Robust Hedging Performance and Volatility Risk in Option Markets

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Outline

- Why ROBUST hedging?
- Nonparametric Volatility Estimation
- Empirical Hedging Performance: SPX & TXO
- Price Limit Effect & Hedging Performance
- An Asymptotic Analysis

Why ROBUST hedging?

Bakshi et al. (1997), Lam et al. (2002) and Yung et al. (2003) documented that **stochastic volatility models, variance gamma models, and EGARCH (GARCH) models,** respectively, are superior in volatility forecast and/or option pricing, but these models perform just comparably or even worse than the ad hoc Black-Scholes model (Dumas et al. (1998)) in option hedging.

AIM: model dependence of option hedging strategies should be minimized.

Two Hedging Categories

- Model-Free: Stop-Loss, adjusted SL

- Volatility-Model-Free*: Delta, adjusted Delta, Delta-Gamma

BUT most of them require VOLATILITY as an input.

*Fouque, Papanicolaou, Sircar (2000)

WHAT VOLATILITY?

- Historical VOL: estimated by Quadratic Variation.

- Instantaneous VOL: estimated by Fourier Transform Method.

- Implied VOL: depend on BS model.

AIM: NONPARAMETRIC ESTIMATION.

Volatility Matrix Estimation Problem *

Suppose that each $u_i(t)$ satisfies

$$du_i(t) = \mu_i(t)dt + \sum_{j=1}^{d} \sigma_{ij}(t)dW_{jt}, i = 1, \cdots, d.$$

Task: Given all return series $\boldsymbol{u}(t)$, estimate the volatility matrix $\Sigma(t)$.

*A discrete time paradigm can be found in Engle's book: Anticipating Correlations. Princeton Univ. Press, 2009.

Fourier Transform Method (Step 1)*

Fourier coefficients of du_i are:

$$a_0(du_i) = \frac{1}{2\pi}\int_0^{2\pi} du_i(t),$$
$$a_k(du_i) = \frac{1}{\pi}\int_0^{2\pi} \cos(kt)du_i(t),$$
$$b_k(du_i) = \frac{1}{\pi}\int_0^{2\pi} \sin(kt)du_i(t).$$

$$\begin{aligned} u_i(t) &= a_0(du_i) \\ &\quad + \sum_{k=1}^{\infty}\left[-\frac{b_k(du_i)}{k}\cos(kt) + \frac{a_k(du_i)}{k}\sin(kt)\right]. \end{aligned}$$

*Malliavin and Mancino (2002, 2009)

Fourier Transform Method (Step 2)

Fourier coefficients of Σ_{ij} are:

$$a_0(\Sigma_{ij}) = \lim_{N\to\infty} \frac{2\pi}{N+1-n_0} \sum_{s=n_0}^{N} \left[a_s^*(du_i)a_s^*(du_j) + b_s^*(du_i)b_s^*(du_j)\right]$$

$$a_k(\Sigma_{ij}) = \lim_{N\to\infty} \frac{2\pi}{N+1-n_0} \sum_{s=n_0}^{N} \left[a_s^*(du_i)a_{s+k}^*(du_j) + b_s^*(du_i)b_{s+k}^*(du_j)\right]$$

$$b_k(\Sigma_{ij}) = \lim_{N\to\infty} \frac{2\pi}{N+1-n_0} \sum_{s=n_0}^{N} \left[a_s^*(du_i)b_{s+k}^*(du_j) - b_s^*(du_i)a_{s+k}^*(du_j)\right],$$

where n_0 is any positive integer.

Fourier Transform Method (Step 3)

$$\Sigma_N(t) := \sum_{k=0}^{N} a_k(\Sigma_{ij}) \cos(kt) + b_k(\Sigma_{ij}) \sin(kt)$$

$$\Sigma(t) = \lim_{N\to\infty} \Sigma_N(t) \text{ in prob.}$$

- **Smoothing** procedure in practical implementation.

- Reno (2008) alerts boundary effect.

Price Correction Scheme*: Bias Reduction

$$\begin{aligned} du_t &\approx \sigma_t dW_{1t} \\ &= \exp\left(h_t/2\right) dW_{1t} \\ &\approx \exp\left(a + b\widehat{h}_t/2\right) dW_{1t} \end{aligned}$$

After discretization,

$$\ln \frac{\left(u_{t+1} - u_t\right)^2}{dt} = a + b\widehat{h_t}(t) + \ln \varepsilon_t^2$$

where ε_t is a standard normal random variable.

*H. Liu, and Chen ('10)

Simulation Study (I) - LV Model

$dS_t = \alpha\,(m - S_t)\,dt + \sigma_t dW_t$ and $\sigma_t = \beta S_t^{\gamma}$.
$S_0 = 0.08$, $\alpha = 0.093$, $m = 0.079$, $\beta = 0.794$ and $\gamma = 1.474$.

	Fourier	Corrected Fourier
MSE	7.5203E-04	7.6117E-06
MAE	0.0435	0.0135

Simulation Study (II) - SV Model

$\sigma_t = \exp(Y_t/2)$ and $dY_t = \alpha' (m' - Y_t)\, dt + \beta' dW'_t$, with $Y_0 = -2$, $m' = -2$, $\alpha' = 5$ and $\beta' = 1$.

	Fourier	Corrected Fourier
MSE	0.0234	0.0016
MAE	0.2902	0.1392

Other Applications of Volatility

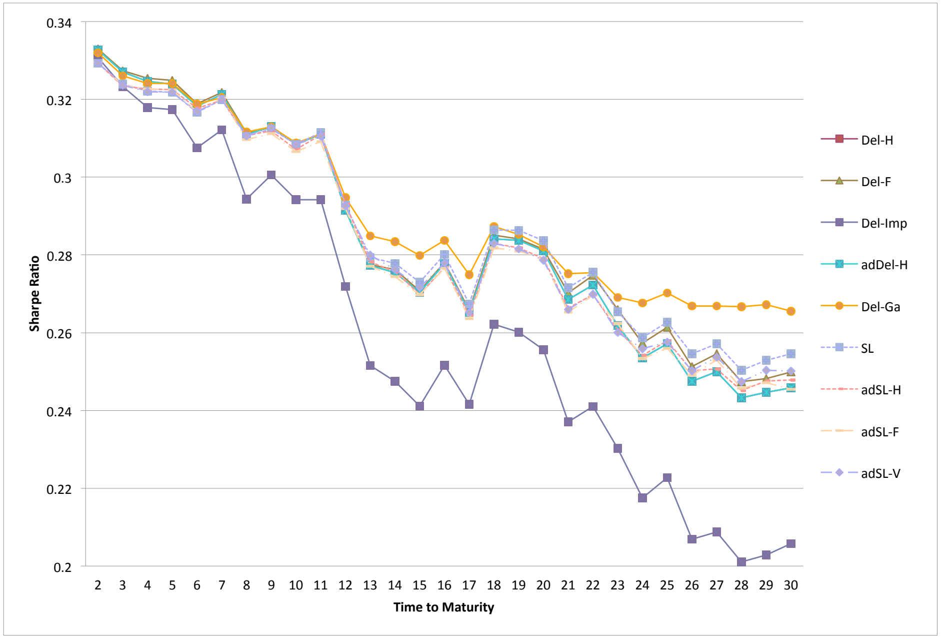
- VaR/CVaR Estimation under Stochastic Volatility*.
- Monte Carlo calibration of implied volatility surface †

⋮

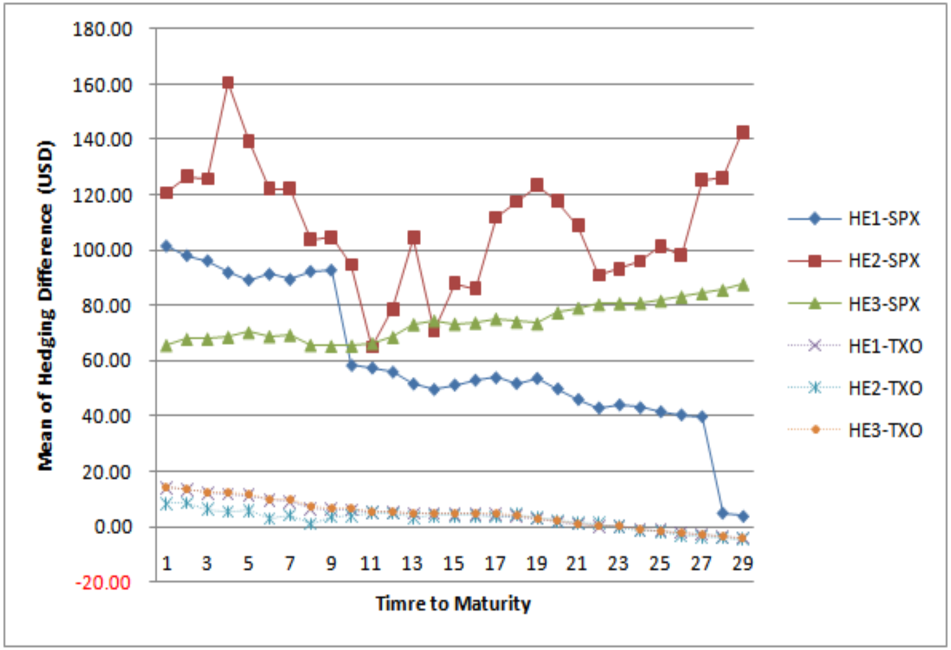
*H. Liu and Chen, 2010, submitted.
†working in progress.

Evolution of Sharpe Ratios of Hedging Strategies on SPX

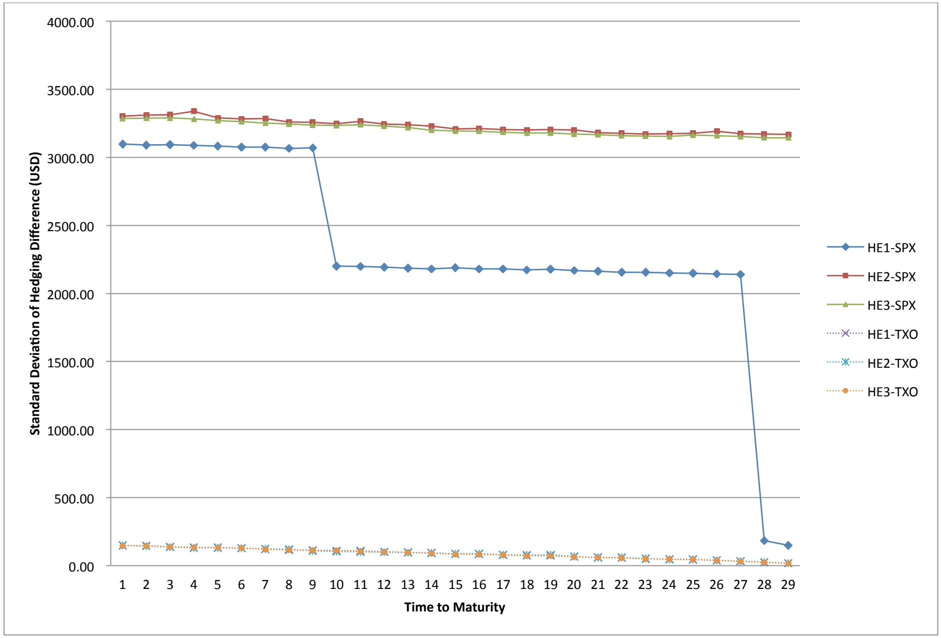
Evolution of Sharpe Ratios of Hedging Strategies on TXO



Hedging Differences: SL VS Delta (mean)



Hedging Differences: SL VS Delta (sd)



Why Such Asymmetric Phenomenon

Empirical Observation on hedging performance of SL and Delta:

SPX: well separated.
TXO: same numeric order.

A Time-Scale Change & Price Limit

A Qualitative Approach

$$\begin{aligned} \frac{dS_t}{S_t} &= \mu\, d\delta t + \sigma\, dW_{\delta t} \\ &= \mu\, \delta\, dt + \sigma \sqrt{\delta}\, dW_t, \end{aligned}$$

$$\mathcal{L}^{\delta} P^{\delta}(t,x) = 0$$

$$\mathcal{L}^{\delta} = \frac{\partial}{\partial t} + \frac{\sigma^2\, \delta\, x^2}{2} \frac{\partial^2}{\partial x^2} + r\, x\, \frac{\partial}{\partial x} - r.$$

Differences of Hedging Portfolios

$$
\begin{aligned}
& HE_T^{(1)} - HE_T^{(2)} \\
= \; & \int_0^T \left(\alpha_t^{(1)} - \alpha_t^{(2)} \right) dS_t - \int_0^T \left(\alpha_t^{(1)} - \alpha_t^{(2)} \right) r \, S_t \, dt \\
= \; & \int_0^T \left(\alpha_t^{(1)} - \alpha_t^{(2)} \right) (\mu \, \delta - r) \; S_t \, dt \\
& + \sigma \sqrt{\delta} \int_0^T \left(\alpha_t^{(1)} - \alpha_t^{(2)} \right) S_t \, dW_t.
\end{aligned}
$$

α_t denotes some hedging strategy.

Asymptotic Moment Estimates

Theorem 1. $E\left\{\left(HE_T^{(1)} - HE_T^{(2)}\right)^n\right\} \leq \frac{C}{\sqrt{\delta}} e^{-1/\delta}$ *for some constant C independent of δ.*

$HE_T^{(1)}$: cumulative SL hedging portfolio value.
$HE_T^{(2)}$: cumulative Delta hedging portfolio value.

Conclusions

- This paper extends previous empirical studies on option hedging performance. Robust hedging strategies and nonparametric volatility estimations are comprehensively studied.
- explain a documented phenomenon by a **time-scale change method**.
- An asymptotic **analysis confirms** estimated moments of hedging portfolio differences with our **empirical finding.**

Thank You