# Regime Switching in Dynamics of Risk Premium: Evidence from SHIBOR

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## ABSTRACT

In the light of regime switching and volatility clustering in the dynamics of SHIBOR, regime-switching CIR model (RSCIR) and regime-switching GARCH CIR model (RSCIR-GARCH) are established by introducing regime-switching and GARCH specifications into CIR model successively. Then, a contrast study among CIR, RSCIR and RSCIR-GARCH models is performed based on SHIBOR sample data, which indicates that the regime-switching and GARCH specifications can improve the model fitness significantly and eliminate the ARCH effect in the volatility dynamics. Furthermore, an empirical research is carried out on the risk premium dynamics of SHIBOR based on the RSCIR-GARCH model, and it is found that there are two regimes, that is, the regime of higher interest rate with higher volatility and the regime of lower interest rate with lower volatility, in the dynamics of the term structure and risk premium of SHIBOR.

**Keywords:** SHIBOR; risk premium; regime switching; GARCH; Cox-Ingersoll-Ross model; Kim filter
**JEL classification:** C58; E43; G12

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## 1 Introduction

Modeling the term structure of interest rates plays a significant role in pricing fixed income derivatives, in risk management and in designing macroeconomic policies. So a wide range of interest rate models, such as Vasicek (1977) model and Cox-Ingersoll-Ross (1985) model, have been put forward to capture the dynamic behavior of the short-term riskless rate. However, there is increasing evidence in the literature that changes in the business cycle conditions or monetary policy may cause interest rates to behave quite differently in different time periods, both in terms of level and volatility, so regime-switching models are suggested instead of single-regime models in capturing the dynamics of the short rate, which is justified by extensive empirical literature on bond yields (Sola and Driffill, 1994, Garcia and Perron, 1996, Dahlquist and Gray, 2000, etc.).

The regime-switching models in the literature are solved under different assumptions about the evolution of the short rate. Naik and Lee (1997) introduce regime-switching into the Vasicek model with the assumption that only the volatility term is allowed to switch, while Bansal and Zhou (2002) adopt a regime-switching CIR model (hereafter RSCIR model) in capturing the dynamics of the short rate, with the assumption that the full set of parameters is allowed to switch. Dahlquist and Gray (2000) use a regime-switching CKLS model to characterize short rates in the European Monetary System. Then Naik and Lee (1997), Landen (2000), and Dai and Singleton (2003, 2007) further develop short rate models into continuous-time regime-switching dynamic term structure models (DTSMs) that yield closed-form solutions for zero-coupon bond prices.

In addition to regime switching, characteristics of volatility clustering and leptokurtosis are also observed in the dynamics of the short rate. So, scholars combine regime switching with GARCH or stochastic volatility to improve the fitness of the model to the historical interest rate data. Among others, Gray (1996) and Bauwens et al. (2010) propose a Markov-switching GARCH model, while Smith (2002) a Markov-switching stochastic volatility model, both of which are applied to capture the dynamics of the short rate of US, leading to the conclusion that the introduction of GARCH or stochastic volatility significantly reduces the volatility persistence of the short rate and thus improves the fitness of regime-switching models.

However, the majority of the studies above focus on the model specification and fitness to the historical data and mainly use US datasets. Recent attempts to apply regime-switching short rate models to countries other than the US include Dahlquist and Gray (2000) using European data, Erlandsson (2002) using Swedish data, Ang and Bekaert (2002) and Christiansen (2008) using UK and German data. These studies justify regime-switching short rate models and investigate the parameter instability of single-regime interest rate models for different countries. This instability in the interest rate process could have important implications not only for capturing the dynamics of the short rate, but also for valuing fixed income securities and interest rate derivatives and for hedging interest rate risk. For example, the level of interest rates is important to the valuation of stocks, bonds, index futures and options, etc. In addition, the volatility of interest rates is a fundamental determinant in the valuation of interest rate derivatives. Moreover, optimal hedging strategies for risk-averse investors are very sensitive to changes in interest rate volatility.

In this paper, we apply a regime-switching GARCH CIR model (hereafter RSCIR-GARCH) to describe the behavior of the risk premium implied in the short-term interest rate in China, using one-month Shanghai Interbank Offered Rate (SHIBOR)[1] as sample data. First, we investigate the regime-switching and leptokurtosis characteristics of the short rate dynamics, based on whose result we establish regime-switching CIR model (RSCIR) and regime-switching GARCH CIR model (RSCIR-GARCH) by introducing regime-switching and GARCH specifications into CIR model successively. Then, a contrast study among CIR, RSCIR and RSCIR-GARCH models is performed based on the one-month SHIBOR sample data, which shows that the regime-switching and GARCH specifications both improve the model fitness significantly and eliminate the ARCH effect in the volatility dynamics. In the light of the above, we perform an empirical research on the risk premium dynamics of SHIBOR using the RSCIR-GARCH model estimated by the Kim filter based maximum likelihood method, finding that there are two distinct regimes, that is, the regime of higher interest rate with higher volatility and the regime of lower interest rate with lower volatility, in the dynamics of both the term structure and risk premium of SHIBOR. The ‘high-volatility’ regime is characterized by periods of extremely high and volatile interest rates, and the corresponding risk premium also has a high level and fluctuation, while in the ‘low-volatility’ regime, in contrast, both the short rate and the risk premium display a low mean and volatility. However, the mean-reversion of the short rate in the low-volatility regime is much stronger than that in the high-volatility regime. Besides, it is also found that the ARCH effect in the volatility dynamics is only significant in the high-volatility regime but not in the low-volatility regime. Furthermore, we make a comparison between the CIR model and the RSCIR-GARCH model in capturing the dynamics of the risk premium implied in the short rate, leading to the conclusion that the RSCIR-GARCH model by far outperforms the CIR model in capturing regime switching in the dynamics of risk premium and thus in reflecting changes in market conditions, which is critical for pricing and hedging of assets.

The remainder of the present paper is organized in the following manner. Section 2 presents the RSCIR model and the RSCIR-GARCH model. Section 3 discusses the Kim filter (Kim, 1994) based maximum likelihood estimator for the models. Section 4 discusses the empirical results and Section 5 presents concluding comments.

## 2 Short Rate Models

In this section, we present the RSCIR models with and without GARCH specification for the short rate volatility based on the regime-switching model proposed by Bansal and Zhou (2002).

[1] Shanghai Interbank Offered Rate (SHIBOR) is a daily reference rate based on the interest rates at which banks offer to lend unsecured funds to other banks in the Shanghai wholesale (or "interbank") money market. There are eight SHIBOR rates, with maturities ranging from overnight to a year. They are calculated from rates quoted by sixteen banks, eliminating the two highest and the two lowest rates, and then averaging the remaining twelve.

Following Bansal and Zhou (2002), we assume the dynamics of the short rate governed by a regime-switching square root diffusion process:

$$r_t - r_{t-1} = k_{s_t}(\theta_{s_t} - r_{t-1}) + \sigma_{s_t}\sqrt{r_{t-1}}u_t \tag{1}$$

where $r_t$ is the short rate at time $t$. $k_{s_t}, \theta_{s_t}$ and $\sigma_{s_t}$ are the regime-dependent mean reversion, long-run mean, and volatility parameters respectively and $u_t$ is the innovation which is conditionally normal given $r_{t-1}$ and $s_t$.

Given $\Phi_{t-1}$ the information set at time $t-1$, the conditional distribution of $r_t$ is a mixture of distributions in all regimes, with the regime probabilities as the weights. Here, we assume there are two states for the short rate regime, that is $s_t = 0$ and $s_t = 1$. Then the conditional distribution of $r_t$ can be written as

$$r_t \mid \Phi_{t-1} \sim \begin{cases} N(\mu_0, V_0) & w.p. \quad p_{0t} \\ N(\mu_1, V_1) & w.p. \quad (1 - p_{0t}) \end{cases} \tag{2}$$

where $\mu_{s_t} = r_{t-1} + k_{s_t}(\theta_{s_t} - r_{t-1})$ and $V_{s_t} = \sigma_{s_t}^2 r_{t-1}$, $s_t = 0,1$, $p_{0t} = \Pr(s_t = 0 \mid \Phi_{t-1})$.

In order to investigate the contributions of regime-switching and GARCH specifications respectively to the improvement of model fitness to SHIBOR data, we assume the volatility parameter $\sigma_{s_t}$ in Equation (1) has two forms as follows: $\sigma_{s_t}^2(t) = c_{s_t}$ and $\sigma_{s_t}^2(t) = \frac{1}{r_{t-1}}\left[c_{s_t} + a_{s_t}\eta_{t-1}^2 + b_{s_t}\sigma^2(t-1)\right]$, where

$\sigma^2(t-1) = \sum_{s_{t-1}} p_{s_{t-1}}\left(\mu_{s_{t-1}}^2 + \sigma_{s_{t-1}}^2(t-1)r_{t-2}\right) - \left(\sum_{s_{t-1}} p_{s_{t-1}}\mu_{s_{t-1}}\right)^2$, $\eta_{t-1} = r_{t-1} - \sum_{s_{t-1}} p_{s_{t-1}}\mu_{s_{t-1}}$, and $p_{s_{t-1}}$ is the priori probability of the regime at time $t-1$, $s_{t-1}$. We name by RSCIR the model with the former volatility specification and by RSCIR-GARCH the model with the latter one.

Suppose the evolution of the regime $s_t$ is governed by the transitional probability matrix of a Markov chain $\Pi = \begin{bmatrix} \pi_{00} & \pi_{01} \\ \pi_{10} & \pi_{11} \end{bmatrix}$, where $\sum_{k=0,1}\pi_{ik} = 1$ and $0 < \pi_{ik} < 1$. Then the yield to maturity of a zero-coupon bond with a maturity of $\tau$ periods at time t is given as

$$R_{s_t}(t,\tau) = \frac{A_{s_t}(\tau)}{\tau} + \frac{B_{s_t}(\tau)}{\tau}r_t + \varepsilon_t, \quad s_t = 0,1 \tag{3}$$

where $$\begin{bmatrix} A_0(\tau) \\ A_1(\tau) \end{bmatrix} = \begin{bmatrix} \pi_{00} & \pi_{01} \\ \pi_{10} & \pi_{11} \end{bmatrix} \begin{bmatrix} A_0(\tau-1) + k_0\theta_0 B_0(\tau-1) \\ A_1(\tau-1) + k_1\theta_1 B_1(\tau-1) \end{bmatrix},$$

$$\begin{bmatrix} B_0(\tau) \\ B_1(\tau) \end{bmatrix} = \begin{bmatrix} \pi_{00} & \pi_{01} \\ \pi_{10} & \pi_{11} \end{bmatrix} \begin{bmatrix} (1-k_0-\lambda_0)B_0(\tau-1) - \frac{1}{2}\sigma_0^2 B_0^2(\tau-1) + 1 \\ (1-k_1-\lambda_1)B_1(\tau-1) - \frac{1}{2}\sigma_1^2 B_1^2(\tau-1) + 1 \end{bmatrix},$$

with initial conditions $A_0(0) = A_1(0) = B_0(0) = B_1(0) = 0$, $\lambda_{s_t}$ is the market price of risk in the regime $s_t$ and the measurement noise $\varepsilon_t \sim N(0,\Omega)$.

Equation (1) and (3) make the state-space form of both the RSCIR model and RSCIR-GARCH model, with different volatility specifications, where (1) serves as the state equation, while (3) serves as the measurement equation.

Note that $-\sigma_{s_t} B_{s_t}(\tau)\sqrt{r_{t-1}}$ is the exposure of the yield to the standardized shock $u_t$ in regime $s_t$. Further, $\lambda_{s_t}\sqrt{r_{t-1}} / \sigma_{s_t}$ is the exposure of the pricing kernel to $u_t$ in regime $s_t$. The covariance between these exposures determine the compensation for risk in regime $s_t$. Hence, the risk premium for regime $s_t$ is the product

$$-\sigma_{s_t} B_{s_t}(\tau)\sqrt{r_{t-1}} \times \lambda_{s_t}\sqrt{r_{t-1}} / \sigma_{s_t} = -B_{s_t}(\tau)\lambda_{s_t} r_{t-1} \tag{4}$$

Given the information regarding $s_{t-1}$, $r_{t-1}$, and the regime transition probabilities, the expected risk premium of the RSCIR model and the RSCIR-GARCH model at time $t$ is given as follows,

$$E_{t-1}\left[-B_{s_t}(\tau)\lambda_{s_t} r_{t-1} \mid r_{t-1}, s_{t-1}\right] = -r_{t-1}E_{t-1}\left[B_{s_t}(\tau)\lambda_{s_t} \mid s_{t-1}\right] \tag{5}$$

In the absence of regime shifts, that is under the CIR model, the risk premium in (5), would simply be $-r_{t-1}B(\tau)\lambda$.

# 3 The Kim Filter Based Maximum Likelihood Estimator

In this paper, we apply the maximum likelihood estimator based on the unscented Kalman filter (Julier and Uhlmann, 1997) to estimate the CIR model, and the one based on the Kim filter to estimate the RSCIR model and RSCIR-GARCH model. The Kim filter is an extension of the Kalman filter, which combines the Kalman filter with the Hamilton filter (Hamilton, 1989) proposed for Markov switching models and a collapsing procedure. The Kim filter is an optimal estimator in the sense that no other estimator based on a linear function of the information set yields a smaller mean square error. A detailed discussion of the Kim filter can be found in Kim (1994) and Kim & Nelson (1999).

The Kim filter is used for generalized state space models where model parameters are allowed to depend on an unobserved, discrete-valued $S$ -regime Markov switching variable $s_t, s_t \in \{1, \cdots, S\}$, with a homogeneous transition probability matrix $\Pi \in \mathbb{R}^{q\times q}$. The generalized spate space model is then

$$\begin{cases} r_t = \alpha_0^{s_t} + \left(\alpha_1^{s_t}\right)' r_{t-1} + v_t^{s_t} \\ R_t = \beta_0^{s_t} + \left(\beta_1^{s_t}\right)' r_t + \varepsilon_t^{s_t} \end{cases} \tag{6}$$

where $r_t \in \mathbb{R}^{m\times 1}$ is the state vector, $R_t \in \mathbb{R}^{n\times 1}$ is the measurement vector, $\alpha_0^{s_t}, \alpha_1^{s_t}$, $\beta_0^{s_t}$, $\beta_1^{s_t}$ are constant within each regime, $v_t^{s_t}$ and $\varepsilon_t^{s_t}$ are the state error vector and measurement error vector respectively, and $\begin{pmatrix} v_t^{s_t} \\ \varepsilon_t^{s_t} \end{pmatrix} \sim N\left(0, \begin{pmatrix} Q_t^{s_t} & 0 \\ 0 & H_t^{s_t} \end{pmatrix}\right)$.

For the convenience of description, we define the notation as follows,

$r_{t|t-1}^{q,l}$ and $\Psi_{t|t-1}^{q,l}$: Estimate of $r_t$ and corresponding mean square error given $R_{t-1}$ and $s_{t-1} = q$, $s_t = l$;

$r_{t|t}^{q,l}$ and $\Psi_{t|t}^{q,l}$: Estimate of $r_t$ and corresponding mean square error given $R_t$ and $s_{t-1} = q$, $s_t = l$;

$r_{t|t}^{q}$ and $\Psi_{t|t}^{q}$: Estimate of $r_t$ and corresponding mean square error given $R_t$ and $s_{t-1} = q$, $s_t = q$.

The basic steps of the Kim filter are then:

1. Run the Kalman filter taking into account the different regimes:

$$\begin{cases} r_{t|t-1}^{q,l} = \alpha_0^l + \left(\alpha_1^l\right)' r_{t-1|t-1}^q \\ \Psi_{t|t-1}^{q,l} = \alpha_1^l \Psi_{t-1|t-1}^q \left(\alpha_1^l\right)' + Q_t^l \\ K_t^{q,l} = \Psi_{t|t-1}^{q,l} \left(\beta_1^l\right)' \left(P_{t|t-1}^{q,l}\right)^{-1} \\ r_{t|t}^{q,l} = r_{t|t-1}^{q,l} + K_t^{q,l} v_t^{q,l} \\ \Psi_{t|t}^{q,l} = \left(I - K_t^{q,l}\beta_1^l\right)\Psi_{t|t-1}^{q,l} \end{cases} \tag{7}$$

where $v_t^{q,l} = R_t - (\beta_0^l + \beta_1^l r_{t|t-1}^{q,l})$, $P_{t|t-1}^{q,l} = \beta_1^l \Psi_{t|t-1}^{q,l} (\beta_1^l)' + H_t^l$.

2. Run the Hamilton filter to compute conditional state probabilities:

$$\begin{cases} f(R_t \mid R_{t-1}) = \sum_{l=1}^{S}\sum_{q=1}^{S} f(R_t \mid s_{t-1}=q, s_t=l, R_{t-1})\Pr(s_{t-1}=q, s_t=l \mid R_{t-1}) \\ \Pr(s_{t-1}=q, s_t=l \mid R_t) = \dfrac{f(R_t \mid s_{t-1}=q, s_t=l, R_{t-1})\Pr(s_{t-1}=q, s_t=l \mid R_{t-1})}{f(R_t \mid R_{t-1})} \\ \Pr(s_t=l \mid R_t) = \sum_{q=1}^{S}\Pr(s_{t-1}=q, s_t=l \mid R_t) \end{cases} \tag{8}$$

where $\Pr\left(s_{t-1}=q, s_t=l \mid R_{t-1}\right) = \Pr\left(s_t=l \mid s_{t-1}=q\right)\sum_{j=1}^{S}\Pr\left(s_{t-2}=j, s_{t-1}=q \mid R_{t-1}\right)$,

$f\left(R_t \mid s_{t-1}=q, s_t=l, R_{t-1}\right) = \left(2\pi\right)^{-\frac{S}{2}} \mid P_{t|t-1}^{q,l} \mid^{-\frac{1}{2}} \exp\left(-\tfrac{1}{2}(v_t^{q,l})'(P_{t|t-1}^{q,l})^{-1}v_t^{q,l}\right)$.

3. Collapse the posteriors:

$$\begin{cases} r_{t|t}^{l} = \dfrac{\sum_{q=1}^{S}\Pr\left(s_{t-1}=q, s_t=l \mid R_t\right) r_{t|t}^{q,l}}{\Pr\left(s_t=l \mid R_t\right)} \\ \Psi_{t|t}^{l} = \dfrac{\sum_{q=1}^{S}\Pr\left(s_{t-1}=q, s_t=l \mid R_t\right)}{\Pr\left(s_t=l \mid R_t\right)}\left(\Psi_{t|t}^{q,l} + \left(r_{t|t}^{l} - r_{t|t}^{q,l}\right)\left(r_{t|t}^{l} - r_{t|t}^{q,l}\right)'\right) \end{cases} \tag{9}$$

The collapsing procedure in Step 3 makes the new posterior estimates $r_{t|t}^{l}$ depend no more on the regime in time $t-1$ and thus conveniently constitute the input of the Kalman filter for the next iteration.

Then we construct the log-likelihood function based on the information the Kim filter provides above:

$$L_R(\theta) = \sum_{t=1}^{J}\ln f\left(R_t \mid R_{t-1}\right) \tag{10}$$

where $J$ is the number of observations and $f(R_t \mid R_{t-1})$ is given in (8).

The maximum likelihood estimator of $\theta$ is then[2]

$$\hat{\theta} = \max_{\theta} L_R(\theta) \tag{11}$$

# 4 Empirical Results

## 4.1 Data Description

We use the one-month SHIBOR rate as the proxy for the short-term interest rate in China to estimate

[2] In this paper, we maximize the likelihood function by a genetic algorithm (Holland (1975)), which uses the evolutionary principle to solve difficult problems with objective functions that do not posses “nice” properties such as continuity and differentiability. The algorithm searches the solution space of a function, and implements a “survival of the fittest” strategy to improve the solutions.

the parameters of the CIR, RSCIR and RSCIR-GARCH models. The data set ranging from October 8, 2006 to February 27, 2009, is obtained from China Foreign Exchange Trading System & National Interbank Funding Center (CFETS). There are total 125 weekly observations. The initial SHIBOR rate is first converted into its continuous compounding form and then its time series and first difference plots are reported in Figure 1, and summary statistics are given in Table 1.

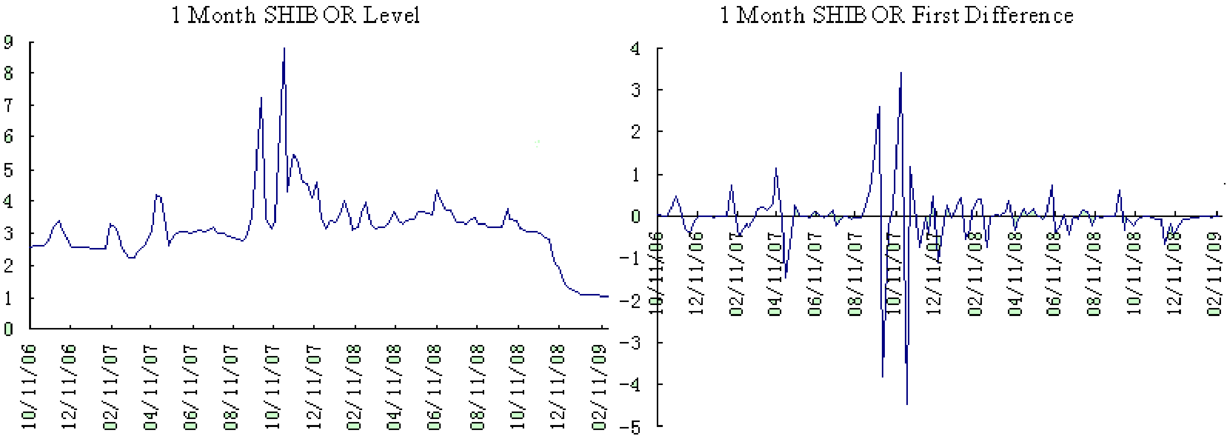


**Figure 1 One-Month SHIBOR and First Difference**

**Table 1 Summary Statistics of One-Month SHIBOR**

| | Mean | Std Dev | Skewness | Kurtosis | Jarque-Bera |
|---|---|---|---|---|---|
| One-Month SHIBOR $R$ | 3.1469 | 1.0613 | 1.4152 | 10.2007 | $311.7806^{***}$ |
| First Difference $\Delta R$ | -0.0124 | 0.7643 | -1.4228 | 19.6136 | $1467.8960^{***}$ |
| Correlation Coefficient | $Corr(R_t, \Delta R_t) = -0.3435$ | | | | |
| Quandt-Andrews Breakpoint Test $H_0$: No breakpoint | Statistic | Date | Value | $p$-value | |
| | Max $LR$ $F$-statistic | 10/29/2007 | $10.1416^{***}$ | 0.0247 | |
| | Max $Wald$ $F$-statistic | | $10.1416^{***}$ | 0.0247 | |

Note: '*', '**' and '***' denote that the null hypothesis $H_0$ is rejected at the significance level of 10%, 5% and 1% respectively. The optimal lag order for the ARCH LM test is determined by taking into account AIC, SBC and HQC all together, and $p$-values in the Quandt-Andrews breakpoint test are calculated by the method introduced in Hansen (1997).

As shown in Table 1, the negative correlation coefficient between one-month SHIBOR rate $R$ and its first difference $\Delta R$ implies mean reversion in the behavior of the short rate. Besides, the kurtosis coefficients of the two variables are both significantly larger than 3, and Jarque-Bera statistics shows that the normal distribution hypothesis is rejected at the significance level of 1% for both of the two variables, which implies that the distributions of one-month SHIBOR rate and its first difference have significantly leptokurtosis characteristics. Furthermore, it is easy to see from Figure 1 that there is a significant volatility clustering phenomenon in the time series of the one-month SHIBOR rate $R$. In the light of the analysis above, there is significantly ARCH effect in the dynamics of the one-month SHIBOR rate $R$.

Figure 1 also shows that the behavior of the short-term interest rate varies dramatically during the data period. During September through December in 2007, the central bank pursued a tight monetary policy and frequently raised the benchmark interest rates to curb inflation, which significantly affected the behavior of SHIBOR. The period saw a sharp increase in both the level and volatility of SHIBOR, and the result of the Quandt-Andrews breakpoint test in Table 1 shows that there is a structure break on the date of October 29, 2007, which indicates that there may be regime shifts during this period.

To further investigate whether there is regime switching in the dynamics of the one-month SHIBOR rate $R$, we apply the likelihood ratio test proposed in Hansen (1992) to its time series, leading to the following result.

**Table 2 Hansen's Likelihood Ratio Test Result for One-Month SHIBOR**

| $LR_T^*$-Statistic | Statistics of $\sup_{\alpha} Q_T^*(\alpha)$ | | | | | $p$-value |
|---|---|---|---|---|---|---|
| | Minimum | Maximum | Mean | Median | Std Dev | |
| 8.536*** | 0.3944 | 4.4766 | 1.8701 | 1.7932 | 0.6317 | 0.0000 |

Note: The hypothesis is $H_0{:}\beta=0$, $H_1{:}\beta<0$, where $\beta=\pi_{00}-1$. The grid considered here is $\pi_{00}=0.2, 0.4, 0.6, 0.8$, $k_1=0.2, 0.4, 0.6, 0.8$, $\theta_1=0.2, 0.4, 0.6, 0.8$, $\sigma_1=0.2, 0.6, 1.0, 1.4$, $\lambda_1=-10, -6, -2, 2, 6, 10$, $\pi_{11}=0.2, 0.4, 0.6, 0.8$, and the likelihood ratio statistic $LR_T^*$ and associated $p$-value are gained using the distribution of $\sup_{\alpha} Q_T^*(\alpha)$ calculated by 1000 Monte-Carlo simulations. '***' denotes significance at the 1% level.

As shown in Table 2, the null hypothesis is rejected at 1% significance level, which implies that there are significantly regime shifts in the dynamics of the one-month SHIBOR rate during the sample period.

## 4.2 Model Estimation and Contrast

With the selected data, we estimate the CIR model using the UKF based maximum likelihood method, while the RSCIR model and RSCIR-GARCH model using Kim filter based method, whose result is reported in Table 3.

As reported in Table 3, the values of the regime classification measure (RCM)[3] for the RSCIR model and RSCIR-GARCH model are 16.13 and 14.91 respectively, which implies that there definitely exist two regimes in the dynamics of one-month SHIBOR rate. Parameter estimation results for the RSCIR model and

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[3] As defined in Ang and Bekaert (2002), the RCM for two states is $RCM = 400 \times \sum_{t=1}^{T} p_t(1-p_t)/T$, where $p_t$ is the smoothed probability for one of the two states. RCM is between 0 and 100. A good regime classification is associated with low RCM statistic values. A value of 0 means perfect classification and a value of 100 implies no information about the regimes is revealed.

**Table 3 Parameter Estimates of Models**

| | CIR | | RSCIR | | RSCIR-GARCH | |
|---|---|---|---|---|---|---|
| | Value | Std Dev | Value | Std Dev | Value | Std Dev |
| $k_0$ | 0.9529*** | 0.0172 | 0.4151*** | 0.0131 | 0.4033*** | 0.0114 |
| $k_1$ | — | — | 0.0108*** | 0.0032 | 0.0109*** | 0.0029 |
| $\theta_0$ | 0.0291*** | 0.0054 | 0.0918*** | 0.0072 | 0.0973*** | 0.0065 |
| $\theta_1$ | — | — | 0.2850*** | 0.0170 | 0.2906*** | 0.0155 |
| $c_0$ | 0.0683*** | 0.0108 | 0.0030*** | 0.0011 | 0.0022*** | 0.0008 |
| $c_1$ | — | — | 0.2956*** | 0.0773 | 0.1755*** | 0.0621 |
| $a_0$ | — | — | — | — | 0.0147 | 0.0154 |
| $a_1$ | — | — | — | — | 1.3824** | 0.5813 |
| $b_0$ | — | — | — | — | 0.0019 | 0.0026 |
| $b_1$ | — | — | — | — | 0.2035* | 0.1061 |
| $\lambda_0$ | -3.6903*** | 0.1291 | -2.9551*** | 0.1130 | -2.8137*** | 0.1022 |
| $\lambda_1$ | — | — | -7.7232*** | 0.3270 | -7.1964*** | 0.2749 |
| $\pi_{00}$ | — | — | 0.8104*** | 0.0007 | 0.8119*** | 0.0005 |
| $\pi_{11}$ | — | — | 0.7094*** | 0.0011 | 0.7105*** | 0.0008 |
| Initial Parameters | $r_0$ | $Q_0$ | $r_0$ | $Q_0$ | $r_0$ | $Q_0$ |
| | 0.0262 | $1.2\times10^{-5}$ | 0.0254 | $9.28\times10^{-6}$ | 0.0257 | $9.65\times10^{-6}$ |
| RCM | — | | 16.13 | | 14.91 | |

Note: '*', '**' and '***' denote significance at the levels of 10%, 5% and 1% respectively.

RSCIR-GARCH model provide further information that $\theta_1 \gg \theta_0$ and $\sigma_1 \gg \sigma_0$ hold under both the two models, which suggests that the two regimes are the 'low-volatility' and 'high-volatility' regimes respectively.

Then, we make a data fitness contrast among the CIR, RSCIR and RSCIR-GARCH models in order to determine the optimal short rate model for the SHIBOR market[4]. The result is reported in Table 4.

**Table 4 Model Fitness Contrast among CIR, RSCIR and RSCIR-GARCH**

| | CIR | RSCIR | RSCIR-GARCH |
|---|---|---|---|
| MAE | 0.004829 | 0.002676 | 0.002237 |
| SAE | 0.004162 | 0.002014 | 0.001681 |
| RMSE | 0.004985 | 0.002731 | 0.002385 |
| Skewness | -2.3589 | -0.0713 | -0.0032 |
| Kurtosis | 14.6686 | 3.8678 | 3.4811 |
| Jarque-Bera | 3960.346*** | 19.3372*** | 5.7874* |
| ARCH LM | 439.0761*** | 58.0242*** | 3.7922* |
| Log likelihood $L_R$ | 419.68 | 593.71 | 611.39 |

Note: The 'MAE' and 'SAE' stand for the mean and standard deviation of the absolute errors respectively. '*', '**' and '***' denote significance at the levels of 10%, 5% and 1% respectively.

First, we carry out a contrast between the CIR model and RSCIR model to investigate the contribution of the regime-switching specification to the improvement of model fitness. As shown in Table 4, the introduction of regime switching greatly improves the data fitness of the short rate model, which can be inferred from the fact that the log likelihood dramatically increases from 419.68 of the CIR model to 593.71 of the RSCIR model, while MAE and RMSE drop sharply from 0.004829 and 0.004985 under the CIR model to 0.002676 and 0.002731 under the RSCIR model respectively. Meantime, the introduction of regime switching also significantly improves the fitting stability of the short rate model, which is reflected in the dramatic decline in the SAE from 0.004162 under the CIR model to 0.002014 under the RSCIR model. Besides, the regime-switching specification partly captures the leptokurtosis and volatility clustering characteristics and thus reduces the ARCH effect in the dynamics of the short rate significantly, which can be seen from the fact that the kurtosis and ARCH LM statistic of the residual series decrease dramatically from 14.6636 and 439.0761 under the CIR model to 3.8678 and 58.0242 under the RSCIR model respectively. Thus the distribution of the residual is made closer to normal, which is reflected in the sharp drop in the Jarque-Bera statistic of the residual series from 3960.346 under the CIR model to 19.3372 under the RSCIR model. However, ARCH LM and Jarque-Bera statistics of the residual series are still significant at the 1% level, which indicates that there is still some part of the ARCH effect is unexplained, making the distribution of the residual significantly different from normal. So it is necessary to introduce the GARCH specification of volatility into the model.

Then, we carry out a contrast between the RSCIR model and RSCIR-GARCH model to investigate the

[4] Following Bansal and Zhou (2002), we first determine the regime state at time $t$ by the smoothed probability for each regime, and then take the estimate of the short rate in the dominate regime as the short rate estimate under the RSCIR model or RSCIR-GARCH model.

contribution of GARCH specification to the improvement of model fitness. As shown in Table 4, the introduction of the GARCH setting of volatility further improves the data fitness of the short rate model, which can be inferred from the fact that the log likelihood further increases from 593.71 of the RSCIR model to 611.39 of the RSCIR-GARCH model, while MAE and RMSE drop further from 0.002676 and 0.002731 under the RSCIR model to 0.002237 and 0.002385 under the RSCIR-GARCH model respectively. Meantime, the introduction of GARCH specification also further improves the fitting stability of the short rate model, which is reflected in the further decline in the SAE from 0.002014 under the RSCIR model to 0.001681 under the RSCIR-GARCH model. Besides, the GARCH specification of volatility also helps capture the leptokurtosis and volatility clustering characteristics and thus further reduces the ARCH effect in the dynamics of the short rate, which can be seen from the fact that the kurtosis and ARCH LM statistic of the residual series decrease further from 3.8678 and 58.0242 under the RSCIR model to 3.4811 and 3.7922 under the RSCIR-GARCH model respectively. Thus the distribution of the residual is made further closer to normal, which is reflected in the further drop in the Jarque-Bera statistic of the residual series from 19.3372 under the RSCIR model to 5.7874 under the RSCIR-GARCH model. With the introduction of the GARCH specification, ARCH LM and Jarque-Bera statistics of the residual series become insignificant at the 5% level, which indicates that the GARCH specification helps explain most of the residual ARCH effect, making the distribution of the residual approximately normal.

Finally, we use the likelihood ratio test to investigate the superiority of the RSCIR-GARCH model over the RSCIR model. The result shows that the likelihood ratio statistic $LR_{GARCH} = 35.36$ is significant at the level of 1%, which indicates that the introduction of the GARCH specification significantly improves the data fitness of the model. Specially, the result of model estimation in Table 3 shows that the GARCH setting is significant in the high-volatility regime but not in the low-volatility regime, which indicates that the introduction of the GARCH specification mainly improve the model fitness in the high-volatility regime, but helps little in the low-volatility regime. That may be because the ARCH effect mainly exists in the high-volatility regime.

### 4.3 Risk Premium Analysis

The model contrast above reveals that RSCIR-GARCH model provides a much better performance than the CIR model and RSCIR model in fitting the data of the one-month SHIBOR rate, so we use the RSCIR-GARCH model to analyze the dynamics of the risk premium in the SHIBOR market.

First, we determine the regime state at each sample point under the RSCIR-GARCH model. As shown in Table 3, there exist the low-volatility regime ($s_t = 0$) and the high-volatility regime ($s_t = 1$) in the dynamics of the one-month SHIBOR rate under the RSCIR-GARCH model, and the two regimes are governed by the transitional probability matrix of a Markov chain $\begin{bmatrix} 0.8119 & 0.1881 \\ 0.2895 & 0.7105 \end{bmatrix}$. Specially, the level and volatility of the short rate in the high-volatility regime are 2.99 and 10.37 times higher than those in the low-volatility regime respectively, while the market price of risk in the high-volatility regime is 2.56 times higher than its counterpart in the low-volatility regime. However, the mean reversion speed in the high-volatility regime is just a thirty-seventh of its counterpart in the low-volatility regime, which implies that the mean-reversion of the short rate in the low-volatility regime is much stronger than that in the high-volatility regime. The smoothed probability for the high-volatility regime is displayed in Figure 2, with the time series plot of one-month SHIBOR rate for better interpretation.

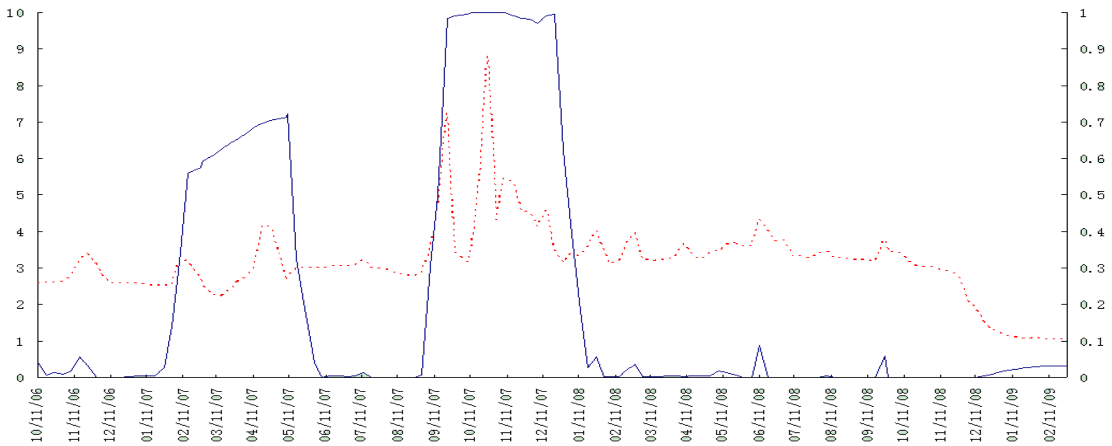

**Figure 2 Smoothed Probability for the High-Volatility Regime**

The solid line stands for the smoothed probability of the high-volatility regime and the dot line for one-month SHIBOR rate.

As shown in Figure 2, both the level and volatility of the short rate soared during the periods February – May and September – December, 2007, and that may imply regime switching in the dynamics of the SHIBOR term structure, which is further proven by the high level of the smoothed probability for the high-volatility regime. There are mainly two reasons for the regime shifts. First, the central bank pursued a tight monetary policy and frequently raised the benchmark interest rates to curb the increasing inflation emerging at the end of 2006, which significantly raised the level and volatility of the short rate. Second, shocks from huge IPOs also make a cause. In the context of the stock market boom, huge IPOs were issued frequently, and the subscribers bridged their financing gap via the interbank market, which posed great shocks to the SHIBOR market, further increasing the volatility of the short rate. A typical case is the IPO of CNPC (China National Petroleum Corp) on October 26, 2007, which froze an acquisition fund up to 3.37 trillion RMB, dramatically driving up the SHIBOR rate.

Dramatic increase in the level and volatility of the interest rates means dramatic increase in investment risk and thus its compensation: risk premium, so the dynamics of the risk premium during the two periods are certainly different from those during the other periods. Figure 3 shows the fluctuation of the actual risk premium[5] and its estimations under the CIR model and the RSCIR-GARCH model.

[5] We calculate the risk-free interest rate by constructing the yield curve using the extended Nelson-Siegel model based on the price data of treasury bonds at each sample point, and then we obtain the actual risk premium as the one-month SHIBOR rate minus the risk-free interest rate.

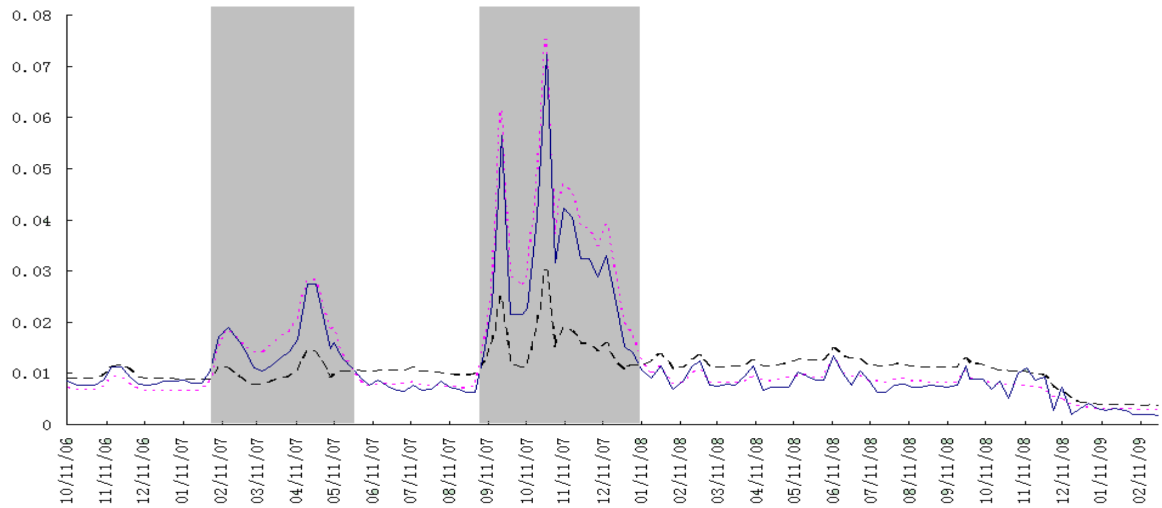


**Figure 3 Contrast of Risk Premium Estimations under CIR and RSCIR-GARCH models**

The solid line stands for the actual risk premium, and the dash line and dot line stand for risk premium estimations under the CIR model and RSCIR-GARCH model respectively. The shaded areas represent the periods for the high-volatility regime.

As shown in Figure 3, the risk premium has a high level and fluctuation in the high-volatility regime, while in the low-volatility regime, in contrast, the risk premium display a low mean and volatility, so there exist significant regime shifts in the dynamics of the risk premium. However, the CIR model, as a single-regime model, fails in capturing nonlinear changes in the market conditions such as the long-term expected rate of return $\theta$ and the volatility of the short rate $\sigma$, and overestimates (underestimates) the market price of risk in the low-volatility (high-volatility) regime, which results in biased estimation of risk premium. As shown in Figure 3, the CIR model not only underestimates the risk premium in the high-volatility regime but overestimates the risk premium in the low-volatility regime, which may cause mispricing of assets and thus failure in hedging. In contrast, the RSCIR-GARCH model not only allows the parameter values shift between regimes but sets the volatility in the GARCH form, capturing the regime-switching and leptokurtosis characteristics of the short rate dynamics, which ensures the estimation accuracy and reliability of the risk premium. The contrast between the statistics of risk premium estimation errors under the CIR model and RSCIR-GARCH model is reported in Table 5.

**Table 5 Contrast between Statistics of Risk Premium Estimation Errors**

| | Mean | Std Dev | MAE | SAE | RMSE |
|---|---|---|---|---|---|
| CIR | -0.001315 | 0.007707 | 0.004726 | 0.006215 | 0.007788 |
| RSCIR-GARCH | 0.000899 | 0.002292 | 0.001778 | 0.001697 | 0.002454 |

Note: The 'MAE' and 'SAE' stand for the mean and standard deviation of the absolute errors respectively.

As shown in Table 5, the mean, MAE and RMSE of risk premium estimation errors under the RSCIR-GARCH model are all significantly smaller than their counterparts under the CIR model[6], which indicates that the RSCIR-GARCH model provides a better estimation accuracy of the risk premium than the CIR model. Meantime, the standard deviation and SAE of risk premium estimation errors under the

[6] This inference is statistically significant at the 1% level.

RSCIR-GARCH model are both much smaller than their counterparts under the CIR model, which implies that the RSCIR-GARCH model also provides a better estimation reliability of the risk premium than the CIR model. So the RSCIR-GARCH model is a preferred short rate model which provides a better model basis for asset pricing and risk hedging.

# 5 Concluding Remarks

Changes in macroeconomic factors, such as business cycle conditions or monetary policy, may cause regime swifts in the dynamics of interest rates, so regime-switching models are suggested instead of single-regime models in capturing the dynamics of the short rate. In this paper, we apply the RSCIR-GARCH model to describe the behavior of the risk premium implied in the short-term interest rate in China, using weekly one-month SHIBOR rate as sample data, whose emphasis is on capturing regime switching in the dynamics of risk premium.

First, we use Hansen's likelihood ratio test and statistical methods to investigate the regime-switching and leptokurtosis characteristics of the short rate dynamics, leading to the finding that there exist significantly regime shifts and ARCH effect in the dynamics of the one-month SHIBOR rate during the sample period.

Second, based on the analysis above, we establish the RSCIR model and RSCIR-GARCH model by introducing regime-switching and GARCH specifications into CIR model successively. Then, a contrast study among CIR, RSCIR and RSCIR-GARCH models is performed based on the one-month SHIBOR sample data, which shows that the regime-switching and GARCH specifications both improve the model fitness significantly and eliminate the ARCH effect in the volatility dynamics. In other words, RSCIR-GARCH model provides a much better performance than the CIR model and RSCIR model in fitting the data of the one-month SHIBOR rate.

Third, in the light of the model contrast result above, we perform an empirical research on the risk premium dynamics of SHIBOR using the RSCIR-GARCH model estimated by the Kim filter based maximum likelihood method, finding that there are two distinct regimes, that is, the regime of higher interest rate with higher volatility and the regime of lower interest rate with lower volatility, in the dynamics of both the term structure and risk premium of SHIBOR. The 'high-volatility' regime is characterized by periods of extremely high and volatile interest rates, and the corresponding risk premium also has a high level and fluctuation, while in the 'low-volatility' regime, in contrast, both the short rate and the risk premium display a low mean and volatility. However, the mean-reversion of the short rate in the low-volatility regime is much stronger than that in the high-volatility regime. Besides, it is also found that the ARCH effect in the volatility dynamics is only significant in the high-volatility regime but not in the low-volatility regime. Furthermore, we make a comparison between the CIR model and the RSCIR-GARCH model in capturing the dynamics of the risk premium implied in the short rate, leading to the conclusion that the RSCIR-GARCH model by far outperforms the CIR model in capturing regime switching in the dynamics of risk premium and thus in reflecting changes in market conditions, which is critical for pricing and hedging of assets.

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