A Comparative Simulation Study of Fund Performance Measures

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ABSTRACT

This study critically reviews current fund performance measures. The performance measure derived from the return-based style analysis (RBSA) by Sharpe (1992) is introduced and compared with other regression-based measures. A comparative simulation is set up to test the robustness, accuracy, and efficiency of the alternative measures. The evidence shows that the RBSA measure is superior to other measures. The performance of the simple Jensen measures is sensitive to fund types. More complicated measures, like market-timing measures and multifactor measures show spurious market timing and wrong fund type information.

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1. Introduction

The investment performance of mutual fund managers has been examined at length in the finance literature. Various performance measures have been proposed since the original papers by Jensen (1968, 1969) and later refined by Black, Jensen, and Scholes (1972) and Blume and Friend (1973), for example, multi-factor measures by Fama and French (1993) and Carhart (1997) and conditional measures by Ferson and Scadt (1996), among others. This paper contributes to the fund performance measurement literature by critically examining current fund performance measures. A comparative simulation is set up and employed to test the robustness, accuracy, and efficiency of the alternative measures.

The paper is organized as follows: section 2 critically reviews alternative traditional performance measures and discusses their limitations, such as benchmark inefficiency, spurious market timing, and unrealistic normality assumption. We then introduce a performance measure derived from the return-based style analysis by Sharpe (1992) and compare it with traditional regression-based measures. Section 3 sets up a simulation experiment to compare and evaluate alternative measures in terms of robustness, accuracy, and efficiency and section 4 discusses simulation results and analyses. Section 5 concludes the paper.

2. Alternative Performance Measures

Most of the traditional fund performance measures are estimated by the regression method and are actually application of the Capital Asset Pricing Model (CAPM). Depending on their assumptions about the measure of fund performance, the measure of fund risk, and the behavior of fund managers, we classify the measures into three general categories: (i) unconditional measures, where it is assumed that there is no market-timing activity; (ii) markettiming measures, where we control the measurement bias caused by the fund manager's market timing behavior; and (iii) conditional measures that control the investment strategies using publicly available macroeconomic information.

2.1. Unconditional Measures

Jensen (1968) develops a single factor measure which implicitly assumes that the market portfolio could capture all the relevant risk of the fund. The model is

$$r_{it} - r_{ft} = \alpha_i + \beta_{im}(r_{mt} - r_{ft}) + \varepsilon_{it}$$

$$(2.1)$$

where r_{it} is the monthly return of fund *i* at period *t*, r_{ft} is the risk-free rate at period *t*, r_{mt} is the monthly return of market portfolio at time *t*, and ε_{it} is the disturbance term. The alpha, α_i , measures the performance of fund *i* during the evaluation period. β_{im} is the covariance of the fund return and market portfolio return, divided by the variance of the market portfolio return. β_{im} is a measure of the fund's systematic risk, i.e. the sensitivity of the fund return to the market portfolio return.

One issue related to the Jensen measure is the difficulty to find a proxy for market portfolio (benchmark inefficiency). This issue has been extensively investigated in the past three decades. Jensen (1968) studies 115 mutual funds from 1955 to 1964 and finds that on average the funds earned 1.1% less annually than what they should have earned given their systematic risk. An analysis of gross returns with expenses added back indicates that 42% of the funds did better than the overall market on a risk-adjusted basis, whereas the analysis of net returns indicates only 34% of the funds outperform the market. Jensen concludes that on average these funds could not beat a buy-and-hold policy - passive investment strategy. Contrary to Jensen's findings, Ippolito (1989) finds that the estimated risk-adjusted return for the mutual fund industry is greater than zero even after accounting for transaction costs and expenses. Ippolito attributes

nonnegative alpha to the existence of informed actions by management. Ippolito uses S&P 500 Index as the benchmark (the proxy for market portfolio) to study the 143 mutual funds during the period of 1965 to 1984. Elton *et al.* (1993) use the same data as the Ippolito's data and notice that returns of S&P stocks, returns of non-S&P stocks, and returns of bonds are significant factors in performance measurement. They argue that Ippolito's conclusions are due to the choice of an inefficient benchmark and that Ippolito's conclusion are reversed after taking account mutual funds' holdings of non-S&P500 stocks and bonds.

Empirical studies, particularly those of Lehmann and Modest (1987) and Grinblatt and Titman (1994), stress the sensitivity of the fund performance to the benchmarks chosen. Lehmann and Modest (1987) employ the standard CAPM benchmarks and a variety of Arbitrage Pricing Theory (APT) benchmarks to investigate this question. They find little similarity between the absolute and relative mutual fund rankings obtained from these alternative benchmarks, which suggest that the conventional measures of abnormal mutual fund performance are sensitive to the benchmarks chosen. Grinblatt and Titman (1994) use a sample of 279 mutual funds that existed from 1974 to 1984 and construct 109 passive portfolios to investigate the sensitivity of the performance to the benchmarks. They find that the measures generally yield similar inferences when using the same benchmark and inferences can vary, even from the same measure, when using different benchmarks.

Roll (1977, 1978) argues that it is practically impossible to find a proxy for the market portfolio. This difficulty poses a serious problem when evaluating fund performance. Since the proxy market portfolio used is not the true market portfolio, the covariance of the return of the fund and the return of proxy market portfolio cannot correctly measure the risk born by the funds. So the alpha derived from the Jensen measure in (2.1) is biased.

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Implicitly admitting that the stock market portfolio cannot capture all the risk factors, Fama and French (1993) propose a three-factor model. The alpha is estimated through the following time series regression:

$$r_{it} - r_{ft} = \alpha_i + \beta_{im}(r_{mt} - r_{ft}) + \beta_{iSMB}r_{SMB,t} + \beta_{iHML}r_{HML,t} + \varepsilon_{it}$$

$$(2.2)$$

where β_{iSMB} is the sensitivity of the excess return of fund *i* to the return of the SMB portfolio and β_{iHML} is the sensitivity of the excess return of fund *i* to HML portfolio. They suggest that securities' returns in excess of risk-free rate are explained by the sensitivity of their return to three factors: (i) the excess return on a broad market portfolio, denoted by $(r_{nut} - r_{ft})$; (ii) the difference between the return on a portfolio of small-cap stocks and the return on a portfolio of large-cap stocks, denoted by $r_{SMB,t}$; (iii) the difference between the return on a portfolio of large-cap stocks, denoted by $r_{SMB,t}$; (iii) the difference between the return on a portfolio of large-cap stocks, denoted by $r_{SMB,t}$; (iii) the difference between the return on a portfolio of bow B/M ratio stocks, denoted by $r_{HML,t}$.

Carhart (1997) finds another significant risk factor: the momentum factor, which can explain the variation of stock returns. After adding this factor to Fama-French three-factor model, he proposes a four-factor model. The additional factor captures the one year momentum anomaly, recognized by Jegadeesh and Titman (1993). The model may also be interpreted as a performance attribution model. The coefficients on the factor-mimicking portfolios, β_{im} , β_{iSMB} , β_{iHML} , and β_{iPR1YR} , indicate the proportion of mean return, attributable to four elementary strategies: high beta stocks versus low beta stocks, large-cap stocks versus small-cap stocks, value stocks versus growth stocks, and one year return momentum stocks versus contrarian stocks. The model is

$$r_{it} - r_{ft} = \alpha_i + \beta_{im}(r_{mt} - r_{ft}) + \beta_{iSMB}r_{SMB,t} + \beta_{iHML}r_{HML,t} + \beta_{iPR1YR}r_{PR1YR,t} + \varepsilon_{it}$$
(2.3)

where $r_{SMB,t}$, $r_{HML,t}$, and $r_{PR1YR,t}$ are returns on value-weighted zero-investment factor-mimicking portfolios for size, B/M ratio, and one year momentum factors at period *t*, and ε_{it} is the disturbance term.

The multi-factor measures, such as the Fama-French three-factor measure and the Carhart four-factor measure, suffer from three problems as they become more refined and complicated. First, they cannot overcome the benchmark inefficiency; they can only reduce the inefficiency effect by adding more risk factors. Second, it is not easy to interpret the coefficients of the risk factors in regression models except for the coefficient of market portfolio. The signs of the coefficients may indicate the fund styles, but they provide no information about the asset allocation of the fund to each asset category or sub-asset groups. Third, the measures are biased, if managers adjust asset allocation or loadings of stocks according to the expectation of market movement or the predetermined market information, such as risk-free rate, dividend yield of the stock market and term structure.

2.2. Market Timing Measures

When fund managers adopt a market-timing strategy, which is common in fund management, the previous unconditional measures are biased. Market timing means that the fund managers change asset allocations or the risk level of stocks on the basis of her/his expectation of future market movement. When managers successfully time the market movement, the measures without controlling market-timing behavior are biased (Ferson and Schadt, 1996). The Treynor and Mazuy (1966) model and the Henriksson and Merton (1981) model are proposed to deal with this issue.

The Treynor-Mazuy model, later refined by Bhattacharya & Pfleiderer (1983), assumes that the risk level of the portfolio varies when managers adopt market-timing strategies. In the original model the beta (measuring the risk of the fund) is a linear function of excess market return: $\beta_i = \beta_{im} + \gamma_i^{TM} (r_{mt} - r_{ft})$. When the expected market return is higher than the risk free rate, the risk of the portfolio is higher in order to obtain a higher expected return. On the other hand, when the expected market return is below the risk free rate, the manager reduces the portfolio's exposure to the market. With this idea, the Jensen model in (2.1) is modified by adding a quadratic term:

$$r_{it} - r_{ft} = \alpha_i + \beta_{im} (r_{mt} - r_{ft}) + \gamma_i^{TM} (r_{mt} - r_{ft})^2 + \varepsilon_{it}$$
(2.4)

where r_{it} , r_{ft} , r_{mt} and β_{im} are as defined in equation (2.1). γ_i^{TM} measures the managers' market timing ability, where a positive γ_i^{TM} indicates that managers have superior market timing ability. α_i measures the fund performance due to mangers' active management after controlling markettiming behavior.

Instead of assuming that the beta is a linear function of excess market return, Henriksson and Merton (1981) propose an alternative model. They assume that managers choose two different levels of risk depending on the managers' forecast of the market return. If the excess market return is positive, a higher risk level is chosen. If the excess market return is negative, a lower risk level is chosen. They modify the Jensen measure in (2.1) by adding a term $MAX(0, r_{mt} - r_{ft})$, which gives

$$r_{it} - r_{ft} = \alpha_i + \beta_{im}(r_{mt} - r_{ft}) + \gamma_i^{HM} MAX(0, r_{mt} - r_{ft}) + \varepsilon_{it}$$
(2.5)

where γ_i^{HM} is used to measure the manager's market timing ability. A positive γ_i^{HM} indicates superior market timing ability. Henriksson and Merton (1981) interpret the term $MAX(0, r_{mt} - r_{ft})$ as the payoff to an option on the market portfolio with an exercise price equal to the risk free asset return. α_i is the fund performance in the Henriksson-Merton market-timing model.

2.3. Conditional Measures

Ferson and Schadt (1996) further explore the market-timing behavior by including lagged macro information into the model. The conditional measure model recognizes that the risk and the expected return of the fund may vary over time given some predetermined public macro information. They argue that mangers, who obtain higher returns using public information, should not be interpreted as superior performance. Unconditional measures and market-timing measures may confuse the performance from the managers' selection ability and market-timing ability with the performance from the managers' responses to the changing macro information.

Ferson and Schadt (1996) argue that the alpha and betas (in unconditional as well as market timing measures) are biased when managers respond to information of the last period, such as risk-free interest rate and term structure. Ferson and Schadt (1996) show that

$$p \lim(\beta_{iu}) = \beta_{ic} + B_i^T Cov((r_{mt} - r_{ft}), z_{t-1}) / \operatorname{var}((r_{mt} - r_{ft}))$$

$$p \lim(a_i) = E(r_{mt} - r_{ft})(\beta_{ic} - p \lim(\beta_i)) + Cov((r_{mt} - r_{ft}), B_i^T z_{t-1})$$
(2.6)

where *T* denotes transpose. β_{iu} is the unconditional measure of fund *i*. β_{ic} is the true beta in the conditional model, and z_{t-1} is an innovation vector of the lagged information variables Z_{t-1} , i.e., $z_{t-1} = Z_{t-1} - E(Z_{t-1})$. B_i^T 's are the response coefficients to the innovations of the lagged information variable. From (2.6), we can see β_{iu} is a biased estimator of β_{ic} . The direction of the bias depends on the covariance between the excess market return and the innovations of the lagged macro information. Ferson and Schadt (1996) use four public information variables that are useful to predict the market: the lagged level of short term Treasury bill rate, the January

dummy, the lagged dividend yield of the stock index, and the lagged measure of the slope of term structure, which were previously observed by Keim and Stambauch (1986), Fama and French (1988), Ferson and Harvey (1991), and Evans (1994).

Market-timing models assume that any information that is correlated to the future market return is superior information. The conditional model assumes public macro information is not superior information. The performance resulting from the public information should be separated from the managers' market-timing ability. The beta of the fund is assumed to be a linear function of public information vector z_{t-1} that captures the changing economic conditions:

$$\beta_{i}(z_{t-1}) = \beta_{im} + \beta_{ic}^{T} z_{t-1}$$

$$z_{t-1} = Z_{t-1} - E(Z_{t-1})$$
(2.7)

The linear specification on time-varying betas are also used in previous studies, such as Ferson (1985), Shanken (1990), Ferson and Harvey (1993), Cochrane (1996), and Jagannathan and Wang (1996), among others. The linearity assumption is used for fund performance measurement for two reasons: it is motivated by theoretical models of managers' behavior, such as in Admati *et al.* (1986) and, it is easy to interpret as illustrated by Ferson and Schadt (1996). Thus we could modify the Jensen model in (2.1) as

$$r_{it} - r_{ft} = \alpha_i + \beta_{im}(r_{mt} - r_{ft}) + \beta_{ic}^T z_{t-1}(r_{mt} - r_{ft}) + \varepsilon_{it}$$
(2.8)

where β_{im} is the unconditional mean of the conditional beta in (2.7). The elements of β_{ic}^{T} are the response coefficients with respect to the innovation of the lagged information variables Z_{t-1} .

When we apply the conditional measure to the Fama-French three-factor model and the Carhart four-factor model, the models become very complicated, since it consumes too many degrees of freedom and thereby leading to less efficient estimation of alpha and betas. Although the conditional measure is theoretically justified by Ferson and Schadt (1996), it makes little sense in practice.

All the measures that we reviewed till now are assuming a constant alpha. However, Christopherson *et al.* (1998) allow a time-varying alpha in their measure, i.e. they treat alpha as a function of Z_{t-1} ,

$$\alpha_i(z_{t-1}) = \alpha_i + A_i^T z_{t-1}$$
(2.9)

where $z_{t-1} = Z_{t-1} - E(Z_{t-1})$, which is the same as we mentioned in (2.6). Allowing a time-varying alpha makes the measures more complicated. It has the same shortcoming as the conditional measure.

Traditional fund performance measures suffer a number of limitations. First, it is difficult to find a proxy for market portfolio (it is called benchmark inefficiency). This difficulty poses a serious problem when evaluating fund performance, because if the market portfolio used is not a perfect market portfolio the covariance of the return of the fund and the return of the market portfolio cannot correctly measure the risk born by the fund. Thus the alpha derived from the measure is biased. Later efforts, like the Fama-French three-factor measure and the Carhart fourfactor measure, attempted to solve this problem by adding more risk factors into the Jensen measure. Although they could reduce the inefficiency problem to some extent, the inefficiency is still material as noted by Grinblatt and Titman (1994). In addition, the complex multi-factor measures brought two other problems along: it consumes more degrees of freedom, making statistical inference of coefficients unreliable. And it is difficult to interpret the beta coefficients. They provide no quantified information about the fund's asset allocations to each asset category, which is valuable for the in-depth analysis of fund risk level. Second, although market timing and conditional measures are theoretically attractive, it is practically impossible to implement them. Admati *et al.* (1986) point out that it is difficult to separate timing from selection ability. Furthermore, when managers invest in options or option-like securities, spurious market-timing ability and selectivity ability may be observed as noted by Jagannathan and Korajczyk (1986). In addition, when managers trade securities in less than one month, which is common in practice, we could also observe spurious market-timing ability as noted by Ferson and Schadt (1996). The correct separation of market-timing ability from selection ability, denoted by alpha, depends on some impractical constrictions. Regarding conditional measures, the measures are complicated in multi-factor models, making the inference about beta coefficients and alpha unreliable within a three-year evaluation period. And we cannot increase the sample size to deal with this problem, because the fund may significantly shift its investment strategy or change fund managers in the longer sample period. But, it is a common practice to use three-year data to evaluate fund performance, see, for example, Cai et al. (1997), Carhart (1997), Elton et al. (1996), and Kosowski et al. (2001).

Third, all these measures are estimated by the regression method. An underlying assumption is that ε_{it} is normally distributed in order to make hypothesis tests on betas and alpha. But many empirical studies have shown this assumption is not likely true, for example, a recent study by Kosowski et al. (2001), where they used bootstrap analysis to assess the p value of alphas.

2.4. A Performance Measure Based on Style Analysis

Sharpe (1992) proposes to measure fund performance based on the return-based style analysis, which overcomes some limitations of traditional measures because of its different rationale and estimation techniques. It attracted a lot of attentions since this pioneering work, see, for example, Buetow, et al. (2000), Christopherson (1995, 1999), Cummisford, et al. (1996), Lieberman (1996), and Mayes, et al. (2000). The model is

$$r_{t} = \beta_{1}f_{1t} + \beta_{2}f_{2t} + \dots + \beta_{k}f_{kt} + \varepsilon_{t}$$
(2.10)

where r_i is the fund return *t*. f_{kt} is the kth index return in period *t*. f_{1t} to f_{kt} are called style indexes. The style of the fund is identified by the coefficients of the style indexes (β_i), which are defined as style exposures. For example, the small-cap growth fund is sensitive to small-cap growth stocks segment of the market and is expected to have an exposure towards this market leading to a relatively large style coefficient on the small-cap growth style index. The model implicitly assumes that the style of the fund is time-invariant in the estimation period. Therefore, it implies that the estimated style exposures are the average style during the evaluation period if the fund changes its factor loadings of assets. The return-based style analysis provides attractive results. It offers valuable insights regarding the fund's investment style. It provides a mechanism to detect asset mix of the fund based on manager's investment style.

Return-based style analysis can be naturally extended to measure fund performance by decomposing the return in model (2.10) into two parts. One is, $\beta_1 f_{1t} + \beta_2 f_{2t} + ... + \beta_k f_{kt}$, attributable to fund styles. The other is attributable to ε_t due to the active management like securities selection and asset allocation. It is defined as the tracking error at period *t*. The expected value of the tracking error, $E(\varepsilon_t)$, is defined as the performance of the fund, alpha. It is the difference between the realized fund return and the return of passive style indexes.

The measure has several advantages compared to traditional measures estimated by the regression method. First, we do not require that ε_{it} should be normally distributed. ε_{it} can be distributed differently. In addition, the expected value of ε_{it} is not even required to be zero. We

interpret the non-zero value of ε_{it} as the management effect, caused by securities selection or asset rotations. The expected value of ε_{it} is the measure of fund performance, a counterpart of the alpha of traditional measures. Second, we circumvent the benchmark inefficiency problem by including all the investable style indexes in the RBSA measure. The only requirements about the style indexes are that they are exhaustive and independent of each other (i.e., orthogonal). These requirements are easily accommodated by a large amount of indexes publicly available in the market. Third, the betas estimated in the RBSA measure provide useful information about fund styles. Fund styles are essential for the decomposed-analysis of the fund's risk level by institutional investors.

3. Setup of Simulation Experiment

We now test the robustness, accuracy, and efficiency of the alternative performance measures, and compare the RBSA measure with traditional measures by a comparative simulation experiment. The fund returns are generated from

$$r_{t} = a + \beta_{1}R_{1t} + \beta_{2}R_{2t} + \beta_{3}R_{3t} + \beta_{4}R_{4t} + \beta_{5}R_{5t} + \varepsilon_{t}$$
(3.1)

where *a* is set at 5% annually. It is possible to change the value of *a* in the simulation, but the results (not reported) show that the selection of *a* does not change our conclusions about the accuracy and efficiency of the measures. In (3.1) R_{1t} , R_{2t} , R_{3t} , R_{4t} and R_{5t} represent three-month Treasury bill rates, Russell Top 200 Growth Index, Russell Top 200 Value Index, Russell 2000 Growth Index, and Russell 2000 Value Index¹, respectively. These five indexes represent the fund's asset allocation to money market, large-cap growth stocks, large-cap value stocks, small-cap value stocks. ε_t is a randomly generated residual with a mean

¹ The definitions of the indexes are available at http://www.russell.com/US/Indexes/US/Definitions.asp.

of zero and standard deviation calculated from the actual style analysis of more than 1000 US domestic well-diversified equity mutual funds, following normal distribution.

To test the measures' ability to measure fund performance and its styles in different situations, we use four sets of beta coefficients as follows:

$$\begin{bmatrix} 0.05 & 0.48 & 0.47 & 0 & 0 \\ 0.05 & 0 & 0 & 0.48 & 0.47 \\ 0.05 & 0.35 & 0.35 & 0.13 & 0.12 \\ 0.05 & 0.13 & 0.12 & 0.35 & 0.35 \end{bmatrix}$$
(3.2)

The four sets of beta coefficients are to mimic the fund return behavior of four general types of funds: large-cap funds, small-cap funds, well-diversified funds with a preference to large-cap stocks, and well-diversified funds with a preference to small-cap stocks. For example, the first set of beta coefficients, [0.05 0.48 0.47 0 0], means that the simulated funds put 5% of assets in Treasury bills, 48% of assets in well-diversified large-cap growth stocks, 47% of assets in well-diversified value stocks, and no assets in small-cap stocks.

With the simulated return series of the fund, we are testing the power of the following performance measures that we reviewed in section 2:

1. RBSA measure that is formulated under the framework of a convex quadratic programming problem (RBSA):

$$r_t = \beta_1 f_{1t} + \beta_2 f_{2t} + \dots + \beta_k f_{kt} + \varepsilon_t$$

subject to $\beta e = 1$ and $\beta \ge 0$

In the simulation setup, the alpha of RBSA measure is simplified as the expected value of the in-sample ε_t .

2. Jensen measure (JS):

$$r_t - r_{ft} = \alpha_t + \beta_m (r_{mt} - r_{ft}) + \varepsilon_t$$

3. Jensen measure with Treynor-Mazuy market-timing adjustment (JS-TM):

$$r_t - r_{ft} = \alpha + \beta_m (r_{mt} - r_{ft}) + \gamma^{TM} (r_{mt} - r_{ft})^2 + \varepsilon_t$$

4. Jensen measure with Henriksson-Merton market-timing adjustment (JS-HM):

$$r_{t} - r_{ft} = \alpha + \beta_{m}(r_{mt} - r_{ft}) + \gamma^{HM} MAX(0, r_{mt} - r_{ft}) + \varepsilon_{t}$$

5. Fama-French three-factor measure (FF3):

$$r_t - r_{ft} = \alpha + \beta_m (r_{mt} - r_{ft}) + \beta_{SMB} r_{SMB,t} + \beta_{HML} r_{HML,t} + \varepsilon_t$$

6. Fama-French three-factor measure with Treynor-Mazuy market-timing adjustment (FF3-TM):

$$r_{t} - r_{ft} = \alpha + \beta_{m}(r_{mt} - r_{ft}) + \beta_{SMB}r_{SMB,t} + \beta_{HML}r_{HML,t} + \gamma^{TM}(r_{mt} - r_{ft})^{2} + \varepsilon_{t}$$

 Fama-French three-factor measure with Henriksson-Merton market-timing adjustment (FF3-HM):

$$r_t - r_{ft} = \alpha + \beta_m (r_{mt} - r_{ft}) + \beta_{SMB} r_{SMB,t} + \beta_{HML} r_{HML,t} + \gamma^{HM} MAX(0, r_{mt} - r_{ft}) + \varepsilon_t$$

where r_t is fund return.

Betas are risk exposures. And in the RBSA measure, betas are style coefficients. The riskfree rate r_{jt} is three-month Treasury bill rate. The market portfolio r_{mt} is S&P 500, the most frequently used proxy for market portfolio. γ^{TM} and γ^{HM} are market-timing coefficients measured by Treynor-Mazuy method and Henriksson-Merton method, respectively. In Fama-French three-factor models, $r_{SMB,t}$ and $r_{HML,t}$ are used to control investment strategies due to size effect and B/M ratio, respectively, where $r_{SMB,t}$ is the difference between the monthly return of Russell 1000 index and Russell 2000 index, and $r_{HML,t}$ is the difference between the monthly return of Russell 3000 Value Index and Russell Growth Index.

4. Simulation Results and Analysis

4.1. Simulation Results and Analysis of Alpha and R²

Table 1 shows simulation results of alpha and R^2 from seven measures, based on 1000 simulation replications of randomly generated fund return series under four sets of style coefficients in (3.2). They are presented in table 1 from panel 1 to panel 4. The alpha and R^2 are the average values of the estimation from 1000 simulation replications. The bias is reported as the difference between the estimated alphas from the measures and the true alpha, which is fixed at 5% in the simulation. To show the efficiency of the performance measurements, we also report the empirical confidence interval at 95% from the simulations. The lower bound is the 5th percentile of the 1000 estimated alphas and the upper bound is the 95th percentile of the 1000 estimated alphas and the upper bound is the 95th percentile of the cross correlations among stocks in the index portfolios (Kosowski et al., 2001), we construct the confidence intervals from simulation instead of constructing them from t values.

[Insert Table 1 about here]

Panel I of table 1 shows alpha estimates of the simulated fund with style coefficients [0.05 0.48 0.47 0 0], meaning 5% of fund asset is allocated to money market, 48% to well-diversified large-cap growth stocks, 47% to large-cap value stocks, and no asset is allocated to small stocks (growth as well as value). We find that RBSA is the most accurate measure with the bias only -0.24% annually. The other measures' accuracy is not comparable to that of the RBSA measure. The biases are larger than 1% as shown in the panel. Using the first set of betas, the three Jensen-based measures, that is, JS, JS-TM, and JS-HM, are less accurate than three FF3-based measures, that is, FF3, FF3-TM, and FF3-HM. The average bias of three JS-based measures is about two times larger than the average bias of three FF3-based measures.

After adjusting market-timing behavior, which actually does not exist in our simulation, with methods suggested by Treynor-Mazuy and Henrikksson-Merton, the biases are even larger, except for FF3-TM. Since there is no market-timing in the simulation, we should not observe any change of biases after adding a market-timing term if the market-timing models are solid. We observe spurious market-timing in the simulation. The spurious market timing is also found empirically by Cai, et al. (1997), Glosten and Jagannathan (1994), and Jagannathan and Korajczyk (1986).

The size of confidence interval indicates the efficiency of measures. JS measure has the smallest size, however since the alphas are severely biased, the efficiency gain has no meaning. The size of RBSA is similar to JS but less biased. The size of confidence interval is largest for FF3-based measures, which are about two times of the size of JS-based measures. This wider confidence interval of FF3 measures is mainly caused by using more variables at the right side of the regression. This kind of correlation may cause inaccurate estimation of alphas in FF3 measures.

We also notice that the R^2 is highest for RBSA measure whose average is 96%. FF3based measures show a little higher R^2 than JS-based measures. Therefore, using the first set of betas that mimics a large-cap fund we find RBSA measure is less biased and has the largest explanatory power and efficiency.

Panel II shows results using another set of betas. The simulated fund behaves like a small-cap fund according to style coefficients that we set in simulation. The magnitude of the bias of RBSA is similar to what we observed in panel I, but now is upwardly biased. And again RBSA has the smallest bias. But now we observe that bias of JS-based measures is much larger and R^2 is quite low, ranging from 52% to 55%. This is because we are using S&P 500 as the

market benchmark, in which most of the stocks are large-cap stocks. This bias clearly illustrates the inability of JS-based measures in capturing performance when funds invest in small-cap securities. FF3-based measures are using the same market benchmark as JS-based measures, but the biases are much smaller, which is due to the explicit incorporation of two risk factors related to size effect and the B/M ratio. We also observe the explanation power of FF3 is comparable to that of the RBSA measure. Therefore, when a fund is a small-cap fund, JS-based measures are not capable of estimating the true alpha. FF3-based measures are more robust than JS-based measures, because they explicitly consider the size effect in the model. RBSA is still the best measure in this case with high R², small bias and efficient estimation.

In panel III, we randomly generate a fund that widely invests in all the stocks in the market, but leans to large-cap stocks. We notice that the bias of RBSA is 0.1, but JS-based measures also have small biases when evaluating this kind of fund. The average is -0.29. The bias, efficiency and R^2 of FF3-based measures are similar to what we observed in panel I and panel II. In this set of style coefficients, JS-based measures are comparable to RBSA in terms of bias and efficiency but RBSA is more powerful to explain the fund's return behavior with the highest R^2 , 0.96.

In panel IV we generate a fund that widely invests in all the stocks in US market, but leans to small stocks with 70% of assets allocated to small stocks. We find RBSA is very accurate with only a 0.02% bias. The magnitude of bias and R^2 for FF3 measures is stable through the four situations. Regarding JS measures, in panel IV we again observe large bias and low explanation power ranging from 66% to 68%, as we observed in panel II.

From the summary panel of table 1, we find that RBSA unanimously has small biases with an average bias of 0.01% annually, high R^2 accounting for 97% of return variation, and

small size of confidence intervals, through the four situations in table 1. FF3-based measures have high R², stable biases, and stable size of confidence intervals, but the average bias is around 2.5%, which is much larger than the average bias of RBSA. JS-based measures have the largest biases and the biases are volatile depending on the type of the simulated fund. Although the size of the confidence intervals of JS-based measures is relatively small, the biases and variation of estimated alphas make the efficiency not meaningful. Adjusting market timing for JS and FF3 only makes the estimation less efficient, and causes biases larger in JS-based measures. Therefore, from simulation results we may say the RBSA is a better measure in measuring fund performance and explaining the fund return variation compared to other traditional measures.

4.2. Simulation Results and Analysis of Style Coefficients (Betas)

Table 2 presents simulation results of style coefficients (betas) in four situations. To test the robustness, accuracy, and efficiency of the seven measures in estimating style coefficients, we simulate four types of funds, that is, large-cap funds, small-cap funds, well-diversified funds with a preference for large-cap stocks, and well-diversified funds with a preference for small-cap stocks. The estimates of betas in the table are average betas of 1000 simulations, and the empirical confidence interval is obtained by setting the 5th percentile of the estimates as the lower bound and 95th percentile as the upper bound.

Panel I shows the estimation results when the simulated fund behaves like a large-cap fund. Our estimates of betas using RBSA are very close to the actual betas. The non-negativity constraints of betas may cause a small upward bias when betas are actually zeros and a small downward bias for other positive betas with the same magnitude. When we use traditional measures: JS-based measures and FF3-based measures, we find that the betas of the market benchmark are uniformly above 0.9. Considering the actual asset allocation where 95% of assets are invested in large-cap stocks, this beta estimation is acceptable. FF3 measures are capable of capturing the style of the fund. We find that β_{smb} is significant in all three cases, indicating a large-cap fund.

[Insert Table 2 about here]

When we study the performance measurement of a fund that behaves like a small-cap fund, which is shown in panel II, we have different results. The estimates based on RBSA are similar to the first panel, but we observe spurious market timing when using JS-based measures. In both JS-TM and JS-HM, we observe significant negative market timing. This may be caused by different return behavior of small-cap stocks from large-cap stocks, because after we control the size effect in FF3-based measures we don't observe market timing behavior of the fund. Again we find that FF3-based measures are capable of capturing the fund style, since β_{smb} is positive and significant, meaning that the fund generally moves in the same direction as the small stocks.

In panel III we investigate the measures' accuracy in measuring a well-diversified equity fund that leans to large-cap stocks. The accuracy in estimation of RBSA is stable as we observed before. But we find that FF3 measures show that the fund is a small-cap fund, which gives a significant positive β_{smb} . The result contradicts the actual asset allocation of the simulated fund, which invests 70% of its assets in large-cap stocks. Therefore, FF3-based measures don't correctly estimate the coefficients in this situation.

Panel 4 gives the estimation results of a well-diversified equity fund that leans to smallcap stocks. The estimates of RBSA are unbiased in this situation. In RBSA, all five estimates of betas are precisely the true values. We again observe the spurious negative market timing in JS- based measures, but no market timing in FF3 measures. The styles from FF3 measures are accurate, which indicates that it is a small-cap fund.

From simulation results of beta estimation, RBSA is quite successful in identifying the true asset allocation no matter whether it is a large-cap fund, a small-cap fund, or a well-diversified fund. FF3-based measures are capable of capturing the true fund style when the fund is exclusively investing in large-cap or small-cap stocks; however, when the fund is a well-diversified fund, FF3-based measures face difficulty in identifying the true styles. Another finding is that FF3-based measures may avoid the spurious market timing that we observed in JS-based measures.

5. Conclusion

From our simulation results of the performance measurement and style identification, we find that the RBSA measure seems to be the best measure among the seven measures. The RBSA measure is accurate, efficient and robust, and its performance does not depend on the type of the fund in the study. The average bias of alphas is around 0.01% annually, whereas the average biases of other measures range from 2.45% to 8.57% in absolute value. The beta coefficients estimation is also satisfactory, very close to the true betas as shown in table 2. However, the beta estimation may be upwardly biased when the beta is actually zero. Since we observed that the bias is quite small around 0.01, it does not pose any difficulty in implementation.

The estimates of JS-based measures are unstable, depending on the fund type. When the fund is a large-cap fund, the results are acceptable. However, when funds invest in small-cap stocks, there are some problems. First, it cannot identify fund styles. Second, it shows spurious negative market-timing, and third, it captures only a relatively small part of return variations, where R^2 is quite small compared to other measures with R^2 well above 90%.

FF3-based measures have stable estimates, not depending on the fund type. We find that using FF3-based measures we may avoid spurious market-timing that we observed in JS-based measures. However, they are unable to identify the true fund style of a well-diversified equity fund, thus the alpha estimates derived from the measures are also questionable. In addition, the accuracy and efficiency of the measures are not comparable with those of RBSA measure. Therefore, based on the criterion of accuracy, efficiency and robustness of the estimation of alpha and betas, RBSA appears to be superior to other measures.

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	TABLE 1					
Simulation I (Alpha and R^2)						
Panel I ($\beta_1 = 0.05, \beta_2 = 0.48, \beta_3 = 0.47, \beta_4 = 0, \beta_5 = 0$)						
<u>Measures</u>	<u>Alpha</u>	<u>Bias</u>	<u>C.I.</u>	<u>Size</u>	\mathbb{R}^2	
RBSA	4.76	-0.24	[2.05 7.59]	5.54	0.96	
JS	1.13	-3.87	[-1.46 3.75]	5.21	0.94	
JS-TM	0.51	-4.49	[-2.83 3.63]	6.46	0.94	
JS-HM	-0.32	-5.32	[-4.43 3.68]	8.11	0.94	
FF3	2.68	-2.32	[-4.03 9.13]	13.16	0.95	
FF3-TM	3.01	-1.99	[-4.20 10.16]	14.36	0.95	
FF3-HM	2.62	-2.38	[-5.15 10.41]	15.56	0.95	

Panel II ($\beta_1 = 0.05, \beta_2 = 0, \beta_3 = 0, \beta_4 = 0.48, \beta_5 = 0.47$)

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<u>Measures</u>	<u>Alpha</u>	<u>Bias</u>	<u>C.I.</u>	<u>Size</u>	$\frac{R^2}{R^2}$
RBSA	5.16	-0.16	[2.45 7.82]	5.37	0.97
JS	13.95	-8.95	[11.51 16.39]	4.88	0.52
JS-TM	24.03	19.03	[20.89 27.23]	6.34	0.55
JS-HM	27.48	22.48	[23.32 31.87]	8.55	0.54
FF3	1.89	-3.11	[-4.71 9.02]	13.73	0.97
FF3-TM	2.19	-2.81	[-4.87 8.91]	13.78	0.97
FF3-HM	2.21	-2.79	[-5.44 9.54]	14.98	0.97

TABLE 1 (Conunueu)								
Simulation I (Alpha and R^2)								
Panel III ($\beta_1 = 0.05, \beta_2 = 0.35, \beta_3 = 0.35, \beta_4 = 0.13, \beta_5 = 0.12$)								
<u>Measures</u>	<u>Alpha</u>	<u>Bias</u>	<u>C.I.</u>	<u>Size</u>	\mathbb{R}^2			
RBSA	5.1	0.1	[2.31 8.03]	5.72	0.96			
JS	4.71	-0.29	[2.04 7.29]	5.25	0.92			
JS-TM	6.75	1.75	[3.42 9.98]	6.56	0.92			
JS-HM	7.03	2.03	[2.72 11.14]	8.42	0.92			
FF3	2.53	-2.47	[-4.17 9.01]	13.18	0.95			
FF3-TM	2.75	-2.25	[-4.33 9.75]	14.08	0.95			
FF3-HM	2.49	-2.51	[-4.99 10.22]	15.21	0.95			

 TABLE 1 (Continued)

Panel IV ($\beta_1 = 0.05, \beta_2 = 0.13, \beta_3 = 0.12, \beta_4 = 0.35, \beta_5 = 0.35$)

<u>Measures</u> RBSA	<u>Alpha</u> 5.02	<u>Bias</u> 0.02	<u>C.I.</u> [2.31 7.81]		$\frac{R^2}{0.97}$
JS	10.64	5.64	[8.15 13.13]	4.98 0	0.67
JS-TM	17.86	12.86	[14.63 21.27]	6.64 0	0.69
JS-HM	20.09	15.09	[15.67 24.44]	8.77 0	0.68
FF3	1.87	-3.13	[-4.53 9.06]	13.59 0	0.96
FF3-TM	2.25	-2.75	[-4.74 9.06]	13.8 0	0.96
FF3-HM	2.14	-2.86	[-5.21 9.62]	14.81 0	0.96

TABLE 1 (Continued)

Simulation I (Alpha and R^2)

The table provides simulation results of alpha and R^2 under four sets of beta coefficients presented in equation (3.2). RBSA is return based style analysis measured by quadratic programming, JS is Jensen measure, JS-TM is JS measure with Treynor-Mazuy market-timing adjustment; JS-HM is the JS measure with Henriksson-Merton market-timing adjustment; FF3 is the Fama-French three-factor measure, FF3-TM is the FF3 measure with Treynor-Mazuy market-timing adjustment; FF3-HM is the FF3 measure with Henriksson-Merton market-timing adjustment; SF3-HM is the FF3 measure with Henriksson-Merton market-timing adjustment; SF3-HM is the FF3 measure with Henriksson-Merton market-timing adjustment. CI is the empirical confidence interval of alpha estimator based on simulations. Size is the length of C.I.

Results Su	immary						
	RBSA	<u>JS</u>	JS-TM	JS-HM	<u>FF3</u>	FF3-Tm	FF3-HM
<u>Alpha</u>							
Panel 1	4.76	1.13	0.51	-0.32	2.68	3.01	2.68
Panel 2	5.16	13.95	24.03	27.48	1.89	2.19	2.21
Panel 3	5.10	4.71	6.75	7.03	2.53	2.75	2.49
Panel 4	5.02	10.64	17.86	20.09	1.87	2.25	2.14
Average	5.01	7.61	12.29	13.57	2.25	2.55	2.36
Bias	0.01	2.61	7.29	8.57	-2.75	-2.45	-2.64
Std. Dev.	0.15	4.99	9.20	10.86	0.37	0.34	0.20
\mathbb{R}^2							
Panel 1	0.96	0.94	0.94	0.94	0.95	0.95	0.95
Panel 2	0.97	0.52	0.55	0.54	0.97	0.97	0.97
Panel 3	0.96	0.92	0.92	0.92	0.95	0.95	0.95
Panel 4	0.97	0.67	0.69	0.68	0.96	0.96	0.96
Average	0.97	0.76	0.78	0.77	0.96	0.96	0.96
Size of C.	<u>I.</u>						
Panel 1	5.54	5.21	6.46	8.11	13.16	14.36	15.56
Panel 2	5.37	4.88	6.34	8.55	13.73	13.78	14.98
Panel 3	5.72	5.25	6.56	8.42	13.18	14.08	15.21
Panel 4	5.50	4.98	6.64	8.77	13.59	13.80	14.81
Average	5.53	5.08	6.50	8.46	13.42	14.01	15.14

				,	
Panel I (β_1	$= 0.05, \beta_2 =$	$= 0.48, \beta_3 = 0.47,$	$\beta_4 = 0, \beta_5 = 0)$		
Measures	$\beta_1 = 0.05$	$\beta_2 = 0.48$	$\beta_3 = 0.47$	$\beta_4 = 0$	$\beta_5 = 0$
RBSA	0.05	0.47	0.47	0.01	0.01
	β_{m}	β_{smb}	β_{hml}	β_{tm}	β_{hm}
JS	0.96				
JS-TM	0.96			0.28	
JS-HM	0.92			[-0.47 1.07]	0.07 [-0.08 0.21]
FF3	0.96	-0.09 [-0.14 -0.04]	-0.01 [-0.11 0.1]		[-0.08 0.21]
FF3-TM	0.96	-0.09 [-0.15 -0.04]	-0.01 [-0.11 0.09	-0.05] [-0.83 0	.75]
FF3-HM	0.96	-0.09 [-0.14 -0.04]	-0.01 [-0.11 0.09]	0.01 [-0.15 0.164
Panel II (β	$\beta_1 = 0.05, \beta_2$	$= 0, \beta_3 = 0, \beta_4 =$	$0.48, \beta_5 = 0.47)$		
Measures	$\beta_1 = 0.05$	$\beta_2 = 0.0$	$\beta_{3} = 0.0$	$\beta_4 = 0.48$	$\beta_{5} = 0.47$
RBSA	0.04	0.01	0.01	0.48	0.46
	β_{m}	β_{smb}	β_{hml}	β_{tm}	β_{hm}
JS JS-TM	0.92 0.87			-3.8	
JS-HM	1.19			[-4.57 -3.02]	-0.6
FF3	0.94	0.95	0.04		[-0.75 -0.46]
FF3-TM	0.94	[0.9 1] 0.95	$[-0.07 \ 0.14]$ 0.04 $[0.07 \ 0.14]$	-0.01	
FF3-HM	0.95	$[0.9 1] \\ 0.95 \\ [0.9 1.01]$	[-0.07 0.14] 0.03 [-0.07 0.13]	[-0.9 0.7]	-0.01 [-0.17 0.15]

TABLE 2Simulation II (Style Coefficients)

			E 2 (Continue II (Style Coeffi					
Panel III ($\beta_1 = 0.05, \beta_2 = 0.35, \beta_3 = 0.35, \beta_4 = 0.13, \beta_5 = 0.12$)								
Measures	$\beta_1 = 0.05$	$\beta_2 = 0.35$	$\beta_3 = 0.35$	$\beta_4 = 0.13$	$\beta_{5} = 0.12$			
RBSA	$\frac{0.05}{\beta_{\rm m}}$	$\frac{0.34}{\beta_{smb}}$	$\frac{0.36}{\beta_{\rm hml}}$	$\frac{0.13}{\beta_{\text{tm}}}$	0.11 β _{hm}			
JS JS-TM	0.95 0.94			-0.79 [-1.57 0.01]				
JS-HM	1				-0.11 [-0.25 0.04]			
FF3	0.95	0.18 [0.13 0.24]	0 [-0.1 0.1]		[-0.25 0.04]			
FF3-TM	0.96	0.18 [0.13 0.24]	0 [-0.11 0.11	-0.07 [] [-0.82 0.	66]			
FF3-HM	0.95	0.18 [0.13 0.24]	0 [-0.1 0.1]		0 [-0.15 0.15]			
Panel IV ($\beta_1 = 0.05, \beta_2$	$= 0.13, \beta_3 = 0.12,$	$\beta_4 = 0.35, \beta_5$	= 0.35)				
Measures	$\beta_1 = 0.05$	$\beta_2 = 0.13$	$\beta_3 = 0.12$	$\beta_4 = 0.35$	$\beta_{5} = 0.35$			
RBSA	0.05	0.13	0.12	0.35	0.35			
	β_{m}	β_{smb}	β_{hml}	β_{tm}	β_{hm}			
JS JS-TM	0.93 0.9			-2.7 [-3.44 -1.92]				
JS-HM	1.12				-0.42 [-0.58 -0.28]			
FF3	0.94	0.68 [0.63 0.73]	0.03 [-0.08 0.13]	1	L-0.20 -0.20]			
FF3-TM	0.94	0.68	0.03	-0.01	1			
FF3-HM	0.95	$\begin{bmatrix} 0.62 & 0.73 \\ 0.67 \\ \\ [0.62 & 0.72] \end{bmatrix}$	-0.02 [-0.08 0.12		0 [-0.15 0.14]			