(27)

As shown in Appendix B, this positive bond risk premium implies an upward sloping yield curve, and the slope also varies counter-cyclically.

5.4 Stock returns

The approximate analytical solution for the log price-dividend ratio is

\[ \log \frac{P_t}{Y_t} = z_t \approx A_0 + A_1 x_t, \quad A_1 = \frac{\lambda + (1 - \frac{1}{\psi}) \alpha}{1 - \kappa_1 \phi}. \] (28)

Here, \( P_t \) is the ex-dividend price of the dividend claim, \( Y_t \) is the dividend, \( \kappa_1 \) is a constant very close to 1, and \( A_0 \) and \( A_1 \) are constants. Since \( A_1 > 0 \), a positive shock to \( x_t \) is also magnified to a rise in the log price-dividend ratio. The magnification factor increases with the persistence parameter \( \phi \).

The return innovation is

\[ r_{t+1} - E_t[r_{t+1}] \approx \kappa_1 A_1 \sigma_x \omega_t \varepsilon_{x,t+1} + \sigma_y \varepsilon_{y,t+1} \]
\[ = \beta_x \sigma_x \omega_t \varepsilon_{x,t+1} + \sigma_y \varepsilon_{y,t+1}. \] (29)
(30)

Since all shocks are assumed to be mutually independent, the dividend claim is only exposed
to the long-run risk in durable consumption growth. Consequently,

\[
E_t[r_{t+1} - r_{f,t}] + \frac{1}{2} \text{var}_t[r_{t+1}] \approx -\text{cov}_t[m_{t+1}, r_{t+1}]
\]

\[
= \beta_x \pi_x \sigma_x^2 \omega_t^2 = \beta_x \pi_x \sigma_x^2 (1 - \zeta x_t).
\]

Hence, the expected excess return varies negatively with \( x_t \).

Note that since \( A_1 > A_{g1} \), the dividend claim has a larger exposure (or beta) to the long-run risk than the consumption claim. This implies that both risk premium and volatility are higher for stock returns than for wealth returns.

The model is also able to generate the volatility feedback effect, or the negative correlation between the return innovation and the conditional volatility innovation [Campbell and Hentschel (1992) and Glosten, Jaganathan, and Runkle (1993)], since

\[
\text{cov}_t \left[ r_{t+1} - E_t[r_{t+1}], \text{var}_{t+1}[r_{t+2}] - E_t[\text{var}_{t+1}[r_{t+2}]] \right] = -\beta_x^3 \sigma_x^4 \zeta \omega_t^2.
\]

The details of this result are presented in Appendix C.

Altogether, these results show that with non-separability, interesting asset market implications can be generated almost solely based on the empirically motivated dynamics of durable consumption growth, even though nondurable consumption growth is modeled as random walk, and there is no variation in consumption shares. Since durable consumption is a large fraction of aggregate consumption, and given the strong empirical evidence documented earlier, long-run durable consumption risk appears to be a plausible channel to explain the asset market phenomena.
6 Model implications

I solve the model numerically using the projection method in Judd (1998) with polynomial expansions. Expectations are evaluated using Gaussian quadrature with 8 nodes. To insure accuracy in the solution, I include up to 5th-order nonlinear terms of $x$ in polynomial expansions. To provide further assurance, I also use cubic splines to approximate the solutions, and find the results to be essentially the same. As demonstrated in the figures presented subsequently, the numerical solutions confirm the intuition illustrated in the approximate analytical solutions. The numerical solutions are obtained for the monthly frequency. Then I simulate monthly series, time-aggregate to the annual frequency, and compare with the observed annual data.

6.1 Solution

Fig. 3 plots the valuation ratios for consumption and dividend claims. Both the log price-total consumption ratio and the log price-dividend ratio are pro-cyclical functions of $x$, consistent with counter-cyclical expected returns.

The risk-free rate, as plotted in Fig. 4, is counter-cyclical. As indicated earlier, this is primarily the result of the effect of goods smoothing dominating that of intertemporal smoothing. When $x_t$ is high, the expected durable consumption growth is high. The tendency to smooth consumption over time will drive the agent to borrow from the future. However, the tendency to smooth between durable and nondurable goods will drive the agent to save so as to increase nondurable consumption in the future. With $\psi = 1.5 > \rho = 1$, the latter dominates the former, resulting in a low risk-free rate.

At $x = 0$, the annualized risk-free rate is 1.67%. The graph also suggests that the volatility of the risk-free rate is small. Fig. 4 also plots the 5-year real bond yield, indicating a counter-cyclical dependence on $x$. The curve is above the risk-free rate, suggesting a term
structure with a small positive slope. In addition, both the level and the slope of the term structure decrease when $x$ increases.

Fig. 4 shows that the risk premia for both consumption and dividend claims are counter-cyclical. As discussed earlier, this ultimately results from the counter-cyclical volatility in the expected durable consumption growth. The level of the consumption risk premium is low. At the annualized level, it is about $2.68\%$ at $x = 0$. On the other hand, the annualized equity premium is $5.17\%$ at $x = 0$. Variations in equity premium are also more sensitive to $x$, as indicated by the steeper slope of the equity premium curve in comparison to that for the consumption claim.

Fig. 5 plots the conditional volatilities and the Sharpe ratios, both annualized. The consumption claim has a much lower volatility than the dividend claim, while its Sharpe ratio is much higher. For example, at $x = 0$, the conditional volatility of the consumption return is about 0.038, in contrast to a level of 0.156 for the dividend claim. The conditional Sharpe ratio of the consumption return is 0.71 at $x = 0$, while that for the dividend claim is 0.33. These results are broadly consistent with the study of Lustig, Nieuwerburgh, and Verdelhan (2008), which shows that wealth returns are much less volatile than stock returns, and consumption risk premium is much lower than equity risk premium.

The maximal Sharpe ratio, which is also the volatility of the log pricing kernel, is 0.775 at $x = 0$. The consumption return is close to achieving the maximal Sharpe ratio, but the dividend claim is far below. As shown in Eq. (20), all three shocks — nondurable consumption growth shocks, random walk shocks to durable consumption growth, and shocks to the expected durable consumption growth — contribute to the variance of the pricing kernel. The volatilities due to the first two short-run risks are 0.033 and 0.050, respectively. These two sources combined contribute less than 1% of the variance of the pricing kernel. The long-run durable consumption risk, on the other hand, makes up more than 99% of the variations in the pricing kernel.
6.2 Simulation results

I turn to simulations to gauge the model’s quantitative performance in matching the key aspects of the observed asset market data. I simulate 10000 samples, each of $56 \times 12$ months, and then time-aggregate to the annual frequency. Table 7 reports the mean and 5th and 95th percentiles of key moments across the simulated samples, and compares them with those obtained from the empirical data.

The model produces an average equity premium of 5.02%, only slightly below the observed value. Note that the only source of equity premium in the model is the long-run durable consumption risk. Hence, the model performance in terms of generating sizable excess returns is impressive. The average return volatility of 0.153 is almost identical to the observed level. The mean risk-free rate is 1.72%, also matching the empirical estimate. The volatility of 0.06% is very low. This, however, is not a serious concern, since the volatility of the ex ante real risk-free rates is most likely much lower than the realized 2.23% for the ex post real rates. Indeed, Campbell and Cochrane (1999) construct their model to generate a constant risk-free rate. For the price-dividend ratio, the model implied moments are somewhat smaller than their empirical counterparts, but are largely consistent.

Overall, Table 7 suggests that the model in this paper is capable of reproducing the key aspects of the asset market data. Equity premium can be further increased by increasing risk aversion and/or the dividend leverage parameter. Additional sensitivity analyses also confirm that decreasing the elasticity of intertemporal substitution results in a higher risk-free rate, and a large durable consumption share is critical — a decrease in $\alpha$ lowers both equity premium and stock return volatility.
6.3 Predictability regressions

In the empirical stock market data, future excess returns are predictable by the price-dividend ratio [Campbell and Shiller (1988)]. In the model of this paper, equity premium varies counter-cyclically with $x_t$, log price-dividend ratio is a pro-cyclical function of $x_t$, and $x_t$ is a persistent process. Hence, excess returns are predictable by the price-dividend ratio with negative slopes in the model. Table 8 reports the results for the predictability regressions in the observed data, and compares with the average results from 10000 simulated samples. In these regressions, cumulative excess returns of horizons of 1 to 5 years are regressed on log price-dividend ratio. The “Data” column shows that for the observed data, the coefficient increases with the horizon, and the $R^2$ values also tend to increase.\(^{23}\) The “Model” column indicates that the simulated, model implied samples largely replicate the empirical results. Both the coefficient and the $R^2$ increase with the horizon. The average coefficients from the simulated samples are somewhat higher than the empirical values, while the $R^2$ values match the empirical estimates well. Overall, the model appears to be capable of capturing the salient features of the observed excess return predictability.

As presented in Section 3, this study also documents a number of empirical predictability relations. An important empirical fact that motivates the model in this paper is that durable consumption growth rates are predicted by the price-dividend ratio with positive slopes. In the model solution, the price-dividend ratio is a pro-cyclical function of $x$, the persistent component in durable consumption growth. Hence, the model solutions imply the same predictive relation as in the empirical data. Column A in Table 9 confirms that the model implied results are consistent with the empirical results presented in Panel B of Table 1.

\(^{23}\) Here, the slopes and $R^2$ obtained for the period 1952–2007 are low in magnitudes in comparison to, for example, those in Bansal and Yaron (2004) for the period 1929–1998. To provide assurance, I run the same predictability regressions for the sample of 1929–2007 and find that, for horizons of 1, 3, and 5 years, the slopes are -0.11, -0.30, and -0.46, while the $R^2$ values are 0.04, 0.14, and 0.25. These results are larger in magnitudes and comparable to those in Bansal and Yaron (2004). Consequently, I conclude that the small magnitudes reported in my paper are specific to the sample period 1952–2007.
Finally, in the model the conditional volatility varies counter-cyclically while the price-dividend ratio is pro-cyclicical. Hence, the model is able to replicate the empirically observed negative predictive relations between the realized volatility and the price-dividend ratio. Columns B and C of Table 9 report the model implied results for these predictability regressions. Log price-dividend ratio predicts the realized volatilities in the future with negative slopes. The realized volatility also predicts future log price-dividend ratios with negative slopes. These model implied results are broadly consistent with the empirical results reported in Panel B of Table 3.

7 Concluding remarks

In this paper I show that the empirical data properties suggest a model for durable consumption growth as containing a long-run risk component with counter-cyclical volatility. I model dividend growth as a levered claim on the long-run risk component, and show that non-separable Epstein-Zin preferences allow the long-run risk dynamics of durable consumption growth to generate interesting asset pricing implications, even though nondurable consumption growth is modeled as a pure random walk, and the consumption shares are constant. In concluding the paper, I point out some limitations and thus possible extensions of this study.

A simplifying assumption in this paper is to set the elasticity of substitution between durable and nondurable consumption to be 1. This yields constant consumption shares and saves an extra state variable. If the elasticity is different from 1, it implies a time-varying share of durable consumption and introduces composition risk, similar to that in Piazzesi, Schneider, and Tuzel (2007). While the effects of this variation on equity premium and return volatility are small as long as the elasticity is not very different from 1, it is interesting to explore the implications further.
A conclusion of this paper is that with non-separability, interesting implications could be generated by durable consumption even if nondurable consumption is modeled as unlikely to do so. One can extend the list of goods, and non-separability suggests that interesting dynamics along these additional dimensions will become reflected in asset prices. These interesting extensions are beyond the scope of the current paper and thus left for future research.

Lastly, following the convention in the consumption based asset pricing literature, this study takes the empirical properties of the durable consumption growth data as given in order to focus on the asset market implications. A general equilibrium model incorporating consumption and production decisions is worth exploring. This merits a separate study, and is also left for future research.

References


**Appendices**

In the appendices I present the approximate analytical solutions of the model. These solutions are based on log-linearization and thus require keeping track of linear terms of $x_t$. For tractability, I ignore quantitatively small contributions from the variance terms in the pricing equations. More discussions of this point are provided in Appendix D.

### A Log price-total consumption ratio

The pricing kernel is

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} + \theta \left(1 - \frac{1}{\psi}\right) \alpha (\Delta d_{t+1} - \Delta c_{t+1}) + (\theta - 1) r_{g,t+1}.$$ 

Also

$$\Delta d_{t+1} - \Delta c_{t+1} = \mu_d + x_t + \sigma_d \omega_t \varepsilon_{d,t+1} - \mu_c - \sigma_c \varepsilon_{c,t+1},$$

$$\Delta g_{t+1} = \Delta c_{t+1}.$$ 

Assume the price-total consumption ratio is

$$\log \frac{P_{g,t}}{G_t} = z_{g,t} \approx A_{g0} + A_{g1} x_t.$$  

(34)
Using the Taylor expansion,

\[ r_{g,t+1} = \log(1 + e^{z_{g,t+1}}) + \Delta g_{t+1} - z_{g,t} \]

\[ \approx \kappa_0 + \kappa_1 z_{g,t+1} + \Delta g_{t+1} - z_{g,t} \]

\[ = \kappa_0 + \kappa_1 A_g + \kappa_1 A_g x_{t+1} + \Delta c_{t+1} - z_{g,t} \]

\[ = \kappa_0 + \kappa_1 A_g + \kappa_1 A_g \phi x_t + \kappa_1 A_g \sigma_x \omega_t \varepsilon_{x,t+1} + \mu_c + \sigma_c \varepsilon_{c,t+1} - A_g - A_g x_t, \]

where \( \kappa_1 < 1 \), but very close to 1. The pricing equation

\[ 1 = E_t[e^{m_{t+1} + r_{g,t+1}}] \]

implies

\[ 0 \approx E_t[m_{t+1} + r_{g,t+1}] + \frac{1}{2} \text{var}_t[m_{t+1} + r_{g,t+1}]. \tag{35} \]

Collect terms linear in \( x_t \) and ignore quantitatively small contributions from the variance term, then

\[ 0 = \theta (1 - \frac{1}{\psi}) \alpha x_t + \theta (\kappa_1 A_g \phi x_t - A_g x_t), \quad A_g = \frac{(1 - \frac{1}{\psi})\alpha}{1 - \kappa_1 \phi}. \]

The return innovation is

\[ r_{g,t+1} - E_t[r_{g,t+1}] = \kappa_1 A_g \sigma_x \omega_t \varepsilon_{x,t+1} + \sigma_c \varepsilon_{c,t+1}. \]

The pricing kernel innovation is

\[ m_{t+1} - E_t[m_{t+1}] \]

\[ = -\frac{\theta}{\psi} \sigma_c \varepsilon_{c,t+1} + \theta(1 - \frac{1}{\psi}) \alpha (\sigma_d \omega_t \varepsilon_{d,t+1} - \sigma_c \varepsilon_{c,t+1}) \]

\[ + (\theta - 1) (\kappa_1 A_g \sigma_x \omega_t \varepsilon_{x,t+1} + \sigma_c \varepsilon_{c,t+1}) \]

\[ = -\gamma \sigma_c \varepsilon_{c,t+1} - (\gamma - 1) \alpha (\sigma_d \omega_t \varepsilon_{d,t+1} - \sigma_c \varepsilon_{c,t+1}) + (\theta - 1) \kappa_1 A_g \sigma_x \omega_t \varepsilon_{x,t+1} \]

\[ = -\gamma \sigma_c \varepsilon_{c,t+1} - (\gamma - 1) \alpha (\sigma_d \omega_t \varepsilon_{d,t+1} - \sigma_c \varepsilon_{c,t+1}) - (\gamma - \frac{1}{\psi}) \frac{\kappa_1 \alpha}{1 - \kappa_1 \phi} \sigma_x \omega_t \varepsilon_{x,t+1}. \]
In addition,

\[
E_t[r_{g,t+1}] = \kappa g_0 + \kappa g_1 A g_0 + \kappa g_1 A g_1 \phi x_t + \mu_c - A g_0 - A g_1 x_t \\
= \kappa g_0 + \kappa g_1 A g_0 + \mu_c - A g_0 - (1 - \frac{1}{\psi}) \alpha x_t,
\]

and

\[
E_t[m_{t+1}] = \theta \log \delta - \frac{\theta}{\psi} \mu_c + \theta (1 - \frac{1}{\psi}) \alpha (\mu_d + x_t - \mu_c) + (\theta - 1) E_t[r_{g,t+1}] \\
= \text{const} + \theta (1 - \frac{1}{\psi}) \alpha x_t - (\theta - 1)(1 - \frac{1}{\psi}) \alpha x_t \\
= \text{const} + (1 - \frac{1}{\psi}) \alpha x_t.
\]

**B Real bond yields**

The pricing equation

\[
1 = E_t[e^{m_{t+1}+r_{f,t}}]
\]

implies

\[
r_{f,t} \approx -E_t[m_{t+1}] - \frac{1}{2} \text{var}_t[m_{t+1}].
\]

Ignoring the quantitatively small variance term, this implies

\[
r_{f,t} \approx -\theta \log \delta + \frac{\theta}{\psi} \mu_c - \theta (1 - \frac{1}{\psi}) \alpha (\mu_d + x_t - \mu_c) - (\theta - 1) E_t[r_{g,t+1}].
\]

Add \((\theta - 1)r_{f,t}\) to both sides, and divide by \(\theta\) (assume \(\theta \neq 0\)),

\[
r_{f,t} \approx -\log \delta + \frac{1}{\psi} \mu_c - (1 - \frac{1}{\psi}) \alpha (\mu_d + x_t - \mu_c) - \theta - 1 \frac{\theta - 1}{\theta} E_t[r_{g,t+1} - r_{f,t}].
\]

Variations in \(r_{f,t}\) are practically driven by the \(x_t\) term in the above.

Assume the log price of a \(n\)-period real zero-coupon bond is

\[
b_{n,t} = B_{n,0} + B_{n,1} x_t.
\]
Then the log holding period return is

\[ hpr_{n,t+1} = b_{n-1,t+1} - b_{n,t}. \]

In

\[ 0 \approx E_t[m_{t+1} + hpr_{n,t+1}] + \frac{1}{2} \text{var}_t[m_{t+1} + hpr_{n,t+1}], \]

collect terms linear in \( x_t \) and ignore quantitatively small contributions from the variance term, then

\[ 0 = (1 - \frac{1}{\psi'})\alpha x_t + B_{n-1,1}\phi x_t - B_{n,1}x_t. \]

With the initial condition \( B_{0,1} = 0 \), it yields

\[ B_{n,1} = (1 - \frac{1}{\psi'})\alpha \frac{1 - \phi^n}{1 - \phi}. \]

Hence the log return innovation is

\[ hpr_{n,t+1} - E_t[hpr_{n,t+1}] = B_{n-1,1}\sigma \omega_t \varepsilon_{x,t+1}. \]

Using zero yields

\[ r_{f,n,t} = -\frac{b_{n,t}}{n}, \]

The excess return can be rewritten as

\[ hpr_{n,t+1} - r_{f,t} = nr_{f,n,t} - (n - 1)r_{f,n-1,t+1} - r_{f,t} \]

Hence

\[ E_t[hpr_{n,t+1} - r_{f,t}] = nr_{f,n,t} - E_t[(n - 1)r_{f,n-1,t+1}] - r_{f,t}. \]

The left hand side is the one-period bond risk premium, while the right hand side is the one period term premium.
C Log price-dividend ratio

Log dividend growth is

$$\Delta y_{t+1} = \lambda x_t + \sigma_y \varepsilon_{y,t+1}. $$

Assume

$$\log P_t = \frac{Y_t}{Y_t} \approx A_0 + A_1 x_t. \tag{36}$$

Using the Taylor expansion,

$$r_{t+1} = \log(1 + e^{z_{t+1}}) + \Delta y_{t+1} - z_t$$

$$\approx \kappa_0 + \kappa_1 z_{t+1} + \Delta y_{t+1} - z_t$$

$$= \kappa_0 + \kappa_1 A_0 + \kappa_1 A_1 x_{t+1} + \Delta y_{t+1} - z_t$$

$$= \kappa_0 + \kappa_1 A_0 + \kappa_1 A_1 \phi x_t + \kappa_1 A_1 \sigma_x \omega t \varepsilon_{x,t+1} + \lambda x_t + \sigma_y \varepsilon_{y,t+1} - A_0 - A_1 x_t. $$

Finally, in

$$0 \approx E_t[m_{t+1} + r_{t+1}] + \frac{1}{2} \text{var}_t[m_{t+1} + r_{t+1}], \tag{37}$$

collect terms linear in $x_t$ and ignore quantitatively small contributions from the variance term, then

$$0 = (1 - \frac{1}{\psi}) \alpha x_t + \kappa_1 A_1 \phi x_t + \lambda x_t - A_1 x_t,$$

$$A_1 = \frac{\lambda + (1 - \frac{1}{\psi}) \alpha}{1 - \kappa_1 \phi}. $$

Hence the log return innovation is

$$r_{t+1} - E_t[r_{t+1}] = \kappa_1 A_1 \sigma_x \omega t \varepsilon_{x,t+1} + \sigma_y \varepsilon_{y,t+1} = \beta_x \sigma_x \omega t \varepsilon_{x,t+1} + \sigma_y \varepsilon_{y,t+1}. $$

The conditional variance is

$$\text{var}_t[r_{t+1}] = \beta_x^2 \sigma_x^2 \omega_t^2 + \sigma_y^2.
Hence, the innovation to the conditional variance is
\[
\text{var}_{t+1}[r_{t+2}] - E_t[\text{var}_{t+1}[r_{t+2}]] = -\beta_x^2 \sigma_x^2 \zeta \sigma \omega \varepsilon_{x,t+1}.
\]
The covariance between the return innovation and the conditional variance innovation is
\[
\text{cov}_t[r_{t+1} - E_t[r_{t+1}], \text{var}_{t+1}[r_{t+2}] - E_t[\text{var}_{t+1}[r_{t+2}]]] = -\beta_x^3 \sigma_x^4 \zeta \omega^2.
\]

D Remarks on variance terms

As presented above, the log-linear approximate analytical solutions involve keeping track of linear \( x_t \) terms in the pricing equations. In doing so, I have ignored the variance terms.

As demonstrated in Bansal and Yaron (2004), variance terms can be critical for deriving approximate analytical solutions. In particular, in their model, volatility is a separate state variable. In order to track this state variable, it is essential to include the variance terms.

My model has only one state variable \( x_t \). Underlying the conditional volatility is \( \omega_t = \sqrt{1 - \zeta x_t} \), an exact function of \( x_t \). In other words, volatility or \( \omega_t \) is not a separate state variable. Consequently, conditional volatility generates time-varying implications solely through \( x_t \) terms. For example, as long as valuation ratios depend on \( x_t \), then the pricing kernel innovation and the return innovation contain \( x \) shocks and exhibit time-varying volatility. Consequently, expected excess return and return volatility vary with time.

Hence, for log-linear approximations, it is important to track the linear \( x_t \) terms properly. The variance terms in the pricing equations also generate linear \( x_t \) terms. For example, the variance term in the pricing equation for the consumption claim [Eq. (35) in Appendix A] is
\[
\text{var}_t[m_{t+1} + r_{g,t+1}] = \text{const} + \theta^2(1 - \frac{1}{\psi})^2 \alpha^2 \sigma^2 \omega_t^2 + \theta^2 \kappa_{g1} A_g \sigma^2 \omega_t \omega_t;
\]
and the variance term in the pricing equation for the dividend claim [Eq. (37) in Appendix C] is
\[
\text{var}_t[m_{t+1} + r_{t+1}] = \text{const} + \theta^2(1 - \frac{1}{\psi})^2 \alpha^2 \sigma^2 \omega_t^2 + ((\theta - 1) \kappa_{g1} A_g + \kappa_1 A_1) \sigma^2 \omega_t \omega_t.
\]
These variance terms contribute linear \( x_t \) terms because \( \omega_t^2 = 1 - \zeta x_t \).

I have ignored these contributions for two reasons. First, they tend to be small, as a result of small variances \( \sigma^2_d \) and \( \sigma^2_x \). Second, including these terms results in much less tractable
equations for the unknowns $A_{g_1}$ and $A_1$, and the solutions will be difficult to interpret.

In the paper I confirm that the approximate analytical solutions are qualitatively consistent with the more accurate numerical solutions, which are plotted in the figures. Hence, ignoring the variance terms does not appear to alter the qualitative nature of the approximate analytical solutions. On the other hand, the numerical solutions also indicate nontrivial nonlinearities. The model’s quantitative performance is ultimately assessed with the numerical solutions and the simulations based on them.
Table 1: Persistent expected component in durable consumption growth

A: Autocorrelation

<table>
<thead>
<tr>
<th>Sample</th>
<th>$AC(1)$ Std Err</th>
<th>$AC(2)$ Std Err</th>
<th>$AC(3)$ Std Err</th>
</tr>
</thead>
<tbody>
<tr>
<td>1952–2007</td>
<td>0.65 (0.08)</td>
<td>0.42 (0.11)</td>
<td>0.07 (0.14)</td>
</tr>
<tr>
<td>1952–1979</td>
<td>0.51 (0.13)</td>
<td>0.40 (0.17)</td>
<td>0.14 (0.17)</td>
</tr>
<tr>
<td>1980–2007</td>
<td>0.79 (0.05)</td>
<td>0.44 (0.09)</td>
<td>0.04 (0.15)</td>
</tr>
</tbody>
</table>

B. Predictability by price-dividend ratio

<table>
<thead>
<tr>
<th>Horizon (year)</th>
<th>$b$</th>
<th>Std Err</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.017</td>
<td>(0.005)</td>
<td>0.10</td>
</tr>
<tr>
<td>3</td>
<td>0.052</td>
<td>(0.015)</td>
<td>0.13</td>
</tr>
<tr>
<td>5</td>
<td>0.092</td>
<td>(0.032)</td>
<td>0.20</td>
</tr>
</tbody>
</table>

In Panel A, autocorrelation coefficients are computed with GMM. In Panel B, cumulative durable consumption growth from $t$ to $t + J$ is regressed on log price-dividend ratio of time $t$. Standard errors are Newey and West (1987) corrected with 10 lags.

Table 2: Skewness of durable consumption growth

<table>
<thead>
<tr>
<th>Sample</th>
<th>Skewness</th>
<th>Bootstrap Confidence Percentiles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>5%</td>
</tr>
<tr>
<td>1952–2007</td>
<td>-0.51</td>
<td></td>
</tr>
<tr>
<td>1952–1979</td>
<td>-0.53</td>
<td>-0.86</td>
</tr>
<tr>
<td>1980–2007</td>
<td>-0.47</td>
<td>-0.96</td>
</tr>
</tbody>
</table>

Skewness estimates are corrected for the small sample bias. Bootstrap confidence percentiles are computed by resampling the original data for 10000 times.
Table 3: Realized volatility of durable consumption growth

A. Variance ratios

<table>
<thead>
<tr>
<th>Horizon (year)</th>
<th>VR</th>
<th>VR adj</th>
<th>Bootstrap Percentiles</th>
<th>Mean</th>
<th>5%</th>
<th>10%</th>
<th>90%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.09</td>
<td>1.11</td>
<td>0.98</td>
<td>0.76</td>
<td>0.80</td>
<td>1.16</td>
<td>1.22</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1.31</td>
<td>1.39</td>
<td>0.94</td>
<td>0.57</td>
<td>0.63</td>
<td>1.28</td>
<td>1.40</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1.48</td>
<td>1.61</td>
<td>0.92</td>
<td>0.50</td>
<td>0.57</td>
<td>1.31</td>
<td>1.46</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>1.51</td>
<td>1.77</td>
<td>0.85</td>
<td>0.36</td>
<td>0.43</td>
<td>1.37</td>
<td>1.59</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1.62</td>
<td>2.01</td>
<td>0.81</td>
<td>0.30</td>
<td>0.36</td>
<td>1.38</td>
<td>1.63</td>
<td></td>
</tr>
</tbody>
</table>

B. Predictability regressions

| Horizon (year) | $|\epsilon_{d}\Delta|$ predicted by $p-d$ | $p-d$ predicted by $|\epsilon_{d}\Delta|$ |
|---------------|---------------------------------|---------------------------------|
|               | $b$ | Std Err | $R^2$ | $b$ | Std Err | $R^2$ |
| 1             | -0.0065 | (0.0026) | 0.104 | -14.0 | (9.7) | 0.079 |
| 3             | -0.0050 | (0.0034) | 0.059 | -18.5 | (8.6) | 0.131 |
| 5             | -0.0048 | (0.0034) | 0.055 | -17.2 | (8.5) | 0.108 |

Realized volatility $|\epsilon_{d}\Delta|$ is the absolute value of the residuals from an AR(5) regression of durable consumption growth.

Panel A presents the variance ratios ($VR$). Bootstrap percentiles are computed under the null hypothesis of no serial correlation by resampling the realized volatility series for 10000 times. “VR adj” is VR divided by the bootstrap mean to adjust for the small sample bias.

In Panel B, realized volatility at $t + J$ is regressed on log price-dividend ratio of time $t$, and log price-dividend ratio at $t + J$ is regressed on realized volatility of time $t$. Standard errors are Newey and West (1987) corrected with 10 lags.
Table 4: Empirical properties of nondurable consumption growth

A: Autocorrelation

<table>
<thead>
<tr>
<th></th>
<th>AC(1) Std Err</th>
<th>AC(2) Std Err</th>
<th>AC(3) Std Err</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.36 (0.11)</td>
<td>0.03 (0.17)</td>
<td>-0.05 (0.15)</td>
</tr>
</tbody>
</table>

B: Skewness

<table>
<thead>
<tr>
<th>Skewness</th>
<th>Bootstrap Confidence Percentiles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5%</td>
</tr>
<tr>
<td>-0.18</td>
<td>-0.49</td>
</tr>
</tbody>
</table>

C. Variance ratios of realized volatility

<table>
<thead>
<tr>
<th>Horizon (year)</th>
<th>VR</th>
<th>VR adj</th>
<th>Bootstrap Percentiles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>2</td>
<td>0.86</td>
<td>0.88</td>
<td>0.98</td>
</tr>
<tr>
<td>4</td>
<td>0.83</td>
<td>0.88</td>
<td>0.94</td>
</tr>
<tr>
<td>5</td>
<td>0.87</td>
<td>0.94</td>
<td>0.92</td>
</tr>
<tr>
<td>8</td>
<td>0.77</td>
<td>0.90</td>
<td>0.86</td>
</tr>
<tr>
<td>10</td>
<td>0.86</td>
<td>1.06</td>
<td>0.81</td>
</tr>
</tbody>
</table>

Autocorrelation coefficients are computed with GMM. Standard errors are Newey and West (1987) corrected with 10 lags.

Skewness estimates are corrected for the small sample bias. Bootstrap confidence percentiles are computed by resampling the original data for 10000 times.

Variance ratios are computed for the realized volatility, or the absolute value of the residuals from an AR(5) regression. Bootstrap percentiles are computed under the null hypothesis of no serial correlation by resampling the realized volatility series for 10000 times. “VR adj” is VR divided by the bootstrap mean to adjust for the small sample bias.
Table 5: Calibrated parameters

Durable

<table>
<thead>
<tr>
<th>$\mu_d$</th>
<th>$\sigma_d$</th>
<th>$\phi$</th>
<th>$\sigma_s$</th>
<th>$\zeta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00325</td>
<td>0.0026</td>
<td>0.99</td>
<td>0.000242</td>
<td>278</td>
</tr>
</tbody>
</table>

Nondurable

<table>
<thead>
<tr>
<th>$\mu_c$</th>
<th>$\sigma_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00173</td>
<td>0.004</td>
</tr>
</tbody>
</table>

Dividend

<table>
<thead>
<tr>
<th>$\mu_y$</th>
<th>$\sigma_y$</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00159</td>
<td>0.0381</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Preferences

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\delta$</th>
<th>$\gamma$</th>
<th>$\psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>0.998</td>
<td>8</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Calibrated parameters for the growth rate dynamics and the preferences at the monthly interval.
Table 6: Empirical and model implied growth rates

<table>
<thead>
<tr>
<th></th>
<th>Data Estimate</th>
<th>Std Err</th>
<th>Model Simulation Mean</th>
<th>5%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A. Durable consumption growth ((\Delta d))</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean (%)</td>
<td>3.93</td>
<td>(0.33)</td>
<td>3.92</td>
<td>2.08</td>
<td>5.37</td>
</tr>
<tr>
<td>Std Dev (%)</td>
<td>1.88</td>
<td>(0.18)</td>
<td>1.88</td>
<td>1.20</td>
<td>2.83</td>
</tr>
<tr>
<td>AC(1)</td>
<td>0.65</td>
<td>(0.08)</td>
<td>0.63</td>
<td>0.38</td>
<td>0.83</td>
</tr>
<tr>
<td>AC(2)</td>
<td>0.42</td>
<td>(0.11)</td>
<td>0.49</td>
<td>0.20</td>
<td>0.75</td>
</tr>
<tr>
<td>AC(3)</td>
<td>0.07</td>
<td>(0.14)</td>
<td>0.38</td>
<td>0.06</td>
<td>0.67</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.51</td>
<td></td>
<td>-0.51</td>
<td>-1.19</td>
<td>0.07</td>
</tr>
<tr>
<td>(</td>
<td>\epsilon_{\Delta d}</td>
<td>-VR(5)</td>
<td>1.48</td>
<td></td>
<td>1.19</td>
</tr>
<tr>
<td>(</td>
<td>\epsilon_{\Delta d}</td>
<td>-VR(10)</td>
<td>1.62</td>
<td></td>
<td>1.26</td>
</tr>
<tr>
<td></td>
<td>B. Nondurable consumption growth ((\Delta c))</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean (%)</td>
<td>2.08</td>
<td>(0.17)</td>
<td>2.08</td>
<td>1.78</td>
<td>2.38</td>
</tr>
<tr>
<td>Std Dev (%)</td>
<td>1.12</td>
<td>(0.09)</td>
<td>1.12</td>
<td>0.94</td>
<td>1.31</td>
</tr>
<tr>
<td></td>
<td>C. Dividend growth ((\Delta y))</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean (%)</td>
<td>1.93</td>
<td>(0.8)</td>
<td>1.93</td>
<td>-1.11</td>
<td>4.83</td>
</tr>
<tr>
<td>Std Dev (%)</td>
<td>10.7</td>
<td>(1.0)</td>
<td>10.7</td>
<td>9.0</td>
<td>12.6</td>
</tr>
<tr>
<td>AC(1)</td>
<td>0.29</td>
<td>(0.11)</td>
<td>0.22</td>
<td>0.01</td>
<td>0.41</td>
</tr>
<tr>
<td>AC(2)</td>
<td>-0.02</td>
<td>(0.10)</td>
<td>-0.02</td>
<td>-0.24</td>
<td>0.20</td>
</tr>
<tr>
<td>corr((\Delta y, \Delta d))</td>
<td>0.11</td>
<td>(0.08)</td>
<td>0.07</td>
<td>-0.19</td>
<td>0.32</td>
</tr>
</tbody>
</table>

Comparison between the properties of the empirical growth rate data and the mean and percentiles from 10000 samples simulated from the models of the growth rates. Empirical estimates are obtained with GMM, and standard errors are Newey and West (1987) corrected with 10 lags.
Table 7: Asset pricing implications

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model Simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>Std Err</td>
</tr>
<tr>
<td>$E[r-r_f]$</td>
<td>5.46% (1.79%)</td>
<td></td>
</tr>
<tr>
<td>$\sigma[r]$</td>
<td>0.154 (0.015)</td>
<td></td>
</tr>
<tr>
<td>$E[r_f]$</td>
<td>1.68% (0.50%)</td>
<td></td>
</tr>
<tr>
<td>$\sigma[r_f]$</td>
<td>2.23% (0.21%)</td>
<td></td>
</tr>
<tr>
<td>$E[P/Y]$</td>
<td>32.2 (4.4)</td>
<td></td>
</tr>
<tr>
<td>$\sigma[p-y]$</td>
<td>0.354 (0.034)</td>
<td></td>
</tr>
<tr>
<td>$AC(1)[p-y]$</td>
<td>0.91 (0.04)</td>
<td></td>
</tr>
<tr>
<td>$AC(2)[p-y]$</td>
<td>0.81 (0.06)</td>
<td></td>
</tr>
</tbody>
</table>

Comparison between the properties of the empirical asset market data and the mean and percentiles from 10000 samples simulated from the model. Empirical estimates are obtained with GMM, and standard errors are Newey and West (1987) corrected with 10 lags.

Table 8: Predictability of excess return by price-dividend ratio

<table>
<thead>
<tr>
<th>Horizon (year)</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$b$</td>
<td>Std Err</td>
</tr>
<tr>
<td>1</td>
<td>-0.096</td>
<td>(0.046)</td>
</tr>
<tr>
<td>3</td>
<td>-0.188</td>
<td>(0.096)</td>
</tr>
<tr>
<td>5</td>
<td>-0.264</td>
<td>(0.124)</td>
</tr>
</tbody>
</table>

Comparison between the empirical results of predictability regressions and the averages of the results from 10000 samples simulated from the model. Cumulative excess stock return from $t$ to $t+J$ is regressed on log price-dividend ratio of time $t$. Standard errors are Newey and West (1987) corrected with 10 lags.
Table 9: Model implied predictability of durable consumption growth, realized volatility, and price-dividend ratio

| Horizon (year) | A. $\Delta d$ by $p-d$ | B. $|\epsilon_{\Delta d}|$ by $p-d$ | C. $p-d$ by $|\epsilon_{\Delta d}|$ |
|---------------|-------------------------|---------------------------------|-------------------------|
|               | $b$          | $R^2$           | $b$          | $R^2$           | $b$          | $R^2$           |
| 1             | 0.045        | 0.19            | -0.0042      | 0.032           | -3.2         | 0.034           |
| 3             | 0.105        | 0.24            | -0.0031      | 0.030           | -5.2         | 0.039           |
| 5             | 0.143        | 0.26            | -0.0023      | 0.028           | -6.4         | 0.047           |

Model implied results are averages from 10000 samples simulated from the model. Realized volatility $|\epsilon_{\Delta d}|$ is the absolute value of the residuals from an AR(5) regression of durable consumption growth. In Column A, cumulative durable consumption growth from $t$ to $t+J$ is regressed on log price-dividend ratio of time $t$. In Column B, realized volatility at $t+J$ is regressed on log price-dividend ratio of time $t$. In Column C, log price-dividend ratio at $t+J$ is regressed on realized volatility of time $t$.

Standard errors are Newey and West (1987) corrected with 10 lags.
Figure 1: Consumption growth

The top panel plots $\omega(x) = \sqrt{1-\zeta x}$ with calibrated $\zeta = 278$. The bottom panel plots the unconditional distribution of $x$. 

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Log price-total consumption ratio \( \log(P/G) \) and log price-dividend ratio \( \log(P/Y) \) as functions of \( x \).
Figure 4: Real rates and expected excess returns

Risk-free rate $r_f \times 12$, 5-year real bond yield $r_f(5y)$, expected excess return to consumption claim $E[r_g - r_f] \times 12$, and expected excess return to dividend claim $E[r - r_f] \times 12$. All are annualized and plotted as functions of $x$. 
The top panel plots the conditional volatilities of consumption claim $\sigma[r_g] \times \sqrt{12}$ and dividend claim $\sigma[r] \times \sqrt{12}$. The bottom panel plots the conditional Sharpe ratios of consumption claim $\frac{E[r_g-r_f]}{\sigma[r_g]} \times \sqrt{12}$ and dividend claim $\frac{E[r-r_f]}{\sigma[r]} \times \sqrt{12}$. All are annualized and plotted as functions of $x$. 

Figure 5: Conditional volatility and Sharpe ratio