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Modelling Overnight and Daytime Returns Using a Multivariate GARCH-Copula Model[1]

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Abstract We introduce a multivariate GARCH-Copula model to describe joint dynamics of overnight and daytime returns for multiple assets. The conditional mean and variance of individual overnight and daytime returns depend on their previous realizations through a variant of GARCH specification, and two Student's *t* copulas describe joint distributions of both returns respectively. We employ both constant and time-varying correlation matrices for the *t* copulas and with the time-varying case the dependence structure of both returns depends on their previous dependence structures through a DCC specification. We estimate the model by a two-step procedure, where marginal distributions are estimated in the first step and copulas in the second. We apply our model to overnight and daytime returns of 15 funds of different types, and illustrate its applications in risk management and asset allocation. Our empirical results show (for most tested assets) higher means, lower variance, fatter tails for overnight returns than daytime returns. The comparison results of dependence between overnight and daytime returns are mixed. Daytime returns are significantly negatively correlated with previous overnight returns. Moreover, daytime returns depend on previous overnight returns in both conditional variance and correlation matrix (through a DCC specification). Most of our empirical findings are consistent with the asymmetric information argument in the market microstructure literature. With respect to econometric modelling, our results show a DCC specification for correlation matrices of *t* copulas significantly improves the fit of data and enables the model to account for time-varying dependence structure.

JEL classification: C32, G12, G14.

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1. Introduction

Modeling the dynamics of overnight and daytime returns is important in at least two aspects. First, a good description of overnight and daytime returns can help to test alternative theories on different features of market microstructure during the day and night. Second, from a practitioner's point of view, many financial instruments (such as index options) are based on the opening prices of underlying assets. Thus, it is necessary to distinguish between overnight and daytime returns for risk management or asset allocation purposes. There is plenty of work documenting different empirical properties of overnight and daytime returns. Some show that the average overnight returns are statistically higher than average daytime returns, while others show higher average returns over trading periods than non-trading periods (see Cliff et al. (2008) and Keim & Stambaugh (1984)). Some show that daytime returns are statistically negatively correlated with previous overnight returns, while others show that the two returns are largely independent (see Gallo *et al*. (2001), Branch & Ma (2006) and Oldfield & Rogalski (1980)). The variance of daytime returns is significantly higher than that of overnight returns (see French & Roll (1986) and Lockwood & Linn (1990)). Overnight returns are more leptokurtic than daytime returns[2] (see Masulis & Ng (1995)). Those empirical patterns also generate interest in proposing theoretical models to explain them (see Admati & Pfleiderer (1988, 1989) and Hong & Wang (2000)).

To the best of our knowledge, current literature on this topic only deals with the univariate case and there is no work on modeling the joint dynamics of multiple assets. Our work attempts to fill this gap. First, we introduce a comprehensive model which can reasonably well capture key empirical aspects of both returns and can be easily implemented by practitioners. Second, by applying the model to the data, we expect to find some different features of the dependence structure between both returns. Moreover, we illustrate how the model can be effectively used for risk management or asset allocation purposes.

With a GARCH-Copula framework, we can more flexibly construct the joint distribution of multiple returns. The dynamics of overnight and daytime returns for each asset are described by a GARCH process, where the conditional mean and variance depend on the previous realizations of both returns and innovations are described by Student's *t* distributions. Then we use two Student's *t* copulas to link overnight and daytime returns of multiple assets respectively. The constant correlation matrices in Student's *t* copulas are assumed and estimated first, and then to describe the time-varying feature of dependence structure we implement a DCC (Dynamic Conditional Correlation) specification for Student's *t* copulas. We estimate the model by a two-step procedure, where the marginal distributions are estimated in the first step and the copulas in the second.

We apply our model to the overnight and daytime returns of 15 funds of different types from September 21, 2006 to March 29, 2011. At the individual level, for most cases daytime returns depend on previous overnight returns negatively. The conditional variance of daytime returns is consistently higher than that of overnight returns. Almost all the ETFs have lower DoF

[2] See Kang & Babbs (2010b) for a comprehensive empirical investigation.

parameters for overnight returns than for daytime returns, which is consistent with the observed higher kurtosis of overnight returns. In addition to their own lags, the conditional variance of overnight (daytime) returns has some degree of dependence on previous daytime (overnight) returns.

With constant Student's *t* copulas, the copula governing overnight returns has a lower DoF than that of daytime returns. The comparison of correlation matrices for the t copulas yield mixed patterns. Meanwhile, time-varying t copulas yield similar results. For most cases and time period (including financial crisis) we tested, overnight returns have significantly higher tail dependence patterns than daytime returns. Moreover, time-varying copula models show that the dependence structure of daytime returns depend on that of previous overnight returns. This adds to observed non-linear dependence between daytime and previous overnight returns. We believe most of our empirical findings are consistent with the asymmetric information argument in theoretical models such as Admati & Pfleiderer (1988, 1989). Moreover, our results show that a DCC specification for correlation matrices of *t* copulas significantly improves fit of data and enables the model to account for time-varying dependence structure.

The rest of the paper is organized as follows. Section 2 introduces the multivariate GARCH-Copula model and its estimation and simulation procedures. Section 3 applies the model to the overnight and daytime returns of 15 funds of different assets or sectors. Section 4 illustrates the applications of the model for risk management and asset allocation purposes. Section 5 concludes.

2. A multivariate GARCH-Copula model

2.1. Individual returns

We model individual returns using a variant of GARCH specification. Let $R_{i,t} = [r_{i,n,t}\ r_{i,d,t}]'$ be overnight and daytime returns for asset i observed at the open and close respectively on day *t*. The individual returns for asset i are

$$r_{i,n,0}, r_{i,d,0}, \ldots, r_{i,n,t-1}, r_{i,d,t-1}, r_{i,n,t}, r_{i,d,t}, \ldots r_{i,n,T}, r_{i,d,T}.$$

We specify the conditional mean as

$$r_{i,n,t} = \alpha_{i,0} + \alpha_{i,1} r_{i,d,t-1} + \alpha_{i,2} r_{i,n,t-1} + \eta_{i,n,t}, \tag{1}$$

$$r_{i,d,t} = \beta_{i,0} + \beta_{i,1} r_{i,n,t} + \beta_{i,2} r_{i,d,t-1} + \eta_{i,d,t}. \tag{2}$$

where $r_{i,n,t}$ depends on previous daytime return $r_{i,d,t-1}$ and previous overnight return $r_{i,n,t-1}$, and similarly $r_{i,d,t}$ depends on $r_{i,n,t}$ and $r_{i,d,t-1}$. We specify residuals as

$$\eta_{i,n,t} = \sqrt{h_{i,n,t}}\,\varepsilon_{i,n,t}, \tag{3}$$

$$\eta_{i,d,t} = \sqrt{h_{i,d,t}}\,\varepsilon_{i,d,t}, \tag{4}$$

where $\varepsilon_{i,n,t}$ and $\varepsilon_{i,d,t}$ are *i.i.d.* innovations with zero mean and unitary variance respectively and are independent between each other at all times, and $h_{i,n,t}$ and $h_{i,d,t}$ are conditional variance described by the following equations.

$$h_{i,n,t} = \theta_{i,0} + \theta_{i,1}\eta_{i,d,t-1}^2 + \theta_{i,2}\eta_{i,n,t-1}^2 + \theta_{i,3}h_{i,n,t-1}, \tag{5}$$

$$h_{i,d,t} = \delta_{i,0} + \delta_{i,1}\eta_{i,n,t}^2 + \delta_{i,2}\eta_{i,d,t-1}^2 + \delta_{i,3}h_{i,d,t-1}. \tag{6}$$

where we require $\theta_{i,j} > 0$ and $\delta_{i,j} > 0$ for $j = 0,...,3$, and the eigenvalues of the matrix $\begin{bmatrix} \theta_{i,2}+\theta_{i,3} & \theta_{i,1} \\ \delta_{i,1}(\theta_{i,2}+\theta_{i,3}) & \delta_{i,1}\theta_{i,2}+(\delta_{i,2}+\delta_{i,3}) \end{bmatrix}$ are less than one in absolute values to ensure the stationarity of the squared errors. Those parameter restrictions guarantee conditional variances are always non-negative and squared residuals are stationary[3]. We assume that the innovations $\varepsilon_{i,n,t}$ and $\varepsilon_{i,d,t}$ have standardized Student's *t* distribution as

$$\varepsilon_{i,n,t} \sim ST(0,1,\nu_{i,n}),\ \varepsilon_{i,d,t} \sim ST(0,1,\nu_{i,d}),$$

where $\nu_{i,n}$ and $\nu_{i,d}$ are degree-of-freedom (DoF) parameters and we have $\nu_{i,n}, \nu_{i,d} > 2$ to ensure the existence of second movements.

It is worth noting that we can use a more sophisticated specification for marginal distributions of individual returns. For instance, we can include more explanatory variables in equations (1) and (2) to possibly better describe the conditional mean, include a GJR specification (see Glosten *et al* (1993)) in equations (5) and (6) to account for the asymmetric effect of stock returns on the conditional variance and specify a distribution with time-varying high-moment parameters (e.g. time-varying Hansen's (1994) skewed t distribution) for residual innovations. Nevertheless, we will focus more on modelling joint distributions here. As we believe the current setup describes the data reasonably well, we will leave those options for further research.

2.2. Copulas

After specifying marginal distributions of returns, we need two copula functions to link overnight and daytime returns respectively across all assets. To formulate the joint distribution of returns for k assets, we are facing the following multiple time series,

$$R_{1,0},...,R_{k,0},...,R_{1,t-1},...,R_{k,t-1},R_{1,t},...,R_{k,t},...,R_{1,T},...,R_{k,T}.$$

[3] A more general VARMA model for the vector of squared overnight and daytime returns can be proposed. The parameter restrictions will depend on the stationartiy and identification of the corresponding VARMA process. See Kang & Babbs (2010a).

Let F_t, $F_{d,t}$ and $F_{n,t}$ be the conditional cumulative distribution function (*c.d.f.*) for $R_{1,t},...,R_{k,t}$, $r_{1,d,t},...,r_{k,d,t}$, and $r_{1,n,t},...,r_{k,n,t}$. At each time t, the conditional distribution of $R_{1,t},...,R_{k,t}$ is given as

$$F_t(R_{1,t},...,R_{k,t} \mid R_{1,t-1},...,R_{k,t-1},...,R_{1,0},...,R_{k,0}) = F_{d,t}(r_{1,d,t},...,r_{k,d,t} \mid \Omega_{d,t-1}) F_{n,t}(r_{1,n,t},...,r_{k,n,t} \left| \Omega_{n,t-1} \right.), \quad (7)$$

where $\Omega_{d,t-1} = \{r_{1,n,t},...,r_{k,n,t}, r_{1,d,t-1},...,r_{k,d,t-1},...,r_{1,d,0},...,r_{k,d,0}, r_{1,n,0},...,r_{k,n,0}\}$ and $\Omega_{n,t-1} = \{r_{1,d,t-1},...,r_{k,d,t-1}, r_{1,n,t-1},...,r_{k,n,t-1},...,r_{1,d,0},...,r_{k,d,0}, r_{1,n,0},...,r_{k,n,0}\}$. Let $F_{i,d,t}$ and $F_{i,n,t}$, for $i=1,...,k$, be the conditional *c.d.f.*'s for $r_{i,d,t}$ and $r_{i,n,t}$. Then the conditional joint distributions $F_{d,t}(r_{1,d,t},...,r_{k,d,t} \mid \Omega_{d,t-1})$ and $F_{n,t}(r_{1,n,t},...,r_{k,n,t} \left| \Omega_{n,t-1} \right.)$ can be modelled using two copulas as,

$$F(r_{1,d,t},...,r_{k,d,t} \left| \Omega_{d,t-1} \right.) = C_d(F(r_{1,d,t} \left| \Omega_{d,t-1} \right.),...,F(r_{k,d,t} \left| \Omega_{d,t-1} \right.) \left| \Omega_{d,t-1} \right.),$$

$$F(r_{1,n,t},...,r_{k,n,t} \left| \Omega_{n,t-1} \right.) = C_n(F(r_{1,n,t} \left| \Omega_{n,t-1} \right.),...,F(r_{k,n,t} \left| \Omega_{n,t-1} \right.) \left| \Omega_{n,t-1} \right.),$$

where C_n and C_d are the two copula *c.d.f.*'s. It is worth noting that the copula function and marginal distributions are all conditional on the previous information set.

There are many choices of copula functions for modelling the dependence structure of multiple variables. We use Student's *t* copula in this paper. The *c.d.f.* of Student's *t* copula is given by

$$C(u_1,...,u_k) = T_{R,\nu}(T_\nu^{-1}(u_1),...,T_\nu^{-1}(u_k)), \quad (8)$$

where $T_{R,\nu}$ is the N-dimensional Student's *t* distribution with correlation matrix R and DoF parameter ν, and $T_\nu^{-1}(\cdot)$ is the inverse of univariate standard Student's *t* distribution[4]. The probability density function (*p.d.f.*) of Student's *t* copula is

$$c(u_1,...,u_k) = \frac{t_{R,\nu}(T_\nu^{-1}(u_1),...,T_\nu^{-1}(u_k))}{\prod_{i=1}^{k} t_\nu(T_\nu^{-1}(u_i))},$$

where $t_{R,\nu}(\cdot)$ is the density function of $T_{R,\nu}$ and $t_\nu(\cdot)$ is the density of T_ν, the standard Student's *t* distribution.

When using Student's *t* copulas for both returns, we can assign constant or time-varying correlation matrices for both *t* copulas. As for the time-varying *t* copulas, we borrow the idea of DCC-GARCH models to make the two sets of correlation matrices depend on past realizations.

[4] In contrast to the previous standardized Student's t distribution, the standard Student's t distribution here has variance as $\nu/(\nu-2)$.

Let $\varsigma_{cn,t} = \left(T_{v_{cn}}^{-1}(u_{1,n,t}),...,T_{v_{cn}}^{-1}(u_{k,n,t})\right)'$ and $\varsigma_{cd,t} = \left(T_{v_{cd}}^{-1}(u_{1,d,t}),...,T_{v_{cd}}^{-1}(u_{k,d,t})\right)'$ according to (8) and $Q_{n,t}$ and $Q_{d,t}$ be the conditional covariance matrices of $\varsigma_{cn,t}$ and $\varsigma_{cd,t}$ respectively. The time-varying correlation matrices $R_{cn,t}$ and $R_{cd,t}$ of t copulas are governed by the dynamics of $Q_{n,t}$ and $Q_{d,t}$ as

$$Q_{n,t} = \Pi_0 + \pi_1(\varsigma_{cd,t-1}\varsigma'_{cd,t-1}) + \pi_2(\varsigma_{cn,t-1}\varsigma'_{cn,t-1}) + \pi_3 Q_{d,t-1} + \pi_4 Q_{n,t-1}, \tag{9}$$

$$Q_{d,t} = \Psi_0 + \psi_1(\varsigma_{cn,t}\varsigma'_{cn,t}) + \psi_2(\varsigma_{cd,t-1}\varsigma'_{cd,t-1}) + \psi_3 Q_{n,t} + \psi_4 Q_{d,t-1}, \tag{10}$$

where $\pi_i \geq 0$ and $\psi_i \geq 0$ for $i = 1,...,4$, and the eigenvalues of the matrix $\begin{bmatrix} \pi_2 + \pi_4 & (\pi_1 + \pi_3) \\ (\psi_1 + \psi_3)(\psi_2 + \psi_4) & (\pi_1 + \pi_3)(\psi_1 + \psi_3) + (\psi_2 + \psi_4) \end{bmatrix}$ are less than one in absolute values to ensure the system of (9) and (10) is valid and stationary[5]. With stationarity, it can be shown that

$$\Pi_0 = (1 - (\pi_2 + \pi_4))S_n - (\pi_1 + \pi_3)S_d, \tag{11}$$

$$\Psi_0 = (1 - (\psi_2 + \psi_4))S_d - (\psi_1 + \psi_3)S_n, \tag{12}$$

where S_n and S_d are the unconditional covariance of $\varsigma_{cn,t}$ and $\varsigma_{cd,t}$. Let $q_{i,j,n,t}$ and $q_{i,j,d,t}$ be the i,j-element of $Q_{n,t}$ and $Q_{d,t}$ respectively, then the i,j-elements of $R_{cn,t}$ and $R_{cd,t}$ are given as

$$\rho_{i,j,n,t} = \frac{q_{i,j,n,t}}{\sqrt{q_{i,i,n,t}q_{j,j,n,t}}}, \tag{13}$$

$$\rho_{i,j,d,t} = \frac{q_{i,j,d,t}}{\sqrt{q_{i,i,d,t}q_{j,j,d,t}}}. \tag{14}$$

With $Q_{n,0} = S_n$ and $Q_{d,0} = S_d$, the equations (9) to (14) completely govern the dynamics of the correlations matrices $R_{cn,t}$ and $R_{cd,t}$. For the correlation matrices to be positive definite, we have the following sufficient conditions.

Proposition 1 *In equations (9) to (14), if*

a) $\pi_i \geq 0$ and $\psi_i \geq 0$ for $i = 1,...,4$,

b) the eigenvalues of the matrix $\begin{bmatrix} \pi_2 + \pi_4 & (\pi_1 + \pi_3) \\ (\psi_1 + \psi_3)(\psi_2 + \psi_4) & (\pi_1 + \pi_3)(\psi_1 + \psi_3) + (\psi_2 + \psi_4) \end{bmatrix}$ are less than one in absolute values,

c) all eigenvalues of S_n and S_d are strictly positive,

d) all eigenvalues of Π_0 and Ψ_0 are strictly positive,

[5] See Appendix for derivation details.

then the correlations matrices $R_{cn,t}$ and $R_{cd,t}$ are positive definite.

Proof: First, a) and b) guarantee the system is stationary and S_n and S_d exist. With $Q_{n,0} = S_n$ and $Q_{d,0} = S_d$, c) guarantees $Q_{n,0}$ and $Q_{d,0}$ are positive definite. With a) and d), $Q_{n,t}$ and $Q_{d,t}$ are the sum of positive semi-definite and positive definite matrices with non-negative coefficients and therefore are positive definite for all t. Based on the proposition 1 in Engle & Sheppard (2001), we prove that $R_{cn,t}$ and $R_{cd,t}$ are positive definite.

2.3. Estimation

We estimate the whole density function by ML estimation procedures. Let F and F_0 be the *c.d.f.*'s for $R_{1,0},...,R_{k,0},...,R_{1,T},...,R_{k,T}$ and $R_{1,0},...,R_{k,0}$. Using Bayes' Theorem, the whole joint distribution of returns can be written as

$$F(R_{1,0},...,R_{k,0},...,R_{1,T},...,R_{k,T}) = F_0(R_{1,0},...,R_{k,0})\prod_{t=1}^{T}(C_d(F_{1,d,t}(r_{1,d,t} \mid \Omega_{d,t-1}),...,F_{k,d,t}(r_{k,d,t} \mid \Omega_{d,t-1}) \mid \Omega_{d,t-1}) \cdot C_n(F_{1,n,t}(r_{1,n,t} \mid \Omega_{n,t-1}),...,F_{k,n,t}(r_{k,n,t} \mid \Omega_{n,t-1}) \mid \Omega_{n,t-1})).$$

Correspondingly, the joint density of returns can be written as

$$f(R_{1,0},...,R_{k,0},...,R_{1,T},...,R_{k,T}) = f_0(R_{1,0},...,R_{k,0})\prod_{t=1}^{T}(c_d(F_{1,d,t}(r_{1,d,t}),...,F_{k,d,t}(r_{k,d,t}))\prod_{i=1}^{k} f_{i,d,t}(r_{i,d,t}) \cdot c_n(F_{1,n,t}(r_{1,n,t}),...,F_{k,n,t}(r_{k,n,t}))\prod_{i=1}^{k} f_{i,n,t}(r_{1,n,t})), \qquad (15)$$

where $\Omega_{d,t-1}$ and $\Omega_{n,t-1}$ are suppressed for notation convenience, f_0, $f_{i,d,t}$ and $f_{i,n,t}$ are the densities for F_0, $F_{i,d,t}$ and $F_{i,n,t}$, and c_n and c_d are the two copula densities for overnight and daytime returns respectively. From equation (15), we can continue to write the density as

$$f(R_{1,0},...,R_{k,0},...,R_{1,T},...,R_{k,T}) = f_0(R_{1,0},...,R_{k,0})\prod_{t=1}^{T}(c_d(F_{\varepsilon_{1,d,t}}(\varepsilon_{1,d,t}),...,F_{\varepsilon_{k,d,t}}(\varepsilon_{k,d,t}))\prod_{i=1}^{k} f_{\varepsilon_{i,d,t}}(\varepsilon_{i,d,t})\frac{1}{\sqrt{h_{i,d,t}}} c_n(F_{\varepsilon_{1,n,t}}(\varepsilon_{1,n,t}),...,F_{\varepsilon_{k,n,t}}(\varepsilon_{k,n,t}))\prod_{i=1}^{k} f_{\varepsilon_{i,n,t}}(\varepsilon_{i,n,t})\frac{1}{\sqrt{h_{i,n,t}}}), \qquad (16)$$

where $F_{\varepsilon_{i,d,t}}$ and $F_{\varepsilon_{i,n,t}}$ are the *c.d.f.*'s for $\varepsilon_{i,d,t}$ and $\varepsilon_{i,n,t}$, and $f_{\varepsilon_{i,d,t}}$ and $f_{\varepsilon_{i,n,t}}$ are corresponding densities.

Let $\Theta = \{\theta_{cn}, \theta_{cd}, \theta_1, ..., \theta_k\}$ be a set of parameters for the two copula densities c_n and c_d, and $R_{i,t}$ for $i = 1,...,k$ respectively. Omitting the first term $f_0(R_{1,0},...,R_{k,0})$, we can write the log-likelihood as

$$L(\Theta) = \sum_{t=1}^{T} \log c_d(F_{\varepsilon_{1,d,t}}(\varepsilon_{1,d,t}),...,F_{\varepsilon_{N,d,t}}(\varepsilon_{k,d,t})) + \sum_{t=1}^{T} \log c_n(F_{\varepsilon_{1,n,t}}(\varepsilon_{1,n,t}),...,F_{\varepsilon_{k,n,t}}(\varepsilon_{k,n,t}))$$
$$+ \sum_{i=1}^{k}\sum_{t=1}^{T} \log f_{\varepsilon_{i,d,t}}(\varepsilon_{i,d,t}) \frac{1}{\sqrt{h_{i,d,t}}} + \sum_{i=1}^{k}\sum_{t=1}^{T} \log f_{\varepsilon_{i,n,t}}(\varepsilon_{i,n,t}) \frac{1}{\sqrt{h_{i,n,t}}}. \tag{17}$$

To estimate all the parameters simultaneously often leads to convergence problem of maximizing (17). Therefore, we maximize the whole log-likehihood by a two-step procedure. First, we estimate the marginal distribution of each asset. For asset i, the log-likelihood is

$$L(\theta_i) = \sum_{t=1}^{T} \log f_{\varepsilon_{i,d,t}}(\varepsilon_{i,d,t}) \frac{1}{\sqrt{h_{i,d,t}}} + \sum_{t=1}^{T} \log f_{\varepsilon_{i,n,t}}(\varepsilon_{i,n,t}) \frac{1}{\sqrt{h_{i,n,t}}}. \tag{18}$$

To further facilitate the estimation, we first estimate the conditional mean in equations (1) and (2) by ordinary least squares (OLS). Then with estimated OLS residuals, we can estimate the variance equations (5) and (6) by maximizing (18). The specific log-likehood becomes

$$L = \sum_{t=1}^{T} \log f_{ST}\left(\frac{r_{i,n,t} - \hat{\alpha}_{i,0} - \hat{\alpha}_{i,1} r_{i,d,t-1} - \hat{\alpha}_{i,2} r_{i,n,t-1}}{\sqrt{h_{i,n,t}}} \mid \nu_{i,n}\right) + \sum_{t=1}^{T} \log f_{ST}\left(\frac{r_{i,d,t} - \hat{\beta}_{i,0} - \hat{\beta}_{i,1} r_{i,n,t} - \hat{\beta}_{i,2} r_{i,d,t-1}}{\sqrt{h_{i,d,t}}} \mid \nu_{i,d}\right)$$
$$+ \sum_{t=1}^{T} \log \frac{1}{\sqrt{h_{i,n,t}}} + \sum_{t=1}^{T} \log \frac{1}{\sqrt{h_{i,d,t}}},$$

where f_{ST} denotes the density of standardized Student's *t* distribution. With all the marginal distributions being estimated, the only component left out in (17) is the copula part. With estimated marginal distribution parameters, we can estimate the two copulas by maximizing

$$L(\theta_{cn}, \theta_{cd}) = \sum_{t=1}^{T} \log c_d(\hat{F}(\varepsilon_{1,d,t}),...,\hat{F}(\varepsilon_{k,d,t})) + \sum_{t=1}^{T} \log c_n(\hat{F}(\varepsilon_{1,n,t}),...,\hat{F}(\varepsilon_{k,n,t})), \tag{19}$$

where $\hat{F}(\cdot)$ is the estimated *c.d.f.* for each innovation. Whether the maximization of (19) is easy or not depends on the specific copula functions. With normal copulas, we can derive analytical ML estimates very easily. With Student's *t* copulas, however, the parameters θ_{cn} and θ_{cd} consist of correlation matrices R_{cn} and R_{cd}, and the DoF parameters ν_{cn} and ν_{cd}, and there is no easy analytical solution for maximizing (19). To smoothly solve this maximization problem, with $\varsigma_{cn,t} = \left(T_{\nu_{cn}}^{-1}(u_{1,n,t}),...,T_{\nu_{cn}}^{-1}(u_{k,n,t})\right)'$ and $\varsigma_{cd,t} = \left(T_{\nu_{cd}}^{-1}(u_{1,d,t}),...,T_{\nu_{cd}}^{-1}(u_{k,d,t})\right)'$, we assign $\hat{R}_{cn}$ and $\hat{R}_{cd}$ as the sample correlation matrices of $\varsigma_{cn,t}$ and $\varsigma_{cd,t}$. $\hat{R}_{cn}$ and $\hat{R}_{cd}$ are functions of DoF parameters ν_{cn} and ν_{cd}. Therefore, we can plug $\hat{R}_{cn}$ and $\hat{R}_{cd}$ into (19) and solve the maximization problem in terms of ν_{cn} and ν_{cd}. Furthermore, as parameters for the two copulas are separate, we can maximize the two components in (19) separately to solve for ν_{cn} and ν_{cd} respectively.

To estimate the time-varying *t* copula, we still maximize the log-likelihood as in (19) except that the time-varying *t* copulas are used. To reduce the number of parameters to directly estimate, we express the time-varying correlation matrices $R_{cn,t}$ and $R_{cd,t}$ as functions of DoF parameters ν_{cn} and ν_{cd} as in the above constant copula case. Specifically, with given ν_{cn} and ν_{cd}, we have the estimated unconditional covariance of $\varsigma_{cn,t}$ and $\varsigma_{cd,t}$ as $\hat{S}_n$ and $\hat{S}_d$, and we set the initial $Q_{n,0}$ and $Q_{d,0}$ equal to $\hat{S}_n$ and $\hat{S}_d$ respectively. Equations (9) to (14) completely describes the dynamics of the correlation matrices $R_{cn,t}$ and $R_{cd,t}$. Then all the parameters to estimate are π_i, ω_i for $i = 1,...,4$, ν_{cn} and ν_{cd}, and the maximization is conducted with the corresponding restrictions.

Generally, this two-step estimation procedure is called inference for the margins (IFM) method. Joe (1997) shows that under regular conditions the IFM estimator is consistent and has the property of asymptotic normality and Patton (2006) also shows that this two-step method yields asymptotically normal parameter estimates.

2.4. Density forecast and simulations

With parameter estimates, we can forecast the joint density and then simulate future overnight and daytime returns. Specifically, with estimated parameters $\hat{\Theta}$ and $\Omega_{n,t-1}$, we can sequentially forecast $f(r_{1,n,t},...,r_{k,n,t} \left| \Omega_{n,t-1}\right.)$, $f(r_{1,d,t},...,r_{k,d,t} \mid \Omega_{d,t-1})$ and then $f(r_{1,d,t},...,r_{k,d,t}, r_{1,n,t},...,r_{k,n,t} \left| \Omega_{n,t-1}\right.)$. Accordingly, we can sequentially simulate $\{r_{1,n,t},...,r_{k,n,t}\}$ and $\{r_{1,d,t},...,r_{k,d,t}\}$. With $\hat{\theta}_{cn}$, we first simulate the copula C_n to get a simulated vector $\{\tilde{u}_1,...,\tilde{u}_k\}$. Using estimated GARCH parameters $\hat{\theta}_1,...,\hat{\theta}_k$, and equations (1), (3) and (5), we can back out $\{\tilde{r}_{1,n,t},...,\tilde{r}_{k,n,t}\}$. Finally, we can use $\hat{\theta}_{cd}$ to simulate copula C_d and then back out $\{\tilde{r}_{1,d,t},...,\tilde{r}_{k,d,t}\}$.

3. An empirical investigation

3.1. Data

We apply our model to returns of 15 funds of different types from September 21, 2006 to March 29, 2011. The 15 symbols are XLY, XLP, XLE, XLF, XLV, XLI, XLB, XLK, XLU, AGG, GSG, USO, RWR, DBV and FXE The first nine symbols are SPDR ETFs representing the sectors of Consumer Discretionary, Consumer Staples, Energy, Financial, Healthcare, Industrial, Materials, Technology and Utilities. The remaining six symbols represent iShares Lehman Aggregate Bond ETF, iShares S&P GSCI Commodity-Indexed ETF, United States Oil Fund LP ETF, SPDR DJ Wilshire REIT ETF, PowerShares DB G10 Currency Harvest Fund, and Currency Shares Euro Trust.

Figure 1 plots overnight log returns of the fifteen funds and figure 2 plots the daytime log returns. The time period we choose for empirical applications features an unprecedented high volatility due to the financial crisis. The open and close prices and dividend payments are directly downloaded from http://finance.yahoo.com/. We calculate log overnight and daytime returns based on the open and close prices and dividend payments.

Table 1 reports the descriptive statistics (mean, standard deviation, Skewness and Kurtosis) for overnight and daytime returns of 15 funds of different types from September 21, 2006 to March 29, 2011. We find that overnight returns are all positive and have consistently higher averages than daytime returns except USO, DBV and FXE. Daytime returns have significantly higher standard deviations than overnight returns except the two currency funds DBV and FXE. Skewness for both returns has mixed signs. Kurtosis is greater than three for both returns and overnight returns tend to have higher Kurtosis except the currency fund FXE. It is not difficult to understand why FXE has higher Kurtosis for its daytime returns as it is a currency fund based on Euros.

<Insert Figures 1 and 2, and Table1 here.>

3.2. Empirical results

Table 2 reports the OLS estimates of conditional mean parameters for each fund. The values in italics are standard errors. Estimates in bold are statistically significant at a 5% confidence level. Among the 15 funds, not many have overnight returns which significantly depend on previous daytime and overnight returns. For significant estimates, the signs are mixed. In contrast, most constant terms for daytime returns are not statistically significant. More funds have daytime returns which statistically depend on previous overnight returns negatively.

Table 3 reports the GARCH estimates of marginal distributions for each fund. The values in italics are robust standard errors. Estimates in bold are statistically significant at a 5% confidence level. Most estimates are statistically significant. Except FXE, we find all the funds have lower DoF parameters for overnight returns than for daytime returns, which is consistent with the observed higher Kurtosis of overnight returns. Figure 3 plots the estimated conditional variance of overnight and daytime returns for each fund. We find that the daytime returns have consistently higher conditional volatility than overnight returns except AGG and FXE.

Table 4 reports the estimates (from the constant correlation model) for the two constant Student's t copulas which govern the dependence structure of overnight and daytime returns respectively. We find that overnight returns have a bit higher (for most cases) values of correlation matrix than daytime returns[6], while daytime returns have a higher DoF parameter than overnight returns. The higher correlation matrix of the t copulas for daytime returns generally indicate that daytime returns are more correlated than overnight returns, even though correlation matrix in t copulas

[6] It is worth noting that in another estimation exercise which is not reported here, we apply the model to a relatively quieter time period (March 11, 2003 and July 19, 2007) with the nine SPDR ETFs and overnight returns tend to have lower values of correlation matrix than daytime returns. This indicates that the comparison results of the dependence structure between overnight and daytime returns depends on both the assets and time period under investigation.

seldom exactly equals to correlation matrix of underlying returns[7]. We also report the log-likelihood of the copula components from the estimation.

Table 5 reports the estimates for the two time-varying Student's *t* copulas. We first observe that the time-varying *t* copula yields significantly higher log-likelihood than its constant case, indicating a better fit of data. Estimates of the parameters in equations (9) and (10) suggest the dependence structure of overnight returns has significant influence on that of the following daytime returns while the dependence of overnight returns appears to be mainly determined by its previous dependence structure. This observation adds to higher moment dependence between daytime and previous overnight returns in a multivariate setting. Similar with the constant copulas, overnight returns have a lower DoF parameter than daytime returns. Figure 4 plots time-varying conditional correlation parameters of the t copulas for four selected fund pairs. We observe that for three pairs the correlation parameter of daytime returns is higher than that of overnight returns. Figure 5 plots the conditional TDC for four selected pairs, where the comparison patterns are mixed. We observe that for all the 4 pairs the TDC of overnight returns is significantly higher than that of daytime returns.

In summary, our empirical results show that for most cases overnight returns have higher mean, lower variance and higher kurtosis than daytime returns. In terms of dependence structure, overnight returns generally have relatively higher correlations than daytime returns. Moreover, daytime returns significantly depend on previous overnight returns in first and second univariate moments and dependence structure. We believe that most of our observations are consistent with the asymmetric information argument in theoretical work such as Admati & Pfleiderer (1988, 1989).

<Insert Tables 2, 3, 4 and 5, and Figures 3, 4 and 5 here.>

4. Applications

4.1. Risk management

Facing overnight and daytime returns, the typical task for risk managers is to forecast the risk measures of portfolios in the following market openings and closings. Specifically, at the end of time $t-1$ and with an information set $\Omega_{n,t-1}$, risk managers are concerned with the return distribution of a portfolio at the market opening of time t and subsequently at the market closing of time t. Let $P_{n,t} = g_{n,t}(r_{1,n,t},...,r_{k,n,t})$ and $P_{d,t} = g_{d,t}(r_{1,d,t},...,r_{k,d,t})$ be the portfolio returns at the market opening and closing of time t respectively. To forecast the risk measures of $P_{n,t}$, we need to specify the conditional density functions

$$f(r_{1,n,t},...,r_{k,n,t} \,|\, \Omega_{n,t-1}) \tag{20}$$

and

[7] They are equal when DoF parameter of the marginal distribution of each asset equals to that of the copulas.

$$f(r_{1,d,t},...,r_{k,d,t} \mid \hat{r}_{1,d,t},...,\hat{r}_{k,d,t},\Omega_{n,t-1}) \quad (21)$$

which is a function of both overnight and daytime returns of underlying assets at time t. From section 2.4, we can forecast $r_{1,n,t},...,r_{k,n,t}$ and $r_{1,d,t},...,r_{k,d,t}$ sequentially. Accordingly, the value-at-risk (VaR) and expected shortfall (ES) can be calculated based on simulated returns.

Let us assume the portfolio returns at the market opening and closing of time t as $P_{n,t} = \frac{1}{k}\sum_{i=1}^{k} r_{1,n,t}$ and $P_{d,t} = \frac{1}{k}\sum_{i=1}^{k} r_{1,d,t}$ and conduct a simple exercise of calculating the VaR and ES of the portfolio. We use the whole period of data (from September 21, 2006 to March 29, 2011) to forecast the conditional density in (20) and simulate 1000 samples for $r_{1,n,t},...,r_{k,n,t}$ first. Then for each scenario of $r_{1,n,t},...,r_{k,n,t}$, we simulate 100 samples for $r_{1,d,t},...,r_{k,d,t}$ based on the forecasted conditional density in (21). Accordingly, the VaR and ES are calculated based on the 1000 samples for $r_{1,n,t},...,r_{k,n,t}$ and 100000 samples for $r_{1,d,t},...,r_{k,d,t}$ respectively. The VaR and ES are calculated in Table 6.

<Insert Table 6 here.>

4.2. Asset allocation

As a simple example of asset allocation, investors solve a one-period investment problem, allocate their wealth among risk-free and risky assets at the beginning of overnight and daytime periods respectively and maximize their one-period expected utility. Let $W_{n,t} = w_{n,t}(r_{1,n,t},...,r_{k,n,t})$ and $W_{d,t} = w_{d,t}(r_{1,d,t},...,r_{k,d,t})$ be the wealth at the end of overnight and daytime periods of time t. The investor maximizes the two expected utilities as

$$\int ...\int U(W_{n,t}) f(r_{1,n,t},...,r_{k,n,t} \left| \Omega_{n,t-1} \right.) dr_{1,n,t}...dr_{k,n,t}, \quad (22)$$

and

$$\int ...\int U(W_{d,t}) f(r_{1,d,t},...,r_{k,d,t} \left| r_{1,n,t},...,r_{k,n,t},\Omega_{n,t-1} \right.) f(r_{1,n,t},...,r_{k,n,t} \left| \Omega_{n,t-1} \right.) dr_{1,d,t}...dr_{k,d,t} dr_{1,n,t}...dr_{k,n,t}, \quad (23)$$

where $U(\cdot)$ is a certain utility function. Usually, the maximization problem is based on the numerical simulations of the expected utility.

As a simple illustration, we assume investors have a CRRA utility function as

$$U(W) = \begin{cases} \frac{W^{1-A}}{1-A} & A > 0 \, and \, A \neq 1 \\ \ln W & A = 1 \end{cases}$$

where A is a CRRA parameter and W is wealth. Let $q_{i,n,t}$ and $q_{i,d,t}$ be weights for the i-th asset during the overnight and daytime periods. We assume $w_{n,t} = 1 + r_{1,n,t}q_{1,n,t} + ... + r_{k,n,t}q_{k,n,t}$ and $w_{d,t} = 1 + r_{1,d,t}q_{1,d,t} + ... + r_{k,d,t}q_{k,d,t}$ where risk-free rate is zero and $q_{i,n,t} > 0, q_{i,d,t} > 0$, $\sum_{i=1}^{N} q_{i,n,t} < 1$

and $\sum_{i=1}^{N} q_{i,d,t} < 1$. We can maximize the expected utilities in (22) and (23) respectively. Based on simulated samples for $r_{1,n,t}, ..., r_{k,n,t}$ and $r_{1,d,t}, ..., r_{k,d,t}$ (as in section 4.1), the optimal portfolio weights for overnight and daytime periods are reported in Table 7. We observe that two funds dominate the portfolio for overnight and daytime respectively.

<Insert Table 7 here.>

5. Conclusion

We introduce a multivariate GARCH-Copula model to describe joint dynamics of both overnight and daytime returns of multiple assets. The conditional mean and variance of individual returns depend on their previous realizations, and two (constant and time-varying) Student's *t* copulas link both returns respectively. We apply the model to 15 funds of different types and illustrate its use in risk management and asset allocation.

There are several possibilities for extensions. First, we can include more explanatory economic variables or factors in the system to better predict joint density. Second, we can investigate how to estimate the model by a Bayesian approach. Finally, we can test the model's performance in an out-of-sample manner with more financial applications. We leave those possibilities for future research.

Appendix

Let $\mathrm{E}_{d,t-1} = \varsigma_{d,t-1}\varsigma'_{d,t-1}$ and $\mathrm{E}_{n,t-1} = \varsigma_{n,t-1}\varsigma'_{n,t-1}$, we can write equations (9) and (10) as

$$\mathrm{E}_{n,t} = \Pi_0 + (\pi_1 + \pi_3)\mathrm{E}_{d,t-1} + (\pi_2 + \pi_4)\mathrm{E}_{n,t-1} + V_{n,t} - \pi_3 V_{d,t-1} - \pi_4 V_{n,t-1}, \tag{24}$$

$$\mathrm{E}_{d,t} = \Psi_0 + (\psi_1 + \psi_3)\mathrm{E}_{n,t} + (\psi_2 + \psi_4)\mathrm{E}_{d,t-1} + V_{d,t} - \psi_3 V_{n,t} - \psi_4 V_{d,t-1}, \tag{25}$$

where

$$V_{n,t} = \mathrm{E}_{n,t} - Q_{n,t}, \tag{26}$$

$$V_{d,t} = \mathrm{E}_{d,t} - Q_{d,t}. \tag{27}$$

We can write equations (24) and (25) in block matrices as

$$\begin{bmatrix} 1 & 0 \\ -(\psi_1+\psi_3) & 1 \end{bmatrix}\begin{bmatrix} \mathrm{E}_{n,t} \\ \mathrm{E}_{d,t} \end{bmatrix} = \begin{bmatrix} \Pi_0 \\ \Psi_0 \end{bmatrix} + \begin{bmatrix} \pi_2+\pi_4 & \pi_1+\pi_3 \\ 0 & \psi_2+\psi_4 \end{bmatrix}\begin{bmatrix} \mathrm{E}_{n,t-1} \\ \mathrm{E}_{d,t-1} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -\psi_3 & 1 \end{bmatrix}\begin{bmatrix} V_{n,t} \\ V_{d,t} \end{bmatrix} + \begin{bmatrix} -\pi_4 & -\pi_3 \\ 0 & -\psi_4 \end{bmatrix}\begin{bmatrix} V_{n,t-1} \\ V_{d,t-1} \end{bmatrix}. \tag{28}$$

Accordingly, we have

$$\begin{bmatrix} \mathrm{E}_{n,t} \\ \mathrm{E}_{d,t} \end{bmatrix} = \begin{bmatrix} \Pi_0 \\ \Psi_0 + (\psi_1+\psi_3)\Psi_0 \end{bmatrix} + \begin{bmatrix} \pi_2+\pi_4 & (\pi_1+\pi_3) \\ (\psi_1+\psi_3)(\psi_2+\psi_4) & (\pi_1+\pi_3)(\psi_1+\psi_3)+(\psi_2+\psi_4) \end{bmatrix}\begin{bmatrix} \mathrm{E}_{n,t-1} \\ \mathrm{E}_{d,t-1} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ (\psi_1+\psi_3)-\psi_3 & 1 \end{bmatrix}\begin{bmatrix} V_{n,t} \\ V_{d,t} \end{bmatrix} + \begin{bmatrix} -\pi_4 & -\pi_3 \\ -\pi_4(\psi_1+\psi_3) & -\pi_3(\psi_1+\psi_3)-\psi_4 \end{bmatrix}\begin{bmatrix} V_{n,t-1} \\ V_{d,t-1} \end{bmatrix}. \tag{29}$$

The above system is a VARMA(1,1) process and we require the eigenvalues of the matrix $\begin{bmatrix} \pi_2+\pi_4 & (\pi_1+\pi_3) \\ (\psi_1+\psi_3)(\psi_2+\psi_4) & (\pi_1+\pi_3)(\psi_1+\psi_3)+(\psi_2+\psi_4) \end{bmatrix}$ be less than one in absolute values for stationarity. Under stationarity, we can take expectations on both sides of (28) and with $E(V_{n,t}) = \mathbf{0}$ and $E(V_{d,t}) = \mathbf{0}$, we have

$$\Pi_0 = (1-(\pi_2+\pi_4))S_n - (\pi_1+\pi_3)S_d,$$

$$\Psi_0 = (1-(\psi_2+\psi_4))S_d - (\psi_1+\psi_3)S_n,$$

where S_n and S_d are the unconditional covariance of $\varsigma_{cn,t}$ and $\varsigma_{cd,t}$.

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Figures and Tables

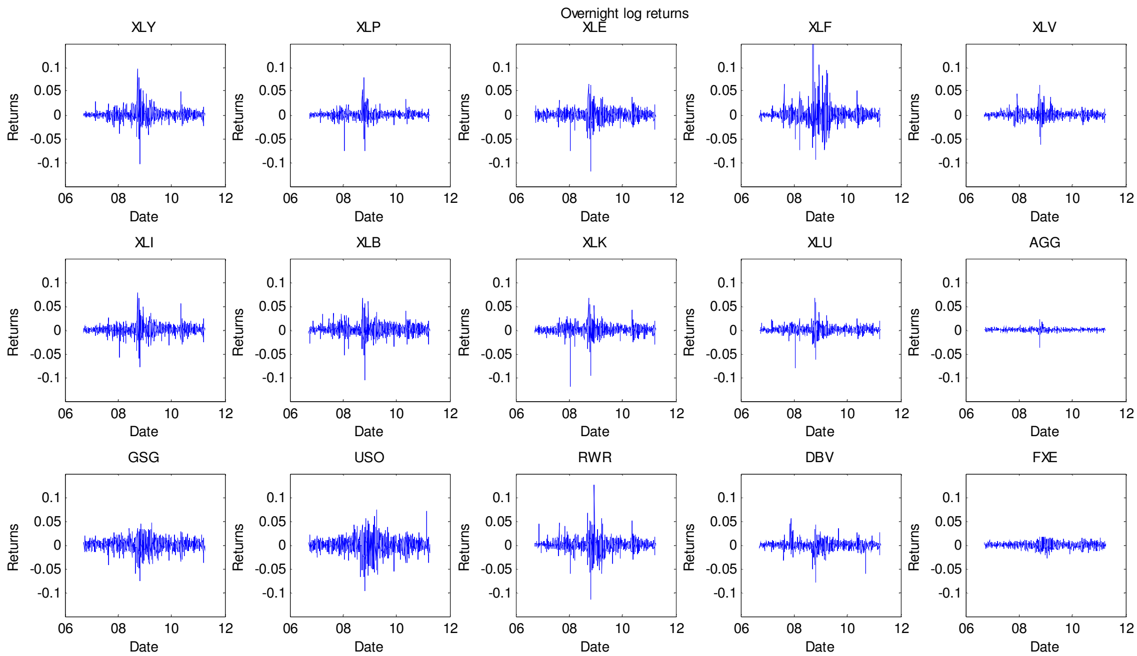

Figure 1 This figure plots the overnight log returns of 15 funds of different types from September 21, 2006 to March 29, 2011. The time period includes an unprecedented high volatility due to the financial crisis. In this paper, we choose this period for applying our copula model to overnight and daytime returns.

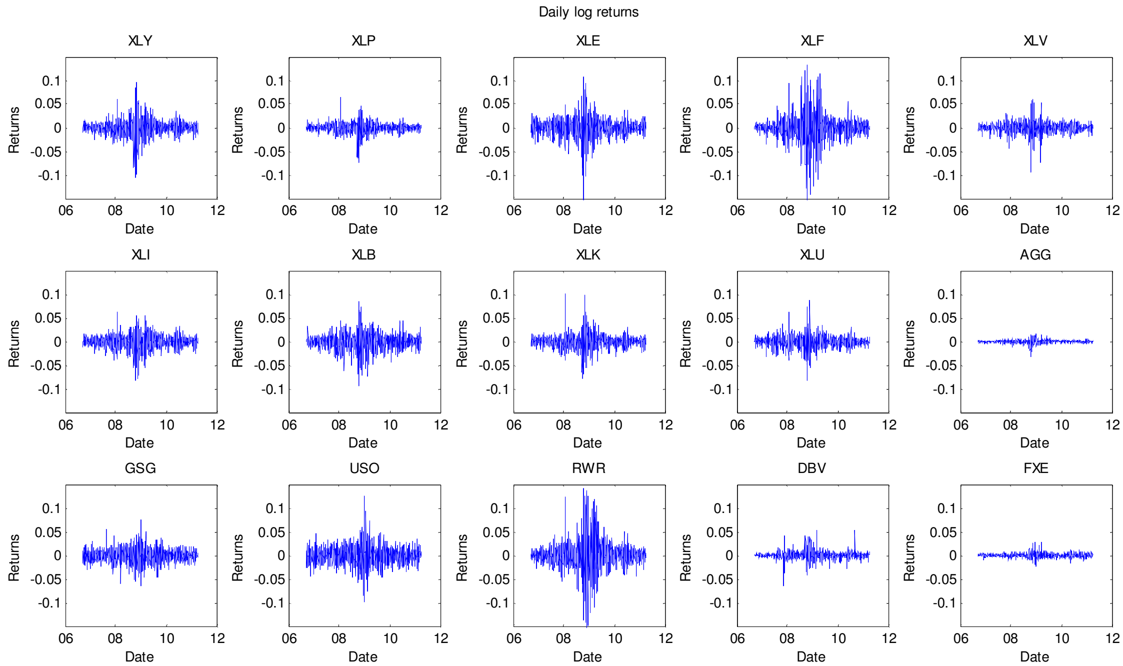

Figure 2 This figure plots the daytime log returns of 15 funds of different types from September 21, 2006 to March 29, 2011. The time period includes an unprecedented high volatility due to the financial crisis. In this paper, we choose this period for applying our copula model to overnight and daytime returns.

		XLY	XLP	XLE	XLF	XLV	XLI	XLB	XLK	XLU	AGG	GSG	USO	RWR	DBV	FXE
Overnight	Mean	0.0005	0.0004	0.0003	0.0003	0.0002	0.0006	0.0006	0.0005	0.0005	0.0003	0.0002	-0.0005	0.0001	-0.0005	-0.000004
	Std. Dev.	0.011	0.008	0.012	0.017	0.008	0.011	0.011	0.010	0.008	0.003	0.012	0.016	0.013	0.009	0.005
	Skewness	0.297	0.013	-1.113	1.568	0.466	0.028	-0.528	-1.783	-0.323	-1.752	-0.657	-0.517	-0.034	-0.842	-0.306
	Kurtosis	19.452	26.746	16.356	22.327	15.621	13.516	12.813	28.180	24.670	42.560	6.240	6.806	18.010	14.303	4.709
Daytime	Mean	-0.0004	-0.00022	0.0002	-0.0009	-0.0001	-0.0004	-0.0003	-0.0003	-0.0005	-0.0001	-0.0003	0.0002	-0.0001	0.0005	0.0001
	Std. Dev.	0.017	0.010	0.019	0.026	0.011	0.015	0.017	0.014	0.013	0.003	0.014	0.019	0.029	0.008	0.004
	Skewness	-0.431	-0.791	-0.761	-0.069	-1.054	-0.616	-0.429	0.098	-0.070	-1.282	-0.075	0.061	-0.138	0.286	0.517
	Kurtosis	9.991	11.857	11.800	9.845	14.850	7.274	6.865	10.778	10.628	17.798	4.809	6.471	10.073	13.687	8.745

Table 1 This table reports the descriptive statistics (mean, standard deviation, Skewness and Kurtosis) for overnight and daytime returns of 15 funds of different types from September 21, 2006 to March 29, 2011. We find overnight returns are all positive and consistently higher than daytime returns except USO, DBV and FXE. Daytime returns have significantly higher standard deviations than overnight returns except the two currency funds DBV and FXE. Skewness for both returns has mixed signs. Kurtosis is greater than three for both returns and overnight returns tend to have higher Kurtosis except currency funds FXE. XLY, XLP, XLE, XLF, XLV, XLI, XLB, XLK, and XLU represent SPDR ETFs of the sectors for Consumer Discretionary, Consumer Staples, Energy, Financial, Healthcare, Industrial, Materials, Technology and Utilities. AGG, GSG, USO, RWR, DBV and FXE represent iShares Lehman Aggregate Bond ETF, iShares S&P GSCI Commodity-Indexed ETF, United States Oil Fund LP ETF, SPDR DJ Wilshire REIT ETF, PowerShares DB G10 Currency Harvest Fund, and Currency Shares Euro Trust.

		XLY	XLP	XLE	XLF	XLV	XLI	XLB	XLK	XLU	AGG	GSG	USO	RWR	DBV	FXE
Overnight	$\alpha_{i,0}$	0.0005	**0.0004**	0.0003	0.0004	0.0002	**0.0006**	**0.0006**	0.0005	**0.0005**	**0.0003**	0.0002	-0.0005	0.0001	-0.0004	-0.00001
		0.0003	*0.0002*	*0.0004*	*0.0005*	*0.0002*	*0.0003*	*0.0003*	*0.0003*	*0.0002*	*0.0001*	*0.0004*	*0.0005*	*0.0004*	*0.0003*	*0.0002*
	$\alpha_{i,1}$	0.026	0.028	-0.009	0.023	0.029	0.035	**0.051**	-0.019	-0.010	**-0.171**	-0.032	-0.028	-0.013	**-0.190**	0.029
		0.02	*0.03*	*0.02*	*0.02*	*0.02*	*0.02*	*0.02*	*0.02*	*0.02*	*0.02*	*0.02*	*0.02*	*0.01*	*0.03*	*0.04*
	$\alpha_{i,2}$	-0.02	0.05	**-0.09**	**-0.09**	-0.02	-0.03	-0.002	-0.02	0.02	**-0.07**	-0.04	-0.05	**-0.11**	-0.05	0.03
		0.03	*0.03*	*0.03*	*0.03*	*0.03*	*0.03*	*0.03*	*0.03*	*0.03*	*0.03*	*0.03*	*0.03*	*0.03*	*0.03*	*0.03*
Daytime	$\beta_{i,0}$	-0.0002	-0.00002	0.0001	-0.0010	-0.0001	-0.0003	-0.0004	-0.0002	-0.0004	**-0.0002**	-0.0003	0.0002	-0.0002	**0.0004**	0.0001
		0.0005	*0.0003*	*0.0006*	*0.0008*	*0.0003*	*0.0004*	*0.0005*	*0.0004*	*0.0004*	*0.0001*	*0.0004*	*0.0006*	*0.0008*	*0.0002*	*0.0001*
	$\beta_{i,1}$	**-0.31**	**-0.49**	**0.18**	-0.05	**-0.33**	**-0.11**	0.01	**-0.20**	**-0.19**	**0.16**	0.01	0.01	0.04	**-0.17**	**0.05**
		0.04	*0.03*	*0.05*	*0.04*	*0.04*	*0.04*	*0.04*	*0.04*	*0.05*	*0.04*	*0.04*	*0.04*	*0.06*	*0.03*	*0.02*
	$\beta_{i,2}$	0.01	**-0.10**	**-0.13**	**-0.15**	-0.04	-0.05	**-0.09**	**-0.07**	**-0.10**	-0.03	-0.05	-0.02	**-0.21**	0.02	0.03
		0.03	*0.03*	*0.03*	*0.03*	*0.03*	*0.03*	*0.03*	*0.03*	*0.03*	*0.03*	*0.03*	*0.03*	*0.03*	*0.03*	*0.03*

Table 2 This table reports the OLS estimates of conditional mean parameters for each fund. The values in italics are standard errors. Estimates in bold are statistically significant at a 5% confidence level. Among the 15 funds, not many have overnight returns which significantly depend on previous daytime and overnight returns. For significant estimates, the signs are mixed. In contrast, most constant terms for daytime returns are not statistically significant. More funds have daytime returns which statistically depend on previous overnight returns negatively.

		XLY	XLP	XLE	XLF	XLV	XLI	XLB	XLK	XLU	AGG	GSG	USO	RWR	DBV	FXE
Overnight	$\theta_{i,0}$	3.50E-07	8.79E-07	8.87E-07	1.13E-07	4.79E-07	5.39E-07	1.44E-06	1.79E-06	1.33E-06	4.94E-07	1.05E-06	1.11E-06	3.41E-06	2.04E-06	2.23E-07
	$\theta_{i,1}$	**0.03**	**0.14**	**0.03**	**0.05**	**0.04**	**0.06**	**0.05**	**0.07**	**0.07**	**0.13**	0.01	0.01	**0.02**	**0.14**	**0.05**
		0.01	*0.09*	*0.01*	*0.01*	*0.02*	*0.01*	*0.01*	*0.02*	*0.03*	*0.02*	*0.01*	*0.01*	*0.01*	*0.03*	*0.02*
	$\theta_{i,2}$	**0.09**	**0.11**	**0.09**	**0.07**	**0.07**	0.05	**0.05**	0.03	**0.08**	**0.09**	**0.06**	**0.05**	0.08	**0.09**	**0.03**
		0.03	*0.05*	*0.02*	*0.02*	*0.02*	*0.03*	*0.02*	*0.04*	*0.03*	*0.04*	*0.01*	*0.02*	*0.05*	*0.03*	*0.01*
	$\theta_{i,3}$	**0.84**	**0.71**	**0.83**	**0.82**	**0.84**	**0.83**	**0.82**	**0.82**	**0.73**	**0.66**	**0.92**	**0.93**	**0.80**	**0.76**	**0.93**
		0.04	*0.15*	*0.04*	*0.02*	*0.04*	*0.04*	*0.04*	*0.03*	*0.09*	*0.07*	*0.02*	*0.01*	*0.04*	*0.06*	*0.02*
	$v_{i,n}$	**5.44**	**4.51**	**9.56**	**5.52**	**4.85**	**6.05**	**5.72**	**4.90**	**4.05**	**7.94**	**7.13**	**7.24**	**4.24**	**5.62**	**15.35**
		0.63	*0.43*	*2.27*	*0.63*	*0.55*	*0.82*	*0.71*	*0.51*	*0.32*	*1.69*	*1.32*	*1.42*	*0.37*	*0.66*	*6.56*
Daily	$\delta_{i,0}$	2.13E-06	1.38E-06	2.71E-06	5.90E-06	3.74E-06	2.30E-06	2.97E-06	2.66E-06	2.29E-06	2.07E-11	3.66E-06	5.58E-06	5.61E-06	7.88E-07	3.62E-08
	$\delta_{i,1}$	**0.14**	**0.07**	0.04	**0.29**	**0.18**	**0.08**	**0.08**	**0.13**	0.12	**0.04**	0.05	0.03	**0.23**	**0.15**	**0.02**
		0.04	*0.02*	*0.03*	*0.08*	*0.09*	*0.03*	*0.04*	*0.05*	*0.07*	*0.01*	*0.04*	*0.02*	*0.09*	*0.04*	*0.01*
	$\delta_{i,2}$	**0.06**	**0.07**	**0.08**	**0.14**	**0.12**	**0.08**	**0.06**	**0.10**	**0.10**	**0.04**	**0.06**	**0.06**	**0.14**	**0.13**	**0.04**
		0.01	*0.02*	*0.02*	*0.03*	*0.03*	*0.02*	*0.02*	*0.02*	*0.02*	*0.01*	*0.03*	*0.02*	*0.03*	*0.03*	*0.02*
	$\delta_{i,3}$	**0.86**	**0.86**	**0.89**	**0.73**	**0.76**	**0.86**	**0.90**	**0.82**	**0.84**	**0.93**	**0.89**	**0.91**	**0.81**	**0.69**	**0.92**
		0.03	*0.03*	*0.03*	*0.05*	*0.07*	*0.03*	*0.04*	*0.04*	*0.05*	*0.00*	*0.07*	*0.04*	*0.04*	*0.06*	*0.02*
	$v_{i,d}$	**13.20**	**12.96**	**13.39**	**13.21**	**7.87**	**13.73**	**10.70**	**11.90**	**8.54**	**8.05**	27.63	40.05	16.27	**6.27**	**7.96**
		4.58	*3.87*	*4.65*	*3.81*	*1.48*	*5.02*	*2.75*	*3.58*	*1.68*	*1.49*	*27.26*	*85.72*	*8.84*	*0.93*	*1.70*

Table 3 This table reports the GARCH estimates of marginal distributions for each fund. The values in italics are robust standard errors. Estimates in bold are statistically significant at a 5% confidence level. Except FXE, we find all the funds have lower DoF parameters for overnight returns than for daytime returns, which is consistent with the observed higher Kurtosis of overnight returns.

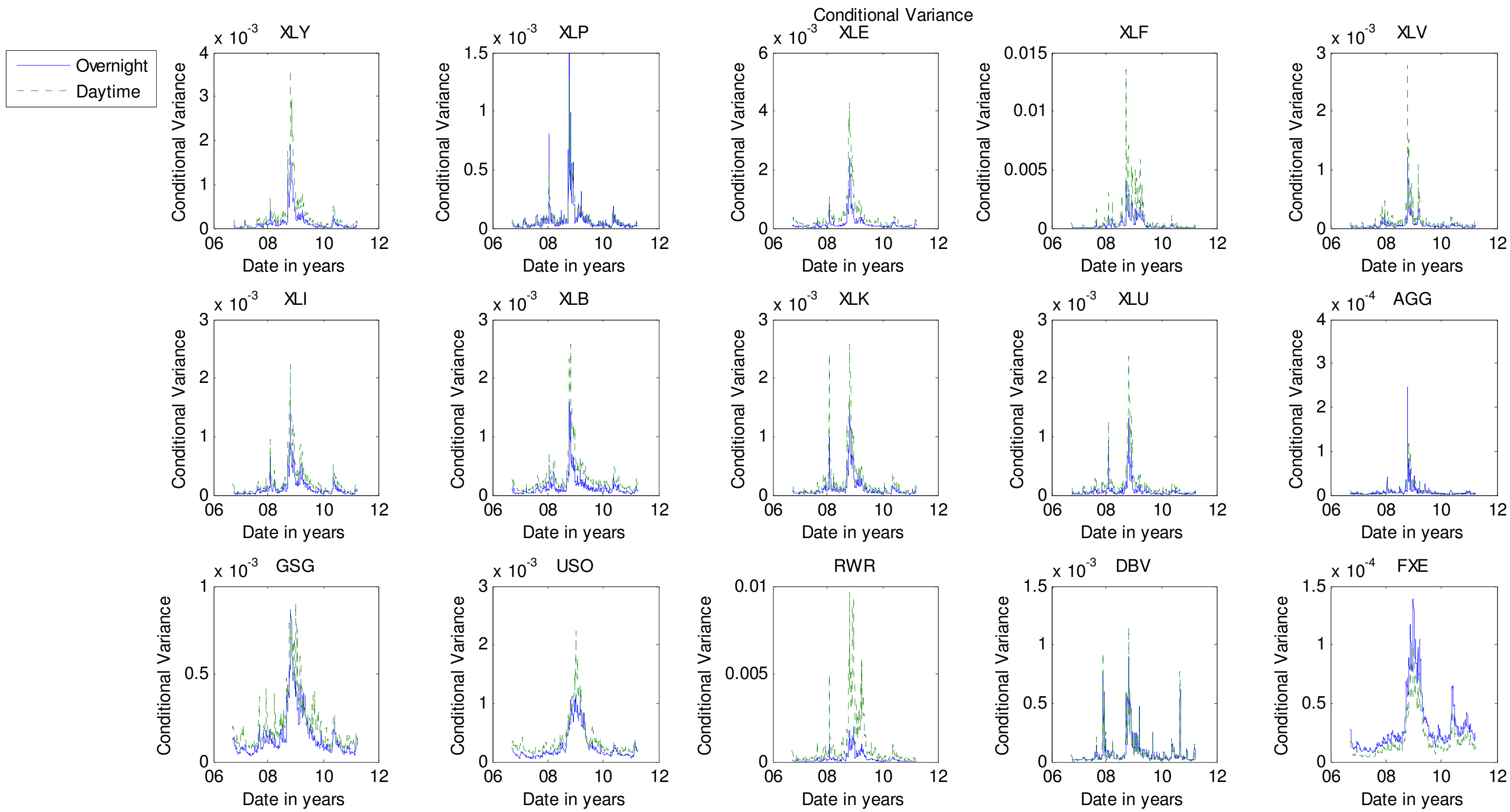

Figure 3 This figure plots the estimated conditional variance of overnight and daytime returns for each fund. We find that the daytime returns have consistently higher conditional volatility than overnight returns except AGG and FXE.

Overnight															
R_{cn}	XLY	XLP	XLE	XLF	XLV	XLI	XLB	XLK	XLU	AGG	GSG	USO	RWR	DBV	FXE
XLY															
XLP	0.66														
XLE	0.65	0.59													
XLF	0.77	0.65	0.69												
XLV	0.65	0.59	0.56	0.66											
XLI	0.77	0.67	0.67	0.76	0.68										
XLB	0.72	0.65	0.75	0.76	0.63	0.73									
XLK	0.75	0.66	0.67	0.74	0.64	0.75	0.74								
XLU	0.65	0.62	0.57	0.66	0.59	0.67	0.67	0.64							
AGG	-0.27	-0.23	-0.22	-0.26	-0.19	-0.24	-0.22	-0.27	-0.23						
GSG	0.38	0.33	0.72	0.40	0.33	0.38	0.52	0.42	0.34	-0.13					
USO	0.34	0.32	0.76	0.38	0.29	0.37	0.50	0.40	0.31	-0.13	0.85				
RWR	0.72	0.64	0.68	0.77	0.63	0.71	0.73	0.70	0.65	-0.27	0.42	0.40			
DBV	0.45	0.41	0.51	0.48	0.40	0.48	0.51	0.46	0.43	-0.24	0.40	0.38	0.49		
FXE	0.25	0.24	0.46	0.26	0.25	0.28	0.38	0.29	0.23	0.06	0.45	0.45	0.30	0.29	
v_{cn}	9.16														
Daytime															
R_{cd}	XLY	XLP	XLE	XLF	XLV	XLI	XLB	XLK	XLU	AGG	GSG	USO	RWR	DBV	FXE
XLY															
XLP	0.67														
XLE	0.53	0.44													
XLF	0.75	0.56	0.55												
XLV	0.63	0.67	0.46	0.57											
XLI	0.80	0.64	0.63	0.73	0.62										
XLB	0.67	0.52	0.75	0.65	0.53	0.74									
XLK	0.78	0.64	0.60	0.71	0.62	0.77	0.68								
XLU	0.52	0.56	0.53	0.45	0.54	0.54	0.52	0.53							
AGG	-0.21	-0.16	-0.17	-0.21	-0.17	-0.20	-0.18	-0.20	-0.08						
GSG	0.19	0.14	0.55	0.22	0.12	0.25	0.38	0.23	0.22	-0.06					
USO	0.19	0.15	0.57	0.23	0.12	0.25	0.37	0.23	0.21	-0.06	0.86				
RWR	0.69	0.50	0.47	0.75	0.49	0.65	0.59	0.64	0.45	-0.19	0.20	0.19			
DBV	0.46	0.41	0.42	0.44	0.41	0.47	0.48	0.47	0.37	-0.17	0.27	0.28	0.40		
FXE	0.20	0.15	0.36	0.19	0.16	0.23	0.32	0.20	0.20	0.05	0.33	0.34	0.18	0.28	
v_{cd}	21.90														
Log-likelihood	13346.47														

Table 4 This table reports the estimates (from the constant correlation model) for the two Student's t copulas which govern the dependence structure of overnight and daytime returns. We find that overnight returns have a bit higher (for most cases) values of correlation matrix than daytime returns, while daytime returns have higher DoF parameter than overnight returns. We also report the log-likelihood of the copula components from the estimation.

	Estimate	SE
π_1	0.007	*0.001*
π_2	0.012	*0.001*
π_3	2.76E-04	*0.026*
π_4	0.97	*0.016*
ψ_1	0.033	*0.004*
ψ_2	0.014	*0.004*
ψ_3	0.365	*0.083*
ψ_4	0.344	*0.107*
ν_{cn}	11.85	*0.69*
ν_{cd}	28.06	*3.11*
Log-likelihood	13945.34	

Table 5 This table shows the ML estimates and their standard errors for the time-varying t copula. Consistent with the constant case, the DoF parameter of the copula for daytime returns is higher than that for overnight returns. We also report the log-likelihood of the copula component. Compared with the constant t copula case, we have significantly higher log-likelihood by making correlation matrices time-varying.

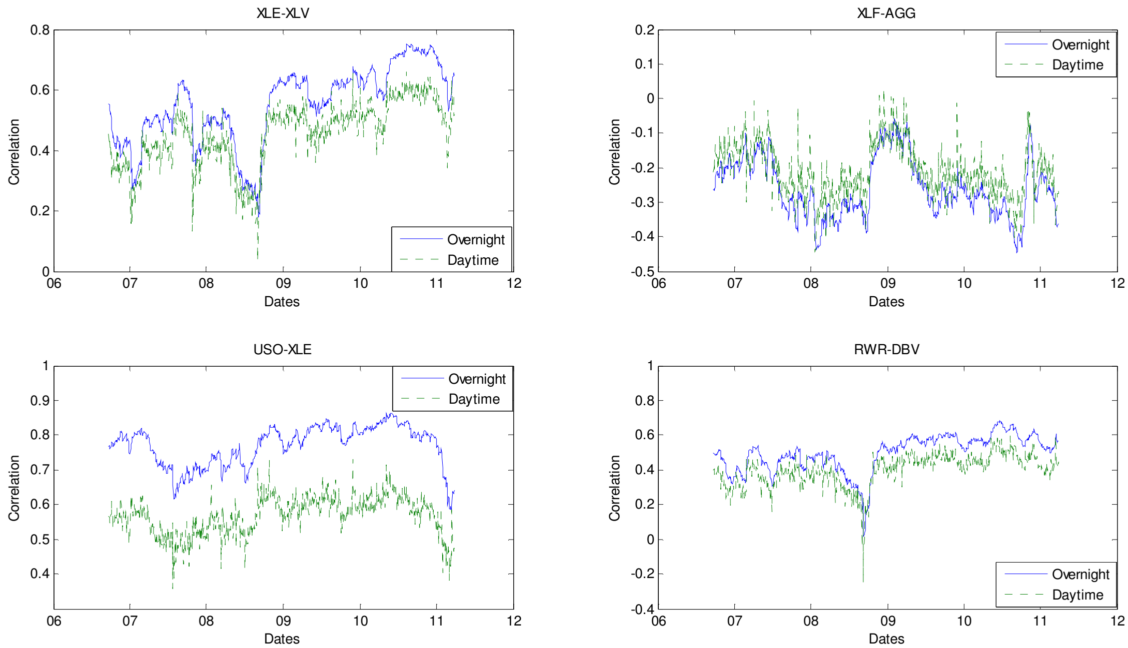

Figure 4 This figure plots the time-varying conditional correlation parameters of the t copulas for the four pairs of the returns. We observe that for three pairs the correlation parameter of daytime returns is higher than that of overnight returns.

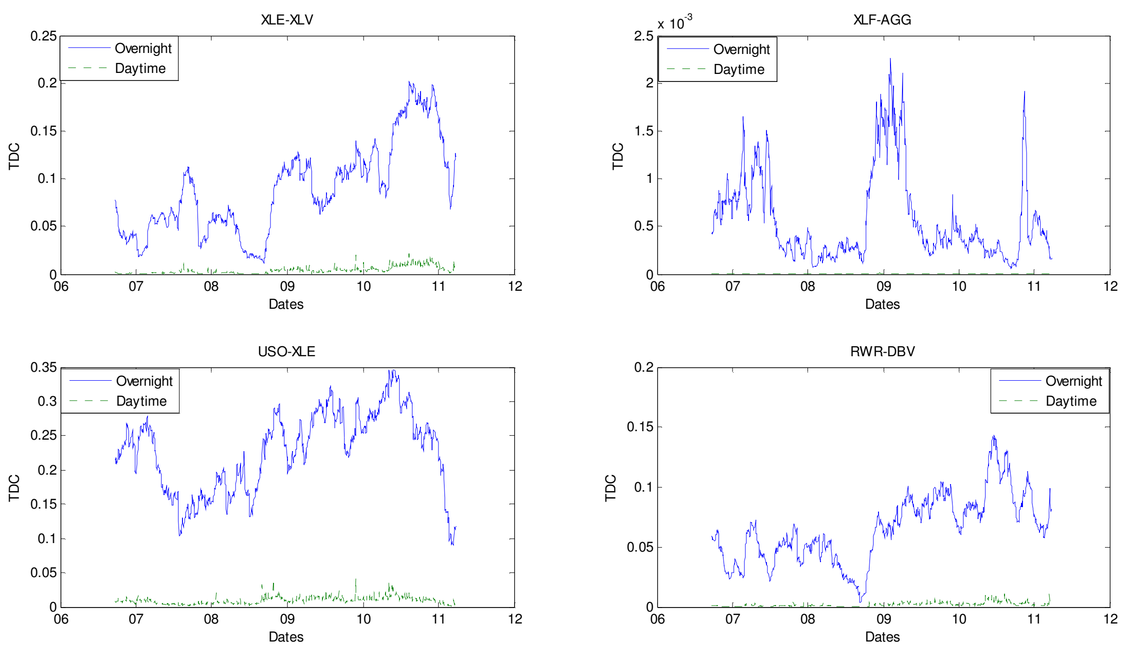

Figure 5 This figure plots the time-varying conditional tail dependence coefficient (TDC) for the four pairs of returns. We observe that for all the 4 pairs the TDC of overnight returns is significantly higher than that of daytime returns.

		VaR	ES
Overnight	1%	0.000122	0.000107
	5%	0.000159	0.00014
	10%	0.000172	0.000153
Daytime	1%	-0.00081	-0.00083
	5%	-0.00076	-0.00079
	10%	-0.00074	-0.00077

Table 6 This table reports VaR and ES forecasts at 1%, 5% and 10% levels for both overnight and daytime periods.

Overnight							
	A=1	A=3	A=5	A=7	A=9	A=12	A=15
XLY	0	0	0	0	0	0	0
XLP	0	0	0	0	0	0	0
XLE	0	0	0	0	0	0	0
XLF	0	0	0	0	0	0	0
XLV	0	0	0	0	0	0	0
XLI	0	0	0	0	0	0	0
XLB	1	1	1	1	1	1	1
XLK	0	0	0	0	0	0	0
XLU	0	0	0	0	0	0	0
AGG	0	0	0	0	0	0	0
GSG	0	0	0	0	0	0	0
USO	0	0	0	0	0	0	0
RWR	0	0	0	0	0	0	0
DBV	0	0	0	0	0	0	0
FXE	0	0	0	0	0	0	0

Daytime							
	A=1	A=3	A=5	A=7	A=9	A=12	A=15
XLY	0	0	0	0	0	0	0
XLP	0	0	0	0	0	0	0
XLE	0	0	0	0	0	0	0

XLF	0	0	0	0	0	0	0
XLV	0	0	0	0	0	0	0
XLI	0	0	0	0	0	0	0
XLB	0	0	0	0	0	0	0
XLK	0	0	0	0	0	0	0
XLU	0	0	0	0	0	0	0
AGG	0	0	0	0	0	0	0
GSG	0	0	0	0	0	0	0
USO	0	0	0	0	0	0	0
RWR	0	0	0	0	0	0	0
DBV	1	1	1	1	1	1	1
FXE	0	0	0	0	0	0	0

Table 7 This table reports optimal portfolio weights for both overnight and daytime periods with different values of CRRA parameter.