The Forecasting Performance of Model Free Implied Volatility: Evidence from an Emerging Market

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Abstract

This paper considers an estimator of the model-free implied volatility (MF-IV) derived by Jiang and Tian (2005) and investigates its information content in index option market in Taiwan. We compare the forecasting performance of MF-IV and other volatility forecasts such as the Black-Scholes implied volatility (BS-IV), historical volatility (HV) and GARCH. The empirical results show that MF-IV outperforms other approaches. The results also reveal that the MF-IV is informational efficient and subsumes all information contained in the HV and GARCH (1,1) in forecasting future realized volatility (RV) on weekly forecast horizon.

1. Introduction

During the past two decades, the study of the implied information from option market, particularly the implied volatility (IV), has been progressing rapidly in finance. Since the IV is obtained from options prices, which reflects market participants' expectations, existing empirical studies seem to support that the Black-Scholes implied volatility (BS-IV) model is a more efficient than time series model such as historical volatility (HV) and GARCH models in measure of future realized volatility (RV).¹

However, the assumptions of the BS-IV model do not completely hold in the real world. As a result, the forecast performance of IV would be unsatisfactory if the model is mis-specified. Britten-Jones and Neuberger (2000) therefore proposed an alternative IV measure named as model-free implied volatility (MF-IV), which is derived entirely from no-arbitrage condition rather than from any specific model. Jiang and Tian (2005) also found the MF-IV model is still valid even if the underlying asset price has jumps. This paper aims to examine the relative performance of the BS-IV, MF-IV, HV and GARCH (1,1) as predictors of the RV over the remaining life of the Taiwan Stock Exchange Capitalization Weighted Stock Index options (TXO) market, particularly in investigating if the MF-IV provides better information content in emerging market.

The analysis of the forecast ability of volatility relies on an accurate measure of the RV. It is increasingly evident that the RV estimator computed from high-frequency

¹ Poon and Granger (2003) reviewed 93 research papers that forecast volatility based on various volatility measures over the last two decades; they found that IV model is better than the HV model in forecasting the RV. Using data from 35 futures options markets from eight separate exchanges, Szakmary et al. (2003) found that the IV, though not a completely unbiased predictor of future volatility, outperforms the HV as a predictor of the subsequently RV in the underlying futures prices over the remaining life of the option.

data such as 5-minute data affords vastly improved the measurement quality for actual volatility and forecast evaluation. In addition, weekly (H1), bi-weekly (H2), tri-weekly (H3), and monthly (H4) forecast horizons are considered as major horizons for option pricing and portfolio management. Therefore, we use sum of square 5-minute return of the TAIEX to calculate the RV, and focus on these four major forecast horizons to test whether forecast accuracy is affected by horizon length over the remaining life of the TXO contract.

We first compare the forecasting performance of the four volatility models based on the forecast errors. Then, we examine their information contents by using univariate and encompassed regression approaches. The encompassing regression will be applied to examine whether the information content of the HV or the GARCH (1,1) is subsumed by the BS-IV or by the MF-IV.

We compare the relative forecast performance of these four models based on the four major horizons for option pricing and portfolio management. Our empirical results, based on high-frequency data such as 5-minute return to calculate the RV, provide a number of interesting findings; for example, the IV model seemly outperforms the time series model, and the MF-IV model is more informational efficiency and subsumes all information contained in the HV and the GARCH (1,1) for the shortest forecast horizon as compared to the BS-IV model. As market efficiency in TAIFEX improved and arbitrage opportunities tend to immediately disappear, the MF-IV provides superior forecasting performance than the BS-IV.

The remainder of this paper is organized as follows. Section 2 describes the institutional setting and data, and Section 3 explains the methodology, followed, in Section 4 by an explanation of the empirical results. Finally, the conclusions drawn from this study are presented in Section 5.

2. Institutional Setting and Data

Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) options contracts, which is traded under the ticker symbol of TXO and is a European-style option, were introduced by the Taiwan Futures Exchange (TAIFEX) on 24 December 2001. Same as the TAIEX futures which is traded under the ticker symbol of TX, the TXO contracts have a monthly expiration cycle, with the expiration day on the first business day after the third Wednesday (the last trading day) of the contract month. There are spot month and the next two calendar months followed by two additional months from the March quarterly cycle (March, June, September, and December) in daily trading. An option that is 'in-the-money' and has not been liquidated or exercised on the last trading day shall be exercised automatically.

Launched in 2001, the TXO market has grown rapidly. Table 1 reports the market volume and average daily trading volume during the period from 2001 to 2005. In 2005, the trading volume reached 80,096,506 contracts, which have increased significantly as compared to 5,137 contracts in 2001. Of this, 45,636,960 were call options and 34,459,546 were put options contracts. Because the trading volume of the call option was larger than that of the put option, this study compares the forecast performance of the BS-IV, MF-IV, HV and GARCH (1,1) models by using the data of nearby TXO call contracts covering the period from 24 December 2001 to 22 December 2005.

Nearby option contracts are selected because they are the most actively traded option contracts within their own classification; this therefore minimizes the problem of infrequent trading. There are 191 observations from various volatility models under the predicting future RV on H1, H2, H3 and H4 forecast horizons covering our study.²

We use high-frequency data such as 5-minute natural log return of the TAIEX to calculate the RV, and use daily natural log return of the TAIEX to calculate the HV and GARCH (1,1). To calculate the BS-IV, considering practical investing phenomenon that investors of the TXO always make investment decisions based on the market situation of the TX, we calculate the implied spot prices by using the closing prices of the corresponding TX contracts, and use them as proxies for the spot indexes of the TXO which are closest to 'at-the-money' of nearby contracts. As for the MF-IV calculation, because 'in-the-money' options are more expensive and less liquid than 'at-the-money' or 'out-of-the-money' options, following Jiang and Tian (2005), we exclude the call options with strike prices less than 97% of the implied spot prices of underlying asset from our samples.

To match the above mentioned volatility calculation, the trading data of the TXO, TX and TAIEX are obtained from the Taiwan Economic Journal (TEJ) databank. The data of the TXO and TX are from 24 December 2001 to 22 December 2005, and the TAIEX is from 1 September 1998 to 22 December 2005. Furthermore, we use fixed rate of the time deposits with one year offered by the First Commercial Bank as a proxy for the risk-free rate.

3. Methodology

As the RV is not directly observable, it must be estimated. Anderson and Bollerslev (1998), Andersen (2000), Andersen et al.(2001), Andreou and Ghysels (2002), and Barndorff-Nielsen and Shephard (2001, 2002) argued that the RV estimator computed from high-frequency data such as 5-minute data provides and

 $^{^2}$ There should be 192 observations under the forecast horizons of H1, H2, H3 and H4 in the overall 48 expiration months covering our study. However, there is no H4 horizon due to only 14 trading days during the period between the expiration months of January and February 2005.

improves vastly in measurement quality for actual yield volatility and forecast evaluation. Bandi and Russell (2003) also argued that 5-minute sampling frequency is close to optimal in the presence of market microstructure noise. Thus, we use sum of square 5-minute return of the TAIEX to calculate the RV.³ Assuming that time is measured in trading days and that there are 252 trading days per year, the RV per annum could be calculated as:

$$\sigma_t^{RV} = \sqrt{\sum_{i=1}^{54} r_{ii}^2} \times \sqrt{252} \tag{1}$$

where r_{it} is the 5-minute intra-day natural log return for the TAIEX at interval *i* of day *t*.

As noted by Ghysels et al. (2006), weekly, bi-weekly, tri-weekly, and monthly forecast horizons are major horizons for option pricing and portfolio management. Therefore, we focus on predicting ability for future RV based on these four nearest the expiration days of the TXO. Four volatility estimators are tested against the RV over the remaining life of the TXO by means of forecast error and regression analysis in this paper. These four volatility estimators are calculated from the time series models such as the HV model and GARCH (1,1) model, and the IV models such as the BS-IV model and the MF-IV model. The former is an econometrics model which is based on historical data; the later, however, is based on options market price.

3.1. Historical Volatility

The HV is perhaps the oldest and simplest volatility model. This model parameterizes current volatility as:

³ For example, in our paper, returns are sampled every 5-minute between the trading hours of 9:00 a.m. and 1:30 p.m. corresponding to 54 intervals of the TAIEX within a trading day.

$$\sigma_{t}^{HV} = \sqrt{\frac{1}{N-1} \sum_{t=1}^{N} r_{t}^{2}} \times \sqrt{252}$$
(2)

where r_t is the natural log of the ratio of the TAIEX from the current day (t) to the previous day (t-1). Any observations inside the window of size N get equal weight of 1/(N-1). In other words, volatility is forecasted to be the same as it was over the last N periods. As noted by Kroner (1996), if too large a data set is used to construct this estimate, there is a risk of clouding the estimate with stale data. On the other hand, if not enough observations are used, there is the risk of having a volatility estimate dominated one or two observations. ap Gwilym (2001) found that the simple 20-day historical estimator performs well for short forecast horizons. Therefore, we use the last 20-day data to calculate the HV in this paper.

3.2. GARCH(1,1)

Financial time series returns frequently exhibit characteristics of time-varying volatilities and volatility cluster which can not be captured by the HV model. Engle (1982) proposed the ARCH model which allows the conditional variances change over time. A practical problem in fitting ARCH (p) models to financial returns data was that in order to obtain a good fitting model, the order p needed to be fairly large. Bollerslev (1986) extended the ARCH model to the GARCH model which gives more parsimonious results than the ARCH model has become a widely used model for effectively dealing with volatility cluster and fat tail phenomena of the equity return, GARCH (1,1) especially. The GARCH (1,1) model can be defined as:

$$r_{t} = \mu_{t} + \varepsilon_{t}, \quad \varepsilon_{t} = \eta_{t}\sigma_{t},$$

$$\sigma_{t}^{2GARCH} = \omega + \alpha\varepsilon_{t-1}^{2} + \beta\sigma_{t-1}^{2GARCH}$$
(3)

where $\omega > 0, \alpha \ge 0, \beta \ge 0$ are sufficient for $\sigma_t^{2_t^{GARCH}} > 0$, and η_t is independently

and identically distributed (i.i.d) random variables with zero mean and unit variance. The GARCH (1,1) is estimated using a rolling window of 866 daily return of the TAIEX in this paper.

3.3. Black-Scholes implied volatility

Black-Scholes (1973) option pricing model (B-S model) provides the foundation for the modern theory of options valuation. One variable in this model that cannot be directly observed is the volatility of the stock price. If option markets are efficient, the BS-IV at time t (σ_t^{BS}) is inverted using the following BS-IV model:

$$\sigma_t^{BS} = f^{-1}(S_t, K, r, \tau, C_t^{MKT})$$
(4)

where S_t is the underlying asset price; K is the strike price; r is the risk-free interest rate; τ is remaining time to maturity; and C_t^{MKT} denotes the market price of the option at time t.

As noted by Lee and Nayar (1993), "market makers in SPX options are continually hedging their positions with the companion S&P 500 futures contracts." Draper and Fung (2002) also argue that, for arbitrageurs, pricing the options contracts directly with the futures contracts could avoid suffering high transaction and market-impact costs, and including stale prices in the index arising from the nontrading of constituent stocks. Therefore, considering the TXO investors generally make investment decisions based prices on the TX rather than that of the TAIEX, we use the implied spot price, which is inferred using the closing prices of the nearby TX contracts discounted at risk-free rate, as a proxy for S_t , and use the closing prices of nearby TXO contracts which are closest to 'at-the-money' as a proxy for C_t^{MKT} , respectively.

If markets are efficient and the option pricing model is correct, then the IV calculated from option prices should be an unbiased and informational efficient estimator of future RV, that is, it should correctly impound all available information including the asset's price history.

3.4. Model-free implied volatility

It is well known that test of the forecast quality of implied volatility is indeed a joint test of the efficiency of the option markets and a specification of option pricing model. Therefore, if the BS-IV model is mis-specified, the forecast performance would be unsatisfactory. Britten-Jones and Neuberger (2000) proposed an alternative IV measure, which is derived entirely from no-arbitrage conditions rather than rely on a specific model. Since it does not impose strong distributional assumptions, the forecast is common to all consistent processes; hence, this model is viewed as model-free implied volatility (MF-IV).

Suppose that call options with a continuum of strike prices (K) for a given maturity (T) are traded on an underlying asset. Let the forward asset price and forward option price be denoted as F_t and $C^F(T,K)$, respectively. Following Dumas et al. (1998) and Britten-Jones and Neuberger (2000), Jiang and Tian (2005) provide a simpler derivation under diffusion assumption for the MF-IV. The integrated return variance between current date 0 and a future date T is fully specified by the set of prices of call options expiring on date T. The MV-IV of BJN is thus defined as an integral of options prices over an infinite range of strike prices:

$$E_{0}^{F}\left[\int \left(\frac{dF_{t}}{F_{t}}\right)^{2}\right] = 2\int_{0}^{\infty} \frac{C^{F}(T,K) - \max(0,F_{0}-K)}{K^{2}} dK$$
(5)

where the superscript F is the forward probability measure. This model is straightforward to be applied for the use of stock prices if assuming that interest rate

and dividends are deterministic. For the case of options on individual stocks or index, let C(T,K) and S_t denote the prices of option and the underlying stock at time t, respectively. We have $F_t = S_t / B(t,T)$ and $C^F(T,K) = C(T,K) / B(t,T)$, where B(t,T) is the time t price of zero coupon bond that pays \$1 at time T. Hence, the MV-IV can be estimated using the following equation:

$$E_0^F \left[\int \left(\frac{dS_t}{S_t} \right)^2 \right] = 2 \int_0^\infty \frac{C(T, K) / B(0, T) - \max(0, S_0 / B(0, T) - K)}{K^2} dK$$
(6)

Because option exchanges only offer limit numbers of strike prices, the numerical integration of the MV-IV can be implemented through the trapezoidal rule:

$$2\int_{K_{\min}}^{K_{\max}} \frac{C^{*}(T,K) - \max(0,S_{0}^{*}-K)}{K^{2}} dK = \sum_{i=1}^{m} [h(T,K_{i}) + h(T,K_{i-1})] \Delta K$$
(7)

where $C^*(T,K) = C(T,K)/B(0,T)$; $S_0^* = S_0/B(0,T)$; $\Delta K = (K_{\max} - K_{\min})/m$, $K_i = K_{\min} + i\Delta K$ for i=0,...,m, and $h(T,K_i) = [C^*(T,K_i) - \max(0,S_0^* - K_i)]/K_i^2$.

In general, the MF-IV has several advantages as compared to the BS-IV. First, without any specific option pricing model, the MF-IV may avoid estimating bias resulted from mis-specified like the BS-IV. Second, subsuming more information by considering all strike prices instead of a single price as the BS-IV, the MF-IV may have better performance in forecast than the BS-IV.

However, if there are many distortions in option prices due to specific demand, the MF-IV may violate the boundary conditions of the options. Besides, there occasionally exists no trading volume at some strike prices. The options contracts violating the boundary conditions or having no trading volume may result in the IV unavailable, and then the MF-IV becomes biased. Therefore, in order to improve price efficiency, reference to Jiang and Tian (2005), we use cubic splines in the curve-fitting of the IV rather than option prices. Prices of listed calls are first translated into the IV based on the B-S model, and a smooth function is then fitted to the IV. We extract the IV at strike prices K_i from the fitted function and the B-S model is used again to invert the extracted IV into call prices. With these call prices excluding the call options with strike prices less than 97% of the implied spot price from our sample, the MF-IV is calculated by using the RHS of equation (7).

3.5. Volatility forecast evaluation criteria

Root mean squared error (RMSE), mean absolute error (MAE), and regression are three dominant methods used to test competing estimates of future volatility. Fair and Shiller (1990) argued that the regression analysis dominates RMSE in comparing alternative forecasts. Therefore, reference to existing research, we employ following univariate in equation (8) and encompassing regressions in equation (9) and (10) to analyze the information content of volatility forecasts of the BS-IV and MF-IV, respectively:

$$\sigma_t^{RV} = \alpha + \beta \sigma_t^{FV} + u_t \tag{8}$$

$$\sigma_t^{RV} = \alpha + \beta_1 \sigma_t^{BS} + \beta_2 \sigma_t^{FV1} + u_t$$
(9)

$$\sigma_t^{RV} = \alpha + \beta_1 \sigma_t^{MF} + \beta_2 \sigma_t^{FV1} + u_t$$
(10)

where σ_t^{RV} is the RV at time t, σ_t^{FV} stands for volatilities estimators of the BS-IV, MF-IV, HV and GARCH (1,1), respectively, and σ_t^{FV1} expresses the HV and GARCH (1,1).

In a univariate regression, the RV is regressed on a single volatility forecast, which examines the forecast ability and information content of one volatility forecast. On the other hand, an encompassing regression, we examine the relative importance of competing volatility forecasts models between the BS-IV and HV, between the BS-IV and GARCH (1,1), between the MF-IV and HV, and between the MS-IV and GARCH (1,1), respectively. If the BS-IV (MF-IV) contains more information as compared to the other volatility measurements, we would expect the null hypothesis $H_0: \beta_2 = 0$. In addition, if a joint hypothesis $H_0: \beta_1 = 1$ and $\beta_2 = 0$, it means that the BS-IV (MF-IV) fully subsumes the information impounded in the other volatility measurement.

As noted by prior studies, volatility in the above equations has measurement errors resulted from heteroskedasticity and serial correlations. Newey and West (1987) proposed a general covariance estimator that is consistent in the presence of both heteroskedasticity and autocorrelation of unknown form. Therefore, we use generalized method of moments (GMM) approach to estimate the above regression models, and then correct heteroskedasticity and serial correlations by using Newey and West (1987) variance-covariance estimator.

4. Empirical Results

4.1. Summary Statistics Analysis

Table 2 provides the summary statistics for the five annualized volatilities on various forecast horizons from 24 December 2001 to 22 December 2005. It shows that the means of all these four measures are higher than that of the RV. Although the HV and RV have roughly equal means, the standard deviations are far between the HV and RV. The means of the BS-IV are the highest one on various forecast horizons, the arguments of Jorion (1995), Fleming (1998), and Bates (2000) that the BS-IV is an upward biased forecast is seemly supported by our results. In addition, from the maximum and minimum of the BS-IV, MF-IV, HV and GARCH (1,1), it is difficult to judge which is nearest the RV. However, it is worth noting that all the maximum

volatility estimators of these four measures occurred on 20 May, 2004, which is the date of inauguration of the 11th-Term President and Vice President of Taiwan together with the expiration day of the Taiwan index derivatives contracts, thereby increasing the expiration day effect in terms of return volatilities.

4.2. Forecast Error Analysis

The results of MAE and RMSE are reported in Table 3. The numbers in parentheses are ranking value. If the ranking value is smaller, the forecast ability of the model is better. Table 3 indicates that the MF-IV performs the best, the second is the GARCH (1,1), and most MAE and RMSE of the BS-IV and the HV produce the same ranking. This is consistent with our conjecture that the MF-IV could have better performance than the BS-IV in emerging derivative markets such as Taiwan index options market, since the effects of market frictions might cause the BS-IV model to be mis-pecified.⁴ In terms of time series, the result of the GARCH (1,1) model outperforms the HV indicates that there exists volatility cluster and fat tail in Taiwan equity market.

As reported in Table 2, the maximum volatility estimators of the BS-IV, MF-IV, HV and GARCH (1,1) occurred on 20 May, 2004. However, we find only the maximum MAE between the RV and the BS-IV occurred on 20 May, 2004, which belongs to monthly (H4) forecast horizon, in Table 3. As compared to the MF-IV, our results seemly imply that the BS-IV rather than the MF-IV could be biased due to a jump. The argument of Jiang and Tian (2005) that the MF-IV model is still valid even if the underlying asset prices have jumps is seemly supported by our result.

Appendix A reports the monthly observations of the RV and forecast volatilities of various models on monthly (H4) forecast horizon from 2002 to 2005; it is worth

⁴ Examples of market frictions in Taiwan stock market include price limit rule, short-sale restriction, transaction costs, and index tracking errors.

noting that the RAEIV, the ratio for absolute error of the MF-IV to the BS-IV, shows that the MF-IV appears to have lower forecast errors after 2004. In order to robust our analysis, we further regress the RAEIV on the Spread:

$$RAEIV_{t} = \alpha + \beta Spread_{t} + \varepsilon_{t}$$
(11)

where *RAEIV*, is the ratio for absolute error of the MF-IV to the BS-IV at time t, *Spread*, stands for the bid-ask spreads of the nearby TXO call contracts with both the last buying and selling prices greater than zero. Table 4 shows that the coefficient of the bid-ask spread ($\hat{\beta}$) is insignificantly different from zero in period 1. However, the coefficient of the bid-ask spread ($\hat{\beta}$) is significantly positive at the 1% level in period 2. Furthermore, the median of bid-ask spreads (Spread) during the period 1 and period 2 are 25.4094 and 13.6654, respectively. The Wilcoxson rank-sum test also supports that the bid-ask spread is significantly decrease after 2004 at the 1 % level. Apparently, the improvement of market efficiency in TAIFEX makes arbitrage opportunities to disappear immediately, so that the MF-IV provides superior forecasting performance than the BS-IV.

4.3. Univariate Regression Analysis

Table 5 reports the GMM regression results of univariate regression. The coefficient of various volatility measures are all significantly different from zero at the 1% level, and the Wald test statistics (χ^2 -statictics) of the BS-IV, HV and GARCH(1,1) are highly significant on various forecast horizons, indicating rejection of the joint null hypothesis of $\alpha = 0$ and $\beta = 1$ in equation (8). This implies that although the BS-IV, HV and GARCH (1,1) volatility measures contain information in forecasting the RV, they are biased estimators in forecasting the RV. On the other hand, the χ^2 -statictics of the MF-IV are insignificant except for the H2 forecast horizon,

indicating unable to reject the joint null hypothesis of $\alpha = 0$ and $\beta = 1$. This implies that, except for the H2 forecast horizon, the MF-IV measure could be regarded as an unbiased estimator in forecasting the RV.

The R^2 -statictics show that the BS-IV has more explanatory power than the others except for monthly (H4) forecast horizon. On the other hand, the HV has less explanatory power than the others. The results in Table 5 thus indicate that although the BS-IV is biased, a strong relationship exists between them and the RV.

4.4. Encompassing Regression Analysis

The results of the univariate regression show that the IV model does well relative to time series model; therefore, we go on conducting an encompassing regression analysis based on the GMM method. Firstly, we explore the informational efficiency of the BS-IV relative to the HV and GARCH (1,1) by respective encompassing regressions in Table 6. Secondly, we examine the informational efficiency of the MF-IV relative to the HV and GARCH (1,1) in Table 7.

Table 6 reports the forecast ability and information content of the BS-IV. If the BS-IV contains more information as compared to the HV and GARCH (1,1), respectively, we would expect the null hypothesis $H_0: \beta_2^{HV} = 0$ in Panel A; and $H_0: \beta_2^{GARCH} = 0$ in Panel B. Table 6 show that the HV and the GARCH (1,1) contains more information only on H4 forecast horizon. For those shorter than H4 forecast horizons, the information of the HV and GARCH (1,1) has been impounded in the BS-IV. In other words, the HV and the GARCH (1,1) are redundant when each of them is regarded as a regressor together with the BS-IV at the same regression. Furthermore, we also find that the explanatory power (\overline{R}^2 -statictics) increases over the forecast horizon.

If the BS-IV is informational efficiency and subsumes all information contained in other volatility forecasts, we would expect the joint null hypothesis of $H_0: \beta_1^{BS} = 1$ and $\beta_2^{FV1} = 0$ (where FV1= HV or GARCH (1,1)) holds in all specifications. Table 6 shows that the Wald test statistics (χ^2 -statictics) are significant for various forecast horizons in all encompassing regressions with the coefficient of the BS-IV are significant different from zero, indicating that the joint null hypothesis of $\beta_1^{BS} = 1$ and $\beta_2^{HV} = 0$ (Panel A) or $\beta_1^{BS} = 1$ and $\beta_2^{GARCH} = 0$ (Panel B) is not hold. Our results imply that the BS-IV is informational efficiency and subsumes part not full information contained in the HV and GARCH (1,1) volatility forecasts.

Table 7 presents the results of encompassing regression when the HV measure (Panel A) or GARCH (1,1) (Panel B) is regarded as a regressor together with the MF-IV at the same regression, respectively. If the MF-IV performs more efficient in forecasting the RV than the HV or GARCH (1,1), we would expect the coefficients of the MF-IV are all significant different from zero but not the coefficients of the HV or GARCH (1,1) in the respective encompassing regression. The results show that only the encompassing regressions on H4 forecast horizons strongly reject the null hypotheses of $H_0: \beta_2^{HV} = 0$ in Panel A; and $H_0: \beta_2^{GARCH} = 0$ in Panel B. It is worth noting that, for the H1 forecast horizon, the joint null hypothesis of $\beta_1^{MF} = 1$ and $\beta_2^{HV} = 0$ (Panel A) or $\beta_1^{MF} = 1$ and $\beta_2^{GARCH} = 0$ (Panel B) are hold. The results support that the MF-IV is informational efficient and subsumes full information contained in the HV and GARCH (1,1) volatility forecasts, respectively, for shorter forecast horizon.

In order to examine if the informational content of the IV model would be biased due to a jump, we exclude the data on 20 May, 2004 from the sample of H4 forecast horizon, which is H4A. From the H4A in Table 6, we find that although the coefficient of the BS-IV is still insignificant when the HV measure is regarded as a regressor together with the BS-IV in Panel A, the coefficient of the BS-IV is becoming significant at the 5% level when the GARCH (1,1) measure is regarded as a regressor together with the BS-IV in Panel B. On the other hand, from the H4A in Table 7, the coefficient of the MF-IV is still insignificant when the HV or GARCH (1,1) measure is regarded as a regressor together with the MF-IV, respectively. Our results seemly support the argument of Jiang and Tian (2005) that the MF-IV model is still valid even if the underlying asset prices have jumps.

In general, the results of the univariate and encompassing regression indicate that the IV models outperform the time series models. The BS-IV is informational efficient and subsumes the information contained in the HV or GARCH (1,1) but not fully. The MF-IV, however, is informational efficient and fully subsumes the information contained in the HV or GARCH (1,1) on H1 forecast horizon. This implies that the MF-IV performs well for shortest forecast horizon over the remaining life of the TXO contracts as compared to the BS-IV.

5. Conclusions

This paper compares the relative forecast performance of the BS-IV, MF-IV, HV, and GARCH (1,1) volatility estimators over four major forecast horizons by using the data of nearby TXO call option contracts covering the period from 24 December 2001 to 22 December 2005. We investigate whether the MF-IV provides better information content than the BS-IV in emerging market.

Following Jiang and Tian (2005), the MF-IV is calculated from observed option prices by employing a curve-fitting method based on cubic smoothing spline and interpolate from endpoint implied volatilities between available strike prices. As noted by Jiang and Tian (2005), the MF-IV considers the aggregative information across options with different strike prices, while the forecasting performance test of the BS-IV generally involves a joint test of market efficiency and the assumed specific option pricing model. Therefore, the MF-IV could provide better information content since no specific price dynamic is required. Our results provide evidence that IV is a more efficient forecast for the RV than time series model. The results of RMSE and MAE show that the MF-IV model is consistent with our conjecture. Univariate regression results show that the MF-IV measure could be regarded as an unbiased estimator in forecasting the RV as compared to the BS-IV, HV and GARCH (1,1). The encompassing regression analyses also suggest that the MF-IV is informational efficiency and subsumes full information contained in the HV and GARCH (1,1) volatility estimators on weekly forecast horizon over the remaining life of the TXO contracts. This is consistent with ap Gwilym (2001) that the forecast accuracy of volatility model is affected by horizon length. On the other hand, we find the BS-IV contains richer information than the other volatility measures; however, it is a biased estimator and subsumes part not full information contained in other measures. The results also show that the MF-IV not the BS-IV is still unbiased when having jumps. The argument of Jiang and Tian (2005) that the MF-IV model is still valid even if the underlying asset price has a jump is supported by our finding.

Since the effects of market frictions such as price limit rule, short-sale restriction, transaction costs, and index tracking errors might cause the BS-IV model to be mis-pecified, our results are particularly informative for options investors in emerging derivative markets. Furthermore, the improvement of market efficiency in TAIFEX causes that arbitrage opportunities tend to disappear immediately; the MF-IV thus

provides superior forecasting performance than the BS-IV.

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	Call			Put	Total		
Year	Volume Avg. Daily		Volume	Avg. Daily	Volume	Avg. Daily	
		Trading Volume		Trading Volume	Trading Volume		
2001	3,519	586	1,618	270	5,137	856	
2002	883,425	3,562	683,021	2,754	1,566,446	6,316	
2003	12,244,366	49,174	9,475,715	38,055	21,720,083	87,229	
2004	25,115,528	100,462	18,708,983	74,835	43,824,511	175,298	
2005	45,636,960	184,765	34,459,546	139,512	80,096,506	324,277	

Table 1 Market volume and daily mean volume of the TXO

Notes: Avg. Daily Trading Volume expresses average daily trading volume which is the ratio of volume to number of trading days. The numbers of trading days during the period from 2001 to 2005 are 6-, 248-, 249-, 200-, and 247-day, respectively.

Н	Ν	Statistics	RV	BS-IV	MF-IV	HV	GARCH(1,1)
H1	48	Mean	0.2096	0.2266	0.2212	0.2120	0.2254
		Std. Dev.	0.0729	0.0722	0.0615	0.0809	0.0676
		Maximum	0.4240	0.3773	0.3401	0.3909	0.3568
		Minimum	0.1160	0.1043	0.1157	0.0980	0.1194
H2	48	Mean	0.2014	0.2418	0.2212	0.2173	0.2291
		Std. Dev.	0.0675	0.0792	0.0653	0.0871	0.0719
		Maximum	0.3750	0.4119	0.3518	0.4110	0.4021
		Minimum	0.1023	0.1096	0.1142	0.0959	0.1165
H3	48	Mean	0.2043	0.2336	0.2100	0.2177	0.2300
		Std. Dev.	0.0680	0.0707	0.0530	0.0928	0.0761
		Maximum	0.3725	0.4151	0.3123	0.4518	0.4035
		Minimum	0.1066	0.1268	0.1282	0.0951	0.1152
H4	47	Mean	0.2063	0.2402	0.2090	0.2237	0.2396
		Std. Dev.	0.0658	0.0745	0.0590	0.0891	0.0819
		Maximum	0.3395	0.4375	0.3536	0.4540	0.4298
		Minimum	0.1025	0.1290	0.1274	0.0990	0.1160

Table 2Summary statistics of various volatility measures

Notes: Forecast horizons are based on the remaining time to maturity days of the TXO, which include a weekly (H1), bi-weekly (H2), tri-weekly (H3), and monthly (H4) forecast horizons.

Н	Obs.	Forecast Error	BS-IV	MF-IV	HV	GARCH(1,1)
H1	48	MAE	0.0406 (3)	0.0381 (1)	0.0434 (4)	0.0403 (2)
		RMSE	0.0534 (3)	0.0498 (1)	0.0596 (4)	0.0527 (2)
H2	48	MAE	0.0524 (4)	0.0404 (1)	0.0426 (3)	0.0421 (2)
		RMSE	0.0642 (4)	0.0485 (1)	0.0597 (3)	0.0539 (2)
H3	48	MAE	0.0441 (3)	0.0386 (1)	0.0487 (4)	0.0417 (2)
		RMSE	0.0529 (2)	0.0476 (1)	0.0673 (4)	0.0573 (3)
H4	47	MAE	0.0442 (4)	0.0367 (1)	0.0385 (2)	0.0421 (3)
		RMSE	0.0575 (3)	0.0468 (1)	0.0565 (2)	0.0587 (4)

 Table 3
 Forecast errors of various forecast methods

Notes: Forecast horizons (H) are based on time to maturity days of the TXO, which include a weekly (H1), bi-weekly (H2), tri-weekly (H3), and monthly (H4) forecast horizons.

Period	â	$\hat{oldsymbol{eta}}$	R^2
Whole Period	1.5490 (4.1578)**	-0.0031 (-0.8256)	0.0041
Period 1 (2002-2003)	2.3370 (3.6158)**	-0.0105 (-1.7175)	0.0421
Period 2 (2004-2005)	0.7585 (5.7777)**	0.0141 (6.6624)**	0.1541

Table 4 Market efficiency and the performance of the MF-IV

Note: The univariate regression model in equation (11) is estimated by using GMM approach. Figures in parentheses are t-values. The reported t-values are corrected for heteroskedasticity and serial correlations using the Newey and West (1987) variance-covariance estimator.

** indicates that the test statistics are significant at the 1 % level.

Н	Model	â	$\hat{oldsymbol{eta}}$	R^2	$\chi^{2 a}$
H1	BS-IV	0.0376 (1.7776)	0.7588 (7.6311)**	0.5644	8.5412*
	MF-IV	0.0136 (0.5477)	0.8860 (7.4925)**	0.5590	2.8251
	HV	0.0761 (4.1784)**	0.6293 (7.3865)**	0.4883	19.0298**
	GARCH (1,1)	0.0296 (1.5049)	0.7986 (8.5853)**	0.5486	7.4974*
H2	BS-IV	0.0416 (2.7754)**	0.6607 (11.7369)**	0.6006	66.1576**
	MF-IV	0.0246 (1.3835)	0.7995 (11.4818)**	0.5975	15.8949**
	HV	0.0759 (4.4196)**	0.5774 (8.8635)**	0.5552	58.0923**
	GARCH (1,1)	0.0341 (1.8450)	0.7300 (9.5606)**	0.6036	30.5079**
H3	BS-IV	0.0259 (1.4811)	0.7638 (11.4177)**	0.6310	32.5041**
	MF-IV	0.0117 (0.4358)	0.9175 (7.9081)**	0.5109	1.1665
	HV	0.0932 (3.7263)**	0.5106 (4.9940)**	0.4852	31.0156**
	GARCH (1,1)	0.0508 (1.9069)	0.6675 (5.9075)**	0.5573	20.9703**
H4	BS-IV	0.0402 (1.6657)	0.6916 (6.7456)**	0.6139	25.3664**
	MF-IV	0.0387 (1.3042)	0.8017 (6.1523)**	0.5172	2.6382
	HV	0.0749 (3.8903)**	0.5870 (7.2939)**	0.6320	34.7466**
	GARCH (1,1)	0.0518 (2.3600)*	0.6445 (6.9978)**	0.6446	40.2663**

Table 5 Results of univariate regression

Note: ^a The univariate regression model in equation (8) is estimated by using GMM approach. χ^2 is the Wald test statistic of the null hypothesis, $H_0:(\alpha,\beta)=(0,1)$. Figures in parentheses are t-values. The reported t-values are corrected for heteroskedasticity and serial correlations by using the Newey and West (1987) variance-covariance estimator.

** and * denote statistical significance at the 1% and 5% levels, respectively.

Panel A: BS-IV and HV								
Н	\hat{lpha}	$\hat{eta}_1^{\scriptscriptstyle BS}$	$\hat{oldsymbol{eta}}_2^{HV}$	\overline{R}^2	χ^{2} (BS) a	χ^{2} (HV) $^{\mathrm{b}}$		
H1	0.0375	0.5580	0.2151	0.5633	11.1993**			
	(1.8166)	(2.0676)*	(0.9479)					
H2	0.0457	0.4590	0.2058	0.5980	83.8110**			
	(3.1699)**	(3.4870)**	(1.5867)					
H3	0.0229	0.8259	-0.0528	0.6156	40.0010**			
	(1.1294)	(3.2649)**	(-0.2630)					
H4	0.0530	0.3113	0.3509	0.6383		4.5273**		
	(2.1524)*	(1.5474)	(2.2766)*					
H4A ^d	0.0366	0.3641	0.3775	0.6813		84.5886**		
	(1.9830)	(1.8961)	(2.5293)*					
Panel B: B	S-IV and GARC	H (1,1)						
Н	\hat{lpha}	$\hat{eta}_1^{\scriptscriptstyle BS}$	$\hat{eta}_2^{\scriptscriptstyle GARCH}$	\overline{R}^2	χ^{2} (BS)	χ^{2} (GARCH) ^c		
H1	0.0176	0.4431	0.4062	0.5912	10.4335*			
	(0.8453)	(1.9683)	(1.8497)					
H2	0.0290	0.3405	0.3930	0.6182	80.3460**			
	(1.7577)	(2.0800)*	(1.9018)					
H3	0.0255	0.6303	0.1373	0.6190	32.0768**			
	(1.5059)	(2.0283)*	(0.4296)					
H4	0.0394	0.2825	0.4131	0.6487		36.5264**		
	(1.7219)	(1.5000)	(2.3813)*					
H4A ^e	0.0237	0.3649	0.4046	0.6832	43.6744**	59.6205**		
	(1.3809)	(2.0173)*	(2.3988)*					

Table 6 Results of encompassing regression of informational efficiency for the **BS-IV**

Note:

Note: ^a The encompassing regression model in equation (9) is estimated by using GMM approach. $\chi^2_{(BS)}$ is the Wald test statistic of the null hypothesis, $H_0: \beta_1^{BS} = 1$ and $\beta_2^{FV1} = 0$ (FV1 = HV, GARCH). ^b $\chi^2_{(HV)}$ is the Wald test statistic of the null hypothesis, $H_0: \beta_1^{BS} = 0$ and $\beta_2^{HV} = 1$. ^c $\chi^2_{(GARCH)}$ is the Wald test statistic of the null hypothesis, $H_0: \beta_1^{BS} = 0$ and $\beta_2^{GARCH} = 1$. ^d The data on H4A is the same as the sample on H4 but deleting the data on 20 May, 2004. In other words, there

are only 46 observations on H4A.

Figures in parentheses are t-values. The reported t-values are corrected for heteroskedasticity and serial correlations using the Newey and West (1987) variance-covariance estimator.

** and * denote statistical significance at the 1% and 5% levels, respectively.

Panel A: MF-IV and HV								
Н	\hat{lpha}	$\hat{oldsymbol{eta}}_1^{MF}$	$\hat{oldsymbol{eta}}_2^{\scriptscriptstyle HV}$	\overline{R}^2	χ^{2} (MF) a	χ^{2} (HV) $^{\mathrm{b}}$		
H1	0.0203	0.6383	0.2269	0.5601	3.5106			
	(0.7636)	(2.1303)*	(1.0373)					
H2	0.0340	0.5373	0.2233	0.5992	22.0522**			
	(2.0429)*	(2.9610)**	(1.4697)					
H3	0.0331	0.5639	0.2430	0.5246	3.1033			
	(1.1652)	(1.9039)	(1.1969)					
H4	0.0725	0.0321	0.5679	0.6154		43.3008**		
	(2.4719)*	(0.1647)	(5.4806)**					
H4A ^d	0.0591	0.0558	0.6145	0.6501		73.4311**		
	(2.3804)*	(0.3093)	(6.5574)**					
Panel B: N	AF-IV and GAR	CH (1,1)						
Н	â	$\hat{oldsymbol{eta}}_1^{MF}$	$\hat{oldsymbol{eta}}_2^{ extsf{GARCH}}$	\overline{R}^2	χ^{2} (MF)	$\chi^{2}_{(GARCH)}$ c		
H1	0.0035	0.5065	0.4173	0.5887	5.3358			
	(0.1470)	(2.0371)*	(2.0025)					
H2	0.0201	0.4036	0.4018	0.6175	23.3168**			
	(1.2432)	(2.0940)*	(1.9540)					
H3	0.0217	0.3845	0.4431	0.5655	5.3866			
	(0.9456)	(1.3731)	(1.8662)					
H4	0.0501	0.0314	0.6244	0.6287		45 6286**		
	(1.8631)	(0.1595)	(4.7435)**			1310200		
H4A ^d	0.0369	0.0859	0.6390	0.6515		50 0438**		
	(1.5557)	(0.4817)	(5.2481)**			2010120		

Table 7 Results of encompassing regression of informational efficiency for the MF-IV

Note: ^a The encompassing regression model in equation (10) is estimated by using GMM approach. $\chi^2_{(MF)}$ is the Wald test statistic of the null hypothesis, $H_0: \beta_1^{MF} = 1$ and $\beta_2^{FV_1} = 0$ (FV1 = HV, GARCH). $\chi^{(HV)}_{(HV)}$ is the Wald test statistic of the null hypothesis, $H_0: \beta_1^{MF} = 0$ and $\beta_2^{HV} = 1$. $\chi^{(2RCH)}_{(GARCH)}$ is the Wald test statistic of the null hypothesis, $H_0: \beta_1^{MF} = 0$ and $\beta_2^{GARCH} = 1$. The data on H4A is the same as the sample on H4 but deleting the data on 20 May, 2004. In other words,

there are only 46 observations on H4A.

Figures in parentheses are t-values. The reported t-values are corrected for heteroskedasticity and serial correlations using the Newey and West (1987) variance-covariance estimator.

** and * denote statistical significance at the 1% and 5% levels, respectively.

Appendix A

	forecast horizon								
DATE	Month	RV	BS-IV	MF-IV	HV	GARCH	RAEIV	Spread	
20020117	2002-02	0.2948	0.3762	0.2901	0.3255	0.2930	0.0580	32.2500	
20020221	2002-03	0.2842	0.3166	0.2498	0.2797	0.2873	1.0643	93.7500	
20020321	2002-04	0.2263	0.2649	0.2098	0.2799	0.2824	0.4284	30.1667	
20020418	2002-05	0.2601	0.2287	0.1398	0.1793	0.2424	3.8261	25.7143	
20020523	2002-06	0.2539	0.2723	0.2505	0.3287	0.3314	0.1894	18.6133	
20020620	2002-07	0.3016	0.2969	0.2559	0.2553	0.2769	9.7377	13.6267	
20020725	2002-08	0.3163	0.3032	0.3011	0.3183	0.3134	1.1597	43.3667	
20020822	2002-09	0.2289	0.3178	0.2952	0.3451	0.2629	0.7457	25.1045	
20020919	2002-10	0.3068	0.3180	0.2767	0.2983	0.3693	2.6905	9.5857	
20021024	2002-11	0.2798	0.3286	0.3063	0.3764	0.4153	0.5426	51.7333	
20021121	2002-12	0.2424	0.3168	0.2841	0.2646	0.2865	0.5612	19.8286	
20021219	2003-01	0.2414	0.2289	0.2016	0.1984	0.2328	3.1914	19.8769	
20030116	2003-02	0.2666	0.2079	0.1954	0.2305	0.2643	1.2137	60.4167	
20030220	2003-03	0.2842	0.2776	0.2479	0.2975	0.3303	5.5175	74.1739	
20030320	2003-04	0.1956	0.3296	0.2827	0.2865	0.3261	0.6500	14.5000	
20030423	2003-05	0.2485	0.2437	0.2074	0.2507	0.2766	8.6688	8.4000	
20030522	2003-06	0.2354	0.2433	0.2185	0.2658	0.2433	2.1078	7.1105	
20030619	2003-07	0.2195	0.2752	0.1875	0.2158	0.2466	0.5736	36.3571	
20030724	2003-08	0.2003	0.2494	0.2069	0.2436	0.2537	0.1334	64.8421	
20030821	2003-09	0.1893	0.2568	0.1978	0.1979	0.2427	0.1247	87.9500	
20030918	2003-10	0.1669	0.2031	0.1742	0.1711	0.2232	0.2012	16.7727	
20031023	2003-11	0.1715	0.1915	0.1711	0.1643	0.2005	0.0200	18.6250	
20031120	2003-12	0.1605	0.2008	0.1803	0.1528	0.1886	0.4908	251.0769	
20031222	2004-01	0.1555	0.1467	0.1435	0.1666	0.1785	1.3666	2.5100	
20040128	2004-02	0.1696	0.1648	0.1524	0.1461	0.1663	3.6258	19.5417	
20040219	2004-03	0.2068	0.1778	0.1432	0.1532	0.1549	2.1923	102.0800	
20040325	2004-04	0.2177	0.2826	0.2781	0.3452	0.3658	0.9310	6.6235	
20040422	2004-05	0.3376	0.2841	0.2311	0.2584	0.2563	1.9895	22.0000	
20040520	2004-06	0.2616	0.4375	0.3536	0.4540	0.4298	0.5229	16.6500	
20040624	2004-07	0.2044	0.3279	0.2752	0.3202	0.3307	0.5735	6.0625	
20040722	2004-08	0.1735	0.2885	0.2511	0.2029	0.2568	0.6744	2.3500	
20040819	2004-09	0.1723	0.2514	0.2263	0.1829	0.1921	0.6826	5.8045	
20040922	2004-10	0.1626	0.1976	0.1768	0.1670	0.1914	0.4068	14.6250	
20041021	2004-11	0.1650	0.2135	0.2011	0.1640	0.1886	0.7436	20.5000	
20041118	2004-12	0.1667	0.2278	0.2088	0.1608	0.1848	0.6887	16.4640	
20041223	2005-01	0.1371	0.1870	0.1689	0.1092	0.1546	0.6360	16.9071	
20050217	2005-03	0.1262	0.1390	0.1313	0.1301	0.1482	0.4009	29.7053	
20050323	2005-04	0.1358	0.1290	0.1276	0.1036	0.1321	1.2076	4.5133	
20050421	2005-05	0.1245	0.1587	0.1538	0.1584	0.1800	0.8566	2.4313	
20050519	2005-06	0.1025	0.1471	0.1415	0.1251	0.1465	0.8747	4.4941	
20050622	2005-07	0.1208	0.1472	0.1340	0.1128	0.1219	0.4996	12.7059	
20050721	2005-08	0.1191	0.1685	0.1590	0.0991	0.1160	0.8072	15.9824	
20050824	2005-09	0.1137	0.1304	0.1274	0.1269	0.1270	0.8210	11.6889	
20050922	2005-10	0.1300	0.1513	0.1472	0.1253	0.1268	0.8040	2.9000	
20051020	2005-11	0.1517	0.1453	0.1444	0.1752	0.1761	1.1422	6.1625	
20051124	2005-12	0.1253	0.1680	0.1528	0.1637	0.1601	0.6437	16.6316	

Table A-1 Realized volatility and forecast volatility of various models on monthly (H4)