Alternative Risk Transfer for Longevity and Mortality

Tomas Cipra
Charles University of Prague

Prof. RNDr. Tomas Cipra, DrSc.
Dept. of Statistics
Charles University of Prague
Sokolovska 83
186 75 Prague 8
Czech Republic

e-mail: cipra@karlin.mff.cuni.cz
phone: (420) 221 913 288

Alternative Risk Transfer for Longevity and Mortality

Abstract: The paper deals with Alternative Risk Transfer (ART) through securitization of longevity and mortality risks in pension plans and commercial life insurance. Various types of such mortality-linked securities are described including methods of their pricing and real examples (e.g. CATM bonds, longevity bonds, mortality forwards and futures, mortality swaps, and others). Hypothetical calculations concerning pricing of potential mortality forwards that correspond to the longevity evolution in the Czech Republic are presented. They make use of the cohort Life Tables constructed by Cipra (1998) for Czech pension funds.

JEL Classification: G10, G23, G28, D8, J11, J26, J32, C15, C32, H55

Keywords: ART, life insurance, life market, longevity risk, mortality risk, pension plans, securitization, tontines

Acknowledgement: The work is a part of research project MSM0021620839 financed by Ministry of Education of Czech Republic

Introduction

This paper deals with important examples of the alternative risk transfer, namely with securitization of longevity and mortality risk, which is one of perspective solutions of the pension and life annuity problem. There is a vast volume of literature devoted to this topic (only a small part of it may be presented here) since it is really a serious problem of future. The paper has ambitions to present the issue in a more economic (or financial) way than as an actuarial problem (there is no doubt that constructions of future pension systems have economic dimensions above all). As various ideas and considerations behind are only hypothetical ones so far, the paper tries to describe some instruments really existing in practice. The investors including banks should be prepared for brand new type of security engineering motivated by pension systems or insurance business. Moreover, the paper shows some calculations that enable to judge consequences of such approaches if applied in the Czech practice. However, first in this Introduction we'll explain the basic concepts which are important from the point of view of further Sections.

The content of this paper is as follows: After introduction of main concepts in *Section 1* we describe catastrophe bonds (CatBonds) as typical ILS securities for non-life insurance. Moreover, a simple mathematical model of CatBonds will be given here which can serve as a general mathematical scheme for ILS. *Section 3* is devoted to ILS securities for life insurance and pension plans (sometimes called mortality-linked securities): mortality catastrophe bonds (CATM bonds) in *Section 3.1* (including a practical example of the bond Vita I), mortality swaps (also called survivor swaps) in *Section 3.2*, longevity bonds (LB bonds) in *Section 3.3* (including a practical example of the bond EIB/BNP Paribas) and mortality forwards and futures in *Section 3.4*. In *Section 4* some demographic facts and actuarial instruments are addressed which are important just in the context of securitization of mortality and longevity risks. In particular, the Cohort Life Tables constructed by Cipra (1998) for Czech pension funds are commented. In *Section 5* some approaches to pricing of mortality-linked securities are briefly mentioned. Finally, *Section 6* suggests hypothetical calculations concerning pricing of mortality forwards that correspond to the longevity evolution in the Czech Republic.

1. Main Concepts

Alternative risk transfer ART are modern methods of insurance industry (both life and non-life one) and pension systems which are more appropriate in nowadays world than the classical cession of insurance risks as e.g. in the classical reinsurance (see Cipra (2004)). If one simplifies the problem, many of the ART methods are motivated by the effort to cede huge insurance risks to capital markets that have a multifold capacity in comparison with insurance markets: e.g. the insurance of oil tankers may be above the capacity of big insurance and reinsurance companies even if they collaborate or even pool in various ways. To obtain an idea how this principle works let's consider e.g. so called catastrophe bonds (see CatBonds below) mitigating the financial stress within insurance companies e.g. in the case of floods: the coupons from such bonds lie so high above a market standard that investors accede to a substantial reduction of coupons (and principals) if the corresponding insurance event (the floods in a given region) incurs. Obviously this mechanism is really a cession of the insurance risk to the capital market. Quite formally, the ART is a product, channel or solution that transfers risk exposures between the insurance industry (including pension funds) and capital markets to achieve stated risk management goals (see Banks (2004)). The ART market is the combined risk management marketplace for innovative insurance and capital market solutions.

The important solution in the framework of ART is a securitization. The *securitization* is the process of removing assets, liabilities or cash flows from the balance sheet (of an insurance company, a pension fund etc.) and conveying them to third parties through tradable securities (so called *insurance-linked securities ILS* including various derivatives). Typical representatives of ILS are just the catastrophe bonds mentioned above. Since the ILS trading is very specialized activity it requires usually a special organizer established just for this single purpose. Such an organizer is usually called a *special purpose vehicle SPV* (e.g. Vita Capital Ltd. in *Figure 1*).

As the securitization is concerned the paper concentrates on securitization of longevity and mortality risks which play very important role among other systematic risks in modern finance (see e.g. van Broekhoven (2002)). In particular, the *longevity risk* should be taken into account by pensions (or life annuities) providers in developed countries, since the growing life expectancy can jeopardize the economy of their pension systems (see e.g. OECD (2006, 2008), Schneider (2009)). The longevity and mortality risks constitute so serious problems that one predicts the origin of other types of capital markets called usually *life markets* (see e.g. Loyes et al. (2007)). The *annuity markets* in the UK and US are working examples of this phenomenon. In addition, the regulation of commercial insurance industry will address this problem in the framework of the regulatory system Solvency II, where the entry denoted as *underwriting risk* in Pillar 1 will contain longevity and mortality risks as its important components (including Solvency II as it is prepared by the Czech National Bank). The private life insurance linked to pension funds (mainly in the contribution defined pension plans) may play a key role in pension systems of the future world, see e.g. CEA (2006), Cipra (2002), IAA (2004), Sandström (2006).

Again to have an idea of the longevity risk securitization let's consider so called longevity bonds (see LB bonds below). While a classical (nominal) bond pays annual or semiannual coupons on a fixed amount and the principal is repaid at the term (maturity), the LB bond provides regular floating payments according to the proportion of an initial population surviving to a future time. This mechanism obviously allows to cede the longevity risk from insurance companies or pension funds (investing to these securities) to LB bond issuers, i.e. from insurance markets to capital markets. In particular, the *tontines* can be mentioned in this context since formally they are one-year zero-coupon LB bonds. Milevsky (2006) explains the principle of tontines by means of a very nice (though a little bit naive) example.[1] There is

[1] A 85-year-old grandmother meets regularly her four best friends of the same age every year on December 31. She proposed to juice up their meetings in such a way that each of five participants deposits $1,000 with 5% interest p.a. and with guarantee that whoever survives till the end of next year gets to split the $5,250 pot. There is a 20% chance that any given member of this club will die during the next year. Therefore the odds are that on average each of four 86-year-old survivors will receive $1,312.5 as the total return on the original investment of $1,000. The 31.25% investment return contains 5% of bank's money and 26.25% of "mortality credits". These credits represent the capital and interest "lost" by the deceased and "gained" by the survivors.

hardly another financial product that guarantees such a high rates of return, even if conditional on survival.

2. Insurance-Linked Securities for Non-Life Insurance

In this Section we'll describe catastrophe bonds (CatBonds) as typical ILS securities for non-life insurance (see e.g. Cox and Pedersen (1998), Cummins (2008), Swiss Re New Markets (1999)) including a simple mathematical model how they work.

CatBonds are highly profitable bonds (their coupon rate is usually much higher than the market average) for which the suspension of coupons and/or principal occurs in the case of a pre-defined natural catastrophe (earthquake, hailstorm, pandemic event and the like). E.g. an annual reinsurance treaty according to which an reinsurer reimburses a sum insured S at the end of the contract year if the catastrophe has occurred can be replaced by the issue of a 1-year catastrophe bond with annual coupon: the *Table 1* contains the appropriate cash flows which comply with requirements of all participating sides: q_{cat} is the probability of the natural catastrophe, i is the annual coupon rate, F is the principal of the bond, P is the reinsurance premium. Moreover, one can use the market price (market quotation) of such a bond to price the reinsurance premium

$$P_b = \frac{1}{1+i} \cdot q_b \cdot S = \frac{S}{1+i} - F^*,$$

where P_b is the reinsurance premium priced by the bond market, F^* is the market price of the given catastrophe bond and

$$q_b = \frac{S - F^* \cdot (1+i)}{S}$$

is the probability of catastrophe priced by the bond market (unlike the estimate q_{cat} by the reinsurance market).

Table 1 Cash flows in a 1-year catastrophe bond

	Time $t = 0$	Time $t = 1$	
		Occurrence of cat. (with prob. q_{cat})	Non-occurrence of cat. (with prob. $1-q_{cat}$)
Insurer	$-P = -\frac{1}{1+i} \cdot q_{cat} \cdot S$	S	0
Reinsurer = Issuer (CatBond)	$P + F$	$-S$	$-S$
Investor (CatBond)	$-F = -\frac{1}{1+i} \cdot (1-q_{cat}) \cdot S$	0	S

3. Insurance-Linked Securities for Life Insurance and Pension Plans

This Section deals with ILS for life insurance and pension plans which may be denoted generally as *mortality-linked securities* (such a terminology does not distinguish between mortality-linked and longevity-linked securities). We'll start with an introduction concerning life markets in general.

The modern practice of risk management requires companies (or governments) to manage mortality and longevity risks as effectively as possible as a part of enterprise risk management rather than to accept it as inevitable. Blake at al. (2006a) and Cairns et al. (2008) mention possible way how to manage mortality and longevity risks:

- insurers can retain these risks as a legitimate business risk;
- insurers can diversify these risks across product ranges, regions and socio-economic groups (an example how to hedge through such a balance of gains and losses on the life and the annuity book is given e.g. in Cox and Lin (2007));
- insurers can enter into various forms of reinsurance (and then the reinsurers can use e.g. the securitization as it is the case in *Table 1*);
- pension plans can arrange a full or partial *buyout* of their liabilities by specialist insurer;
- insurers can securitize a line of business (see e.g. Cowley and Cummins (2005));
- mortality and longevity risks can be managed through the application of mortality-linked securities and derivatives (this approach differs from the securitization of a line of business from the previous point since such securities have cash-flows that are purely linked to the future value of a mortality index, rather than being a complex package of business risks).

To establish a new flourishing capital market (a life market in our case) several conditions should be fulfilled (see Corkish et al. (1997), Loyes et al. (2007)). First, the market must provide effective exposure, or hedging, to a state of the world. This state of the world must be economically important and cannot be hedged sufficiently through existing market instruments. Further, the market must use a homogeneous and transparent contract to permit exchange between agents.

Let's give some examples of successful and unsuccessful capital markets for product innovations in the framework of financial risks:

- *Successful* products: credit default swaps (CDS), inflation-linked bonds, interest rate swaps (IRS), mortgage-backed securities (MBS), real estate investment trusts (REIT).
- *Unsuccessful* products: GDP derivatives (the corresponding market should be an analogy to the markets with inflation-linked bonds), residential real estate derivatives (they should diversify risk of the tremendous financial wealth concentrated in family dwelling).

The market trading mortality or longevity risks (via new life markets) meets these criteria if one considers *systematic* parts of these risks. *Systematic* mortality or longevity risks are undiversifiable, since they affect all individuals in the same way. In particular, *systematic mortality risk* is an increased exposure to a catastrophic mortality deterioration (e.g. in the whole life insurance or in the term insurance). On the contrary, *systematic longevity risk* consists in growing costs to meet increasing life expectancy due to improvement in health conditions across the word (e.g. in pension funds). *Unsystematic* mortality or longevity risks can be diversified by pooling the individuals to large portfolios (the larger the portfolio, the smaller unsystematic risk).

This paper deals only with systematic mortality or longevity risks since unsystematic ones can be managed (at least for the time being) by classical insurance instruments. In the remaining part of this Section we'll describe typical representatives of mortality(or longevity)-linked securities:

3.1. Mortality Catastrophe Bonds

Mortality catastrophe bonds (*CATM bonds*) are similar to CatBonds from *Section 2*, see e.g. Bauer and Kramer (2007), Cairns et al. (2008), Cowley and Cummins (2005), Krutov (2006), Lin and Cox (2008). They help to reduce exposure to a catastrophic mortality deterioration (i.e. to extreme mortality). Catastrophes impose a big potential problem for life insurers since fatalities from natural and man-made disasters can be tremendous (such as a repeat of the 1918 Spanish Flu pandemic, a major terrorist attack using weapons of mass destruction, the earthquake and tsunami in southern Asia and eastern Africa in 2004, and the like).

CATM bonds are market-traded securities whose payments are linked to a *mortality index*. The CATM bonds issued to date have been structured as principal-at-risk notes with a fixed tenor, where the principal repayment is contingent on a catastrophic outcome for the value of a customized mortality index. Such a catastrophic outcome is defined as an extreme rise in mortality beyond a particular baseline. The CATM bonds have been issued mostly by reinsurers looking to free up capital related to the extreme mortality risk they face in their life insurance book.

The first bond of this type was the three-year life catastrophe bond Vita I which came to market in December 2003 maturing on 1 January 2007. It was designed to securitize Swiss Re's own exposure (one of the leading reinsurers all over the world) to certain catastrophic mortality events: a severe outbreak of influenza, a terrorist attack or a natural catastrophe. To carry out the transaction, Swiss Re set up a special purpose vehicle Vita Capital Ltd. that enabled to keep the corresponding cash-flows off Swiss Re's balance sheet. The principal of $400m was at risk if during any single calendar year the mortality index exceeded 130 % of the base 2002 level, and would be exhausted if the index exceeded 150 %. In return for having their principal at risk, investors received quarterly coupons of three-month US LIBOR plus 135 basis points. It means that only the principal was unprotected, and the principal repayment depended on what happened to a specifically constructed mortality index. This mortality index was constructed as weighted average of mortality rates (deaths per 100,000) over age, sex (male 65 % and female 35 %) and nationality (US 70 %, UK 15 %, France 7.5 %, Italy 5 % and Switzerland 2.5 %). The bonds Vita I have been successful, and soon further CATM bonds have followed due to strong investor demand (Vita II and Vita III by Swiss Re, Tartan by Scottish Re, OSIRIS by AXA). E.g. the last one issued in 2006 should cover extreme mortality in France, Japan and US. In 2008 Munich Re (another leading reinsurer) established a bond program (with SPV managed by JPMorgan) in value of $1.5 billion for the transfer of catastrophic mortality risk to capital markets (see www.artemis.bm).

A scheme of Vita I is given in *Figure 1*. Usually the SPV (i.e. Vita Capital Ltd. in this case) makes use of a swap counterparty to exchange fixed returns for LIBOR returns necessary for bond holders as coupons (see *Figure 1*). The payoff function $f_t(\cdot)$ (t = 1, 2, 3) for bond holders depends on experienced extreme mortality:

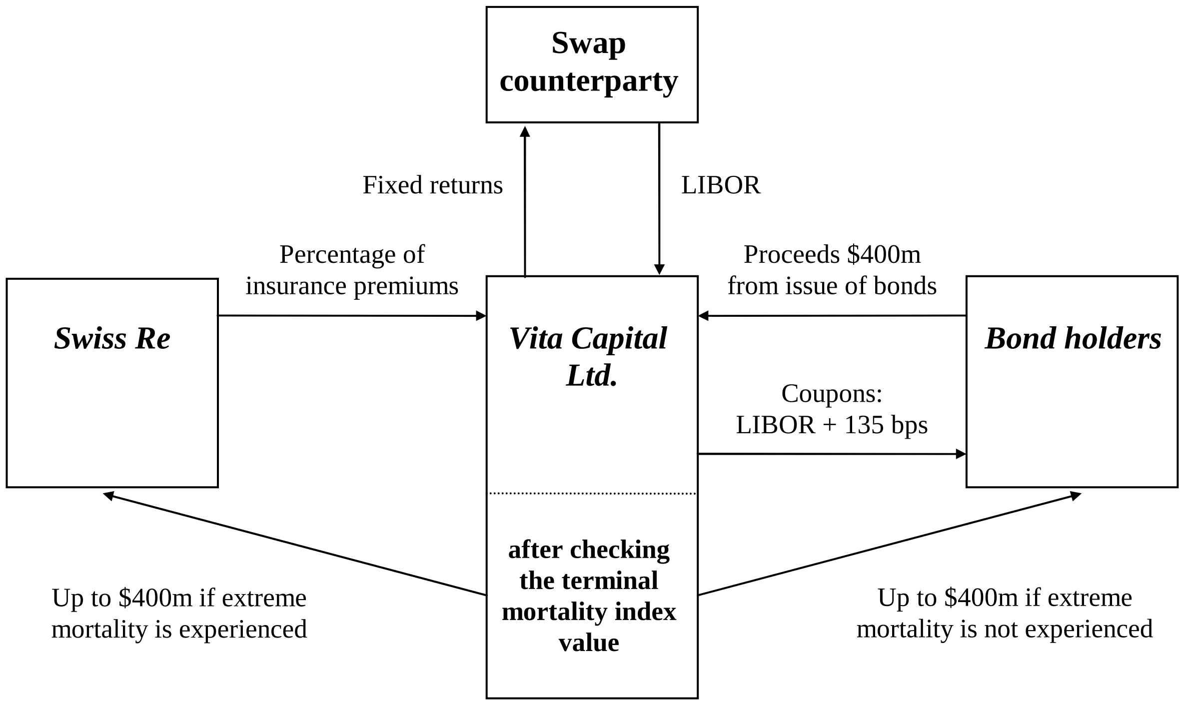
$$f_t(\cdot) = \begin{cases} LIBOR + 1.35\%\,, & t = 1, 2 \\ LIBOR + 1.35\% + \max(0\,; 100\% - \sum_{s=1}^{3} L_s)\,, & t = 3 \end{cases},$$

where

$$L_t = \begin{cases} 0\%\,, & M_t < 1.3M_0 \\ [(M_t - 1.3M_0)/0.2M_0]\cdot 100\%\,, & 1.3M_0 \le M_t \le 1.5M_0 \\ 100\%\,, & 1.5M_0 < M_t \end{cases} \quad \text{for } t = 1, 2, 3$$

and M_0 is the base 2002 level of mortality index and M_t is the mortality index for year t.

Figure 1 Scheme of CATM bond Vita I



3.2. Mortality Swaps

Mortality swaps (also called *survivor swaps*) are derivative securities where counterparties swap fixed series of payments in return for series of payments linked to the number of survivors in a given cohort or linked to the outcome of a mortality index, see e.g. Blake et al. (2006a), Cairns et al. (2008), Dowd et al. (2006), Lin and Cox (2005). It is just the random leg (i.e. the number of survivors or the outcome of a mortality index) that discriminates the mortality swaps from the classical swaps (e.g. from the interest rate swaps IRS used in *Figure 1*). Even if the mortality swaps bear a similarity to reinsurance contracts (both of them exchange anticipated for actual payments), the mortality swaps are not insurance contracts in the legislative sense (e.g. they may be used for speculative purposes without existence of an insurable interest).

E.g. in 2007 Goldman Sachs launched a monthly index *QxX.LS* (www.qxx-index.com) in combination with standardized 5 and 10-year mortality swaps. The index was based on pools of approximately 46,000 lives of individual ages 65 and older with a primary impairment other than AIDS or HIV. The second index *QxX.LS2* was launched in 2008 starting with a pool of 65,655 individuals over age of 65 with impairments that included cancer, cardiovascular conditions and diabetes.

3.3. Longevity Bonds

There are various types of *longevity bonds LB* (or *survivor bonds*), see e.g. Antolin and Blommestein (2007), Blake and Burrows (2001), Blake et al. (2006a, 2006b, 2010), Brown and Orszag (2006), Collet-Hirth and Haas (2007), Kabbaj and Coughlan (2007), Krutov (2006), Leppisaari (2008), Levantesi and Torri (2008), Lin and Cox (2005), Reuters (2010), Richards and Jones (2004), Thomsen and Andersen (2007). In general, these bonds are designed to protect companies (or governments) from unexpected increase in the life span of their annuitants, i.e. from the systematic longevity risk.

LBs are bonds, whose payoffs $f_t(\cdot)$ ($t = 1, ..., T$) depend on a survivor index S_t. This index represents the proportion of initial population surviving to a future time. While a classical (nominal) bond pays annual or semiannual coupons on a fixed amount and the principal is repaid at the term, the LB provides regular floating payments which depend on the number of cohort survivors translated again via a selected survivor index (survivor indices may be obtained similarly as mortality indices from *Section 3.1* and *3.2*)

LBs may be divided into several categories:

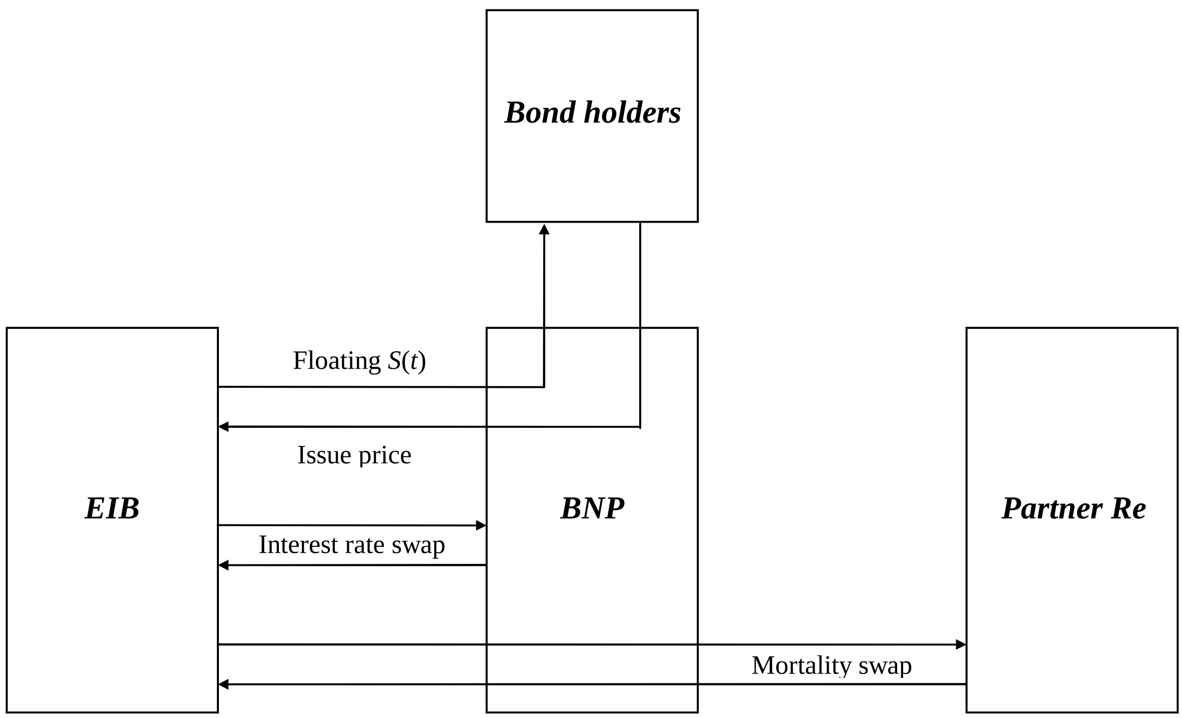
- *Standard LBs*: They are coupon-bearing bonds whose coupon payments fall over time proportionally to a survivor index, i.e. $f_t(\cdot) = k \cdot S_t$ for a positive constant k.
- *Inverse LBs*: Their coupons are inversely related to a survivor index, i.e. rising over time instead of falling with $f_t(\cdot) = k \cdot (1 - S_t)$.
- *Longevity zero bonds*: They are zero-coupon bonds (see e.g. Cipra (2010)) where the principals are functions of a survivor index.
- *Principal-at-risk LBs*: In this case not the coupons (fixed or floating ones) but the principal is linked to a survivor index.
- *Survivor bonds*: Unlike the standard LBs they have no specified maturity but they continue to pay the coupons as long as the last member of the reference population is alive (in particular, they have no principal payment).
- Further types of LBs exist but they are not mentioned here.

The first LB was the EIB/BNP Paribas bond in 2004 (see e.g. Collet-Hirth and Haas (2007)). This bond was to be issued by the European Investment Bank (EIB) with the commercial bank BNP Paribas as its structurer and manager, and Partner Re (Bermuda) as the longevity risk reinsurer (see *Figure 2*). The issue size was £540m, the initial coupon £50m and maturity 25 years. The corresponding survivor index was based on the realized mortality experience of the population of English and Welsh males aged 65 in 2003: if $m(t,x)$ denotes age-specific death rate at age x in year t (see *Section 4*) then

$$
\begin{aligned}
&S(0)=1\ ,\\
&S(1)=S(0)\cdot(1-m(2003,65))\ ,\\
&\quad\vdots\\
&S(t)=S(0)\cdot(1-m(2003,65))\cdot(1-m(2004,66))\cdot\ldots\cdot(1-m(2002+t,64+t))
\end{aligned}
\tag{1}
$$

and at times t = 1, 2, ..., 25 the bond pays coupon payments of £50m $\times\, S(t)$. It means that the bond was an *annuity bond* with floating coupon payments linked to realized mortality rates of English and Welsh males aged 65 in 2002 and with initial coupon set at £50m.

Figure 2 Scheme of EIB/BNP Paribas longevity bond



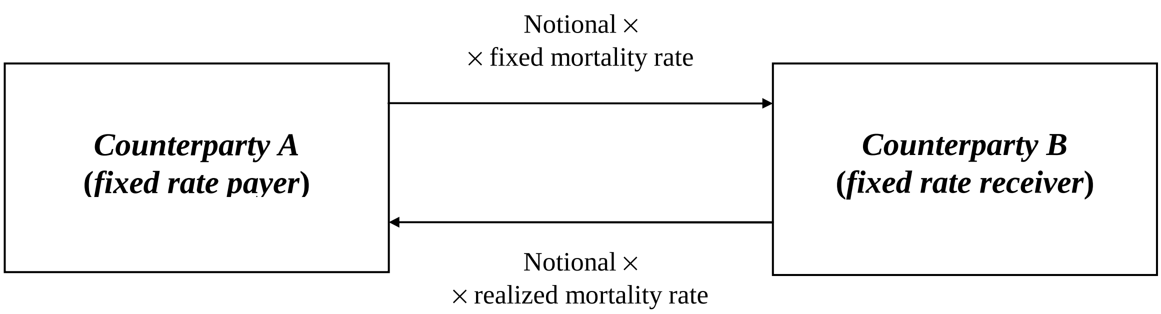
Practically, this LB was made up of three components (see *Figure 2*). The first one is a floating rate (annuity) bond issued by the EIB with a commitment to pay floating coupons in

€s. The second one is a (cross-currency) interest rate swap IRS (see also *Section 3.2*) between the EIB and BNP Paribas, in which the EIB pays floating €s and receives fixed £s. The third component is the key one since it is a mortality swap (see *Section 3.2*) between the EIB and Partner Re in which the EIB exchanges the fixed payments in £s for floating £50m $\times S(t)$ payments. In particular, the first and the third components were structured and organized via the BNP Paribas (see *Figure 1*). Unfortunately, the EIB/BNP Paribas bond was only partially subscribed and later withdrawn due to inadequate design.

3.4. Mortality forwards and futures

Mortality forwards (*q-forwards*) resemble interest rate forwards (see e.g. Cipra (2010)). They are forward contracts linked to a future mortality rate (the standard actuarial notation in *Section 4* uses the symbol *q* for the mortality rate), see e.g. Cairns et al. (2008), Coughlan et al. (2007a, 2007b), Loyes et al. (2007). The *q*-forward exchanges at time *T* a realized (i.e. "delivered") mortality rate $q(T-1, x)$ in return for a fixed mortality rate which is agreed at the beginning of the contract at time $T-1$ (of course, this exchange is made in financial terms, see *Figure 3*). In practice they may be used to hedge mortality swaps from *Section 3.2* which are also important for financial engineering of LBs (see e.g. *Figure 2*). For instance, JPMorgan announced the launch of *q*-forwards in 2007 (see also the corresponding business system called *LifeMetrics* in Coughlan et al. (2007a)).

Figure 3 Scheme of *q*-forwards



Mortality futures (*q-futures*) are mortality forward contracts standardized to be marketable on exchanges, see e.g. Blake et al. (2006a).

4. Population and actuarial instruments and methods

In this Section we remind some basic concepts of population mathematics, which are important in the context of mortality-linked securities (see e.g. Cipra (2010)).

The *age-specific death rate* $m(t, x)$ mentioned in *Section 3.3* is defined as the relative number of deaths in the given age x and period t in the mid-population of this period

$$m(t,x) = \frac{D(t,x)}{E(t,x)} = \frac{\text{number of deaths during calendar year } t \text{ aged } x}{\text{mid-population during calendar year } t \text{ aged } x}\,. \tag{2}$$

The (*age-specific*) *mortality rate* $q(t, x)$ is the probability that a life aged x at time t will die within one year. It can be calculated approximately (for forces of mortality remaining constant in particular years) as

$$q(t,x) = 1 - \mathrm{e}^{m(x,\,t)} \tag{3}$$

(the approximate relation (3) can be compared with the exact relation (5) using the concept of force of mortality). The corresponding survival probability $p(t, x) = 1 - q(t, x)$ can be generalized over n years by chain relation

$${}_n\,p(t,x) = p(t,x)\cdot p(t+1,x+1)\cdot\ldots\cdot p(t+n-1,x+n-1)\,. \tag{4}$$

The survivor index $S(t)$ in (1) may be taken as the estimated survival probability ${}_t\,p(2003, 65)$.

The *force of mortality* $\mu(x, t)$ is the instantaneous death rate for lives aged x at time t. The rigorous form of the relation (3) is then

$$q(t,x) = 1 - \exp\left\{-\int_0^1 \mu(t+\tau, x+\tau)\,\mathrm{d}\tau\right\}. \tag{5}$$

Another important concept is the *life expectancy* $e(t, x)$ for lives aged x at time t

$$e(t,x) = \int_0^\infty \tau\cdot{}_\tau\, p(t,x)\cdot\mu(t+\tau, x+\tau)\,\mathrm{d}\tau\,. \tag{6}$$

In practice the observed values of these variables are arranged in *Life Tables LT*. In particular, the so-called cohort (or generation) LT are suitable if one must do calculations over long time horizons as it is usual e.g. in pension calculations. The *cohort LT* can be used as records of the actual lifetimes of particular generations or cohorts (while the so-called period LT display mortality for people of different ages at one point in time so that they include people born in different years, i.e. belonging to different cohorts). Moreover, the cohort LT enable projections of mortalities and life expectancies over long time horizons (see e.g. Lee and Carter (1992)) and may be adjusted to respect the corresponding selection principles. E.g. the cohort LT constructed by Cipra (1998) are suitable for pension annuities since they take into account the selection approach by potential annuitants. Some results due to these LT (including the volatility of survival projections, see also Blake et al. (2008)) are applied just in the framework of longevity securitization in *Section 6*.

Above one mentions the pension annuities (or life annuities). E.g. the (fair) value of such an annuity with unit payments in arrear for lives aged x at time t is

$$a(t,x) = \sum_{n=1}^{\infty} d(0,n) \cdot {}_n p(t,x)\,, \tag{7}$$

where ${}_1 p(t,x) = p(t,x)$ and $d(0,t)$ is the corresponding discount factor (i.e. the price at time 0 for a unit payment payable with certainty at time t).

5. Pricing of mortality-linked securities

Mortality-linked securities involve significant valuation problems that are mostly solved using the stochastic modeling, see e.g. Barbarin (2007), Bauer and Kramer (2007), Bauer and Russ (2006), Blake et al. (2006b), Cairns et al. (2006), Cox and Lin (2007), Cox and Pedersen (1998), Dahl (2004), Dahl and Møller (2006), Denuit et al. (2007), Hári et al. (2008), Leppisaari (2008), Levantesi and Torri (2008), Lin and Cox (2005, 2008), Wang (2002) and others.

This section describes very briefly and without any technical details two approaches how to price e.g. standard LBs from *Section 3.3* (more practical approach to price systematic longevity risks is shown in *Section 6*):

The first of them is the *distortion approach* by Wang (see e.g. Wang (2002)) which distorts the distribution of the survivor index to obtain suitable risk-adjusted expected values of this index. For a distribution function $F(t)$ the corresponding Wang transform is

$$F^*(t) = \Phi[\Phi^{-1}(F(t)) - \lambda]\,, \tag{8}$$

where $\Phi(\cdot)$ is the standard normal distribution function and the parameter λ is the market price of risk. After such a transform the survivor index can be discounted at the risk-free rate assuming that mortality and interest rate risk are independent. It means that the (fair) value $V(\text{LB})$ of a standard LB with unit initial coupon can be obtained as

$$V(\text{LB}) = \sum_{t=1}^{T} d(0,t) \cdot \mathrm{E}^*(S(t))\,, \tag{9}$$

where $\mathrm{E}^*(S(t))$ is the expected cash-flow under the transformed distribution $F^*(t)$ of the corresponding survival index $S(t)$ starting at age x (see (1)) and $d(0,t)$ is the risk-free discount factor (i.e. the price at time 0 for a unit payment payable with certainty at time t, see also (7)). Moreover, the parameter λ reflecting the level of systematic longevity risk can be calibrated by means of market prices of this risk for corresponding assets existing in the market place, i.e. one looks for λ solving equations of the type

$$a^{market}(t,x) = \sum_{n=1}^{\infty} d(0,n) \cdot \Phi[\Phi^{-1}(S(t)) - \lambda] \tag{10}$$

for quoted annuity values at the market.

The second approach is the one based on the *risk-neutral pricing* which is popular in finance in general. Assuming an arbitrage-free environment there exists a risk-neutral measure Q allowing risk-free discounting using the same discount factor $d(t,0)$ as in (9):

$$V(\mathrm{LB}) = \sum_{t=1}^{T} d(0,t) \cdot \mathrm{E}_Q(S(t) \,|\, \Omega_0)\,, \tag{11}$$

where $\mathrm{E}_Q(S(t) \,|\, \Omega_0)$ is the expected value of $S(t)$ under the risk-neutral measure Q conditional on the information Ω_0 available at time 0. However, so far due to non-existence of regular quotations of LBs at the markets the corresponding measures Q cannot be calibrated.

6. Practical pricing of mortality forwards

Mortality forwards have been described in *Section 3.4.* as contracts linked to a future mortality rate in such a way that they exchange a realized (delivered) mortality rate q in return for a fixed mortality rate which is agreed at the beginning of the contract.

As an example of possible practical approach how to price such securities (see Loyes et al. (2007)) let's consider a 10-year forward for the 75-year old cohort of males in the Czech Republic that is aged of 65 at the beginning of the contract in 2010. *Table 2* shows the male and female mortality rates $q(t, x)$, t = 2010, ..., x = 65, ... (see *Section 4*) for the corresponding male and female cohort born in 1945 according to the cohort Life Tables constructed by Cipra (1998). These LT respect the corresponding selection principle in the framework of pension systems and life annuity markets, i.e. they take into account the selection approach by potential annuitants.

Table 2 Male and female mortality rates for the corresponding male (x) and female (y) cohort born in 1945 (Czech Republic), i.e. aged x, y = 65, … in t = 2010, …

x	$q(x, t)$	y	$q(y, t)$
65	0.014425	65	0.005139
66	0.015771	66	0.005692
67	0.017345	67	0.006345
68	0.019146	68	0.007109
69	0.021134	69	0.007999
70	0.023320	70	0.009047
71	0.025659	71	0.010244
72	0.028102	72	0.011560
73	0.030615	73	0.012968
74	0.033220	74	0.014438
75	0.035828	75	0.015929

Source: Cipra (1998, Table 3 and 4)

The mortality forward can be practically implemented in such a way that an investor buy a 10-year zero coupon bond with a principal of 100 monetary units and simultaneously enters a mortality forward contract of notional value 100. This investment may earn 100 + 100· $(q_{index} - q_{forward})$ at the maturity, where q_{index} is the mortality index (see *Section 3.1*) delivered at the maturity by a suitable agency (similarly to security indices of the type S&P 100) and $q_{forward}$ is the contracted forward price (a more general payoff may be 100 + $100 \cdot k \cdot (q_{index} - q_{forward})$ where k is a suitable *leverage coefficient*). It means that the investor makes a profit in this forward contract when $q_{index} - q_{forward} > 0$ (i.e. when the longevity risk does not occur) and suffers a loss when $q_{index} - q_{forward} < 0$ (i.e. when the counterparty of the issuer faces the longevity risk).

In order to find $q_{forward}$ (i.e. to price this mortality forward) and at the same time to take into account the volatility of future mortality rates one can make use of Sharpe ratio (excess return divided by volatility) that should attain a reasonable value for such investments (Loyes et al. (2007) recommend the value of 0.25 in view of longer-term returns of bonds and equities). Hence the calibrated value $q_{forward}$ should fulfill

$$\frac{(q_{projection} - q_{forward})/10}{volatility} = 0.25\,, \tag{12}$$

where $q_{projection}$ is the mortality rate (in our case it is $q(2020, 75)$) projected by means of the cohort LT (see *Table 2*), the nominator in (12) is the annualized excess return (ignoring compounding effects) and the denominator of (12) is the annualized risk (i.e. the annual volatility of projections of mortality rates). From (12) one obtains a simple formula

$$q_{forward} = q_{projection} - 10 \cdot 0.25 \cdot volatility\,. \tag{13}$$

The numerical value corresponding to our example can be obtained using *Table 2* for mortality rate projections and *Table 3* for volatilities. The annual volatilities in *Table 3* following from the construction of projections in the framework of the cohort LT are given as the percentage of the corresponding mortality rate; they are slightly higher than the ones presented in Loyes et al. (2007) for population in England & Wales and in US (see *Table 3*).

Table 3 Annual volatilities for selected ages as the percentage of the corresponding mortality rates (England & Wales, US, Czech Republic)

Male volatility (%)				Female volatility (%)			
x	E & W	US	CZ	*y*	E & W	US	CZ
45	2.96	2.31	3.10	45	2.82	2.41	2.93
55	2.57	1.53	2.69	55	2.90	1.61	3.01
65	2.64	1.01	2.78	65	2.36	1.52	2.45
75	3.03	1.47	3.15	75	2.81	1.66	2.90

Numerically according to (13) and *Tables 2* and *3* (for the Czech Republic) we'll obtain for males

$$q_{forward} = (1 - 10 \cdot 0.25 \cdot 0.0315) \cdot 0.035828 = 0.03301 \approx 3.30\,\% \,.$$

It means that the forward needs to be 0.28 % below the projected future mortality of 3.58 % (it is $3.30 - 3.58 = -0.28$ %), which is discount of $0.28/3.58 \approx 7.82$ % on the projected mortality. What does it mean numerically? Let the corresponding forward contract with the volume of 5 billions CZK be negotiated with $q_{forward}$ = 3.30%, but the mortality index achieves the real value q_{index} = 3.52% (i.e. 6 basis points below the projected value $q_{projection}$ = 3.58%). Then the profit margin of investors amounts to $(0.0352 - 0.0330) \cdot 5 \cdot 10^9 = 11 \cdot 10^6$ = 11 millions CZK. Obviously the investors' profit decreases with declining mortality index q_{index}, i.e. with growing longevity of population, since the investors are not averse against the longevity risk.

Conclusions

The paper shows that some risks of contemporary and future world (risk of catastrophes, ecological damage, terrorism, but also "positive" risks of longevity) cannot be covered by classical insurance instruments. Therefore alternative ways of risk transfer are experimented how to manage such problems. There are many examples of successful and less successful experiments of such type in economic practice.

A relative successful risk transfer consists in securitization process where one conveys risks from insufficient insurance institutions by means of tradable securities to financial or capital markets. The practical examples of such securities shown in the paper outline that the investors including banks may expect a new generation of financial instruments (securities, financial derivatives, annuities, credits and others) which are linked to insurance or pension systems. Naturally, a responsible risk evaluation will be the key assumption of such investing which on the other hand can make for lucrative profits (see e.g. footnote 1).

A very hopeful area for applications of these approaches seems to be future pension systems with a substantial risk of longevity (in addition to demographic, migration, labor, tax and other problems). So far such applications are only experimental and confined to countries with "effective" annuity markets (mainly UK and US, but also Australia, Chile, Singapore, Switzerland, see e.g. Cannon and Tonks (2008)). On the other hand, some ideas and principles of alternative risk transfers may be instructive even for pension reforms in Central Europe with expected transfer of responsibility from governments to other subjects.

The work is a part of research project MSM0021620839 financed by Ministry of Education of Czech Republic.

REFERENCES

Antolin P, Blommestein H (2007): *Governments and the Market for Longevity-Indexed Bonds.* OECD Working Paper on Insurance and Private Pensions, No. 4, OECD Publishing, Paris.

Banks E (2004): *Alternative Risk Transfer*. Wiley, Chichester.

Barbarin J (2007): *Heath-Jarrow-Morton Modelling of Longevity Bonds and the Risk Minimization of Life Insurance Portfolios.* Working Paper, Université Catholique de Louvain.

Bauer D, Kramer FW (2007): *Risk and Valuation of Mortality Contingent Catastrophe Bonds.* Working Paper, Ulm University.

Bauer D, Russ J (2006): *Pricing Longevity Bonds using Implied Survival Probabilities.* Working Paper, Ulm University.

Blake D, Boardman T, Cairns A (2010): *Sharing Longevity Risk: Why Governments Should Issue Longevity Bonds.* Discussion Paper PI-1002, The Pension Institute, Cass Business School, City University, London.

Blake D, Burrows W (2001): *Survivor Bonds: Helping to Hedge Mortality Risk.* Journal of Risk and Insurance, 68: 339–348.

Blake D, Cairns AJG, Dowd K (2006a): *Living with Mortality: Longevity Bonds and other Mortality-Linked Securities*. British Actuarial Journal, 12: 153–228.

Blake D, Cairns A, Dowd K (2008): *Longevity Risk and the Grim Reaper's Toxic Tail: The Survivor Fan Charts.* Insurance: Mathematics & Economics, 42: 1062–1066.

Blake D, Cairns AJG, Dowd K, MacMinn R (2006b): *Longevity Bonds: Financial Engineering, Valuation, and Hedging*. Journal of Risk and Insurance, 73: 647–672.

Brown JR, Orszag PR (2006): *The Political Economy of Government Issued Longevity Bonds.* Journal of Risk and Insurance, 73: 611–631.

Cairns AJG, Blake D, Dowd K (2006): *Pricing Death: Frameworks for the Valuation and Securitization of Mortality Risk*. ASTIN Bulletin, 36: 79–120.

Cairns AJG, Blake D, Dowd K (2008): *Modelling and Management of Mortality Risk: A Review*. Scandinavian Actuarial Journal, 2008(2–3): 79–113.

Cannon E, Tonks I (2008): *Annuity Markets*. Oxford University Press, Oxford.

CEA (2006): *Working Document on the Standard Approach for Calculating Solvency Capital Requirement.* European Insurance and Reinsurance Federation.

Cipra T (1998): *Cohort Life Tables for Pension Insurance and Pension Funds.* Pojistné rozpravy, 3: 31–57 (in Czech).

Cipra T (2002): *Capital Adequacy in Finance and Solvency in Insurance.* Ekopress, Prague (in Czech).

Cipra T (2004): *Reinsurance and Alternative Risk Transfer.* Grada Publishing, Prague (in Czech).

Cipra T (2010): *Financial and Insurance Formulas.* Physica Verlag/Springer, Heidelberg.

Collet-Hirth O, Haas S (2007): *Longevity Risk. The Longevity bond.* Technical Report (November 2007), Patner Re, Bermuda.

Corkish J, Holland A, Vila AF (1997): *The Determinants of Successful Financial Innovations: An Empirical Analysis of Future Innovations on LIFFE.* Bank of England, London.

Coughlan G et al. (2007a): *LifeMetrics: A Toolkit for Measuring and Managing Longevity and Mortality Risks.* Technical Document, JPMorgan Pension Advisory Group (March 2007).

Coughlan G, Epstein D, Sinha A, Honig P (2007b): *q-Forwards: Derivatives for Transfering Longevity and Mortality Risks.* JPMorgan Pension Advisory Group (July 2007).

Cowley A, Cummins JD (2005): *Securitization of Life Insurance Assets and Liabilities.* Journal of Risk and Insurance, 72: 193–226.

Cox SH, Lin Y (2007): *Natural Hedging of Life and Annuity Risks.* North American Actuarial Journal, 11: 1–15.

Cox SH, Pedersen HW (1998): *Catastrophe Risk Bonds.* Actuarial Research Clearing House, 1998(1): 421–452.

Cummins JD (2008): *CAT Bonds and other Risk-Linked Securities: State of the Market and Recent Developments.* Risk Management and Insurance Review, 11: 23–47.

Dahl M (2004): *Stochastic Mortality in Life Insurance: Market Reserves and Mortality-Linked Insurance Contracts.* Insurance: Mathematics & Economics, 35: 113–136.

Dahl M, Møller T (2006): *Valuation and Hedging of Life Insurance Risks with Systematic Mortality Risk.* Insurance: Mathematics & Economics, 39: 193–217.

Denuit M, Devolder P, Goderniaux A-C (2007): *Securitization of Longevity Risk: Pricing Survival Bonds with Wang Transform in the Lee-Carter Framework.* Journal of Risk and Insurance, 74: 87–113.

Dowd K, Blake D, Cairns AGJ, Dawson P (2006): *Survivor Swaps.* Journal of Risk and Insurance, 73: 1–17.

Hári N, De Waegenaere A, Melenberg B, Nijman TE (2008): *Longevity Risk in Portfolios of Pension Annuities.* Insurance: Mathematics & Economics, 42: 505–519.

IAA (2004): *Global Framework for Insurer Solvency Assessment*. International Actuarial Association.

Kabbaj F, Coughlan G (2007): *Managing Longevity Risk through Capital Markets.* De Actuaris (September 2007): 26–29.

Krutov A (2006): *Insurance-Linked Securities: An Emerging Class of Financial Instruments.* Financial Engineering News, 48: 7–16.

Lee RD, Carter LR (1992): *Modelling and Forecasting U.S. Mortality.* JASA, 87 (419): 659–675.

Leppisaari M (2008): *Managing Longevity Risk with Longevity Bonds.* Helsinki University of Technology (Mat-2.4108, August 2008).

Levantesi S, Torri T (2008): *Setting the hedge of Longevity Risk through Securitization.* Proceedings of the 10th Italian-Spanish Congress of Financial and Actuarial Mathematics, Cagliari.

Lin Y, Cox SH (2005): *Securitization of Mortality Risks in Life Annuities.* Journal of Risk and Insurance, 72: 227–252.

Lin Y, Cox SH (2008): *Securitization of Catastrophe Mortality Risks.* Insurance: Mathematics & Economics, 42: 628–637.

Loyes J, Panigirtzoglou N, Ribeiro RM (2007): *Longevity: A Market in the Making*. JPMorgan Technical Report (Global Market Strategy, July 2007).

Milevsky MA (2006): *The Calculus of Retirement Income.* Cambridge University Press, Cambridge.

OECD (2006): *Pension Markets in Focus*. Issue 3 (October 2006), Paris.

OECD (2008): *Pension Markets in Focus*. Issue 5 (December 2008), Paris.

Reuters (2010): *Factbox: How Longevity Bonds May Work.* Thomson Reuters (Wed, Apr 7, 2010).

Richards S, Jones G (2004): *Financial Aspects of Longevity Risk.* Staple Inn Actuarial Society (October 2004), London.

Sandström A (2006): *Solvency. Models, Assessment and Regulation.* Chapman & Hall/CRC, London.

Schneider O (2009): *Reforming Pensions in Europe: Economic Fundamentals and Political Factors.* Czech Journal of Economics and Finance, 59(4): 292–308.

Swiss Re New Markets (1999): *Insurance Linked Securities.* Zürich 1999.

Thomsen GJ, Andersen JV (2007): *Longevity Bonds – A Financial Market Instrument to Manage Longevity Risk.* Monetary Review, 4th Quarter: 29–44.

van Broekhoven H (2002): *Market Value of Liabilities Mortality Risk: A Practical Model.* North American Actuarial Journal, 6: 95–106.

Wang SS (2002): *A Universal Framework for Pricing Financial and Insurance Risks.* ASTIN Bulletin, 32, 213–234.