The Structural Agency Problem under Credit Risk

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Abstract

This paper examines the agency problem caused by credit risk. Different from the literature, a new kind of agency problem, structural agency problem under credit risk, is investigated, and a compound option pricing method is applied to quantify the agency cost in this study. Under the specific setting of this model, the shareholders will take advantage of the debt holders by taking more risky projects under a situation where the firm cannot issue new equity or debt and yet is capable of making debt payments. Even under no asymmetric information between shareholders and debt holders, the agency problem exists since it results from a structural difference under credit risk. In this case, applying Geske's compound option pricing model (1977 & 1979)[1] is able to quantify the magnitude of structural agency cost under credit risk in a multi-period setting. Several possible solutions are also proposed to alleviate this agency problem. Finally, a case study of Lucent Technologies Inc. is provided to demonstrate the existence of the structural agency problem under credit risk.

[1] A correction note, Geske and Johnson (1985), is made for Geske (1977).

1 Introduction

This paper examines the agency problem caused by credit risk. In particular, it looks at the problem where the debt holders are taken advantage of by the shareholders in the situation where the firm should have been default but the shareholders still have the control of the firm.

The dazzling observation by Black and Scholes (1973) and Merton (1974) that the equity resembles a call option is widely applied in the finance literature. When debts are issued, equity holders are effectively selling the assets of the firm to the debt holders in return for cash and a call option. When the debts are due (so is the call option), the equity holders will face the choice whether to buy the assets back from the debt holders (whether to exercise the call option) and will do so only if the value of the assets exceed the redemption value of the debts. Under the situation where the equity holders are unwilling or unable to redeem the assets of the firm, the debt holders must takeover the firm. This method lays also the foundation of the structural credit risk models, such as all kinds of default barrier models and compound option models.

The implication of option pricing method for capital structure is the debt holder wealth expropriation hypothesis. If the equity is thought as a call option, the optimal choice for the equity holders is to support as many risky projects as possible no matter whether the NPV of the projects is positive or not since the option value is positively correlated with the variance of returns of firm assets. Obviously, this is not of the debt holders' interest and an agency problem emerges from this framework. Under no asymmetric information, the debt holders are fully informed of this agency problem and consequently will lower their loan to the equity holders at the time of debt issuance.

Barnea, Haugen, and Senbet (1980) demonstrate that it is then *not* at the best interest of the equity holders to take on risky projects. They conclude that the agency problem will only exist under asymmetric information. However, this result, as we shall see shortly, will not hold in a multi-period setup.

Under perfect market assumptions, financing choices should have no impact on the value of the firm, as so suggested by Modigliani and Miller (1958) and generalized by Stiglitz (1974). However, in reality, the irrelevancy theorem seems unable to consistently explain the complicated capital structures of the current firms. Therefore, by relaxing the perfect market assumptions, various expanded theorems are offered to explain the determinants of the optimal capital structure. In the finance literature, agency problems are firstly raised and discussed by Jensen and Meckling (1976). Seeing current corporate structures as "nexuses of contracts", Jensen and Meckling (1976) expand the original framework of Modigliani and Miller (1958), who argue that financing choices should have no impact on the value of the firm under perfect markets, by introducing the incentive problems of members of the firm and argued the existence of optimal capital structure when the firm minimizes the total agency costs of the firm (trading-off between the agency costs of outside equity and the agency costs of outside debt).

In the literature, two well known agency problems can be applied in the situation when the firm is facing financial distress. The asset substitution problem is first raised by Jensen and Meckling (1976). It mainly describes the situation that when default is very likely to happen, shareholders will have nothing to lose and will tend to pursue extremely risky but not necessarily positive NPV investment projects. The under-investment problem is another kind of agents' incentive problems associated with leverage originally described by Myers (1977). This kind of problems happen when the shareholders have incentive to reject the projects which are beneficial to both debt holders and the whole company by using the equity holders' stakes or just not beneficial to the equity holders.

In order to mitigate the agency problems with financial distress, various methods are suggested by previous papers. Secured debt, debt is collateralized by tangible assets of the firm, is one of the methods suggested by Scott (1976) and Stulz and Johnson (1985). Smith and Warner (1979), Asquish and Wizman (1990), Crabbe (1991), and Bae, Klein, and Padmaraj (1994) show that making protective bond covenants can also be an efficient method to avoid some strategic actions from shareholders (managers). Moreover, Myers (1977) indicates that debt maturity choice can mitigate the under-investment problems. The equity holders can pay off the debt holders' fixed claim and obtain all of the benefits of the project by funding it themselves with debt that matures before investment opportunities expire. Barnea, Haugen, and Senbet (1980) also demonstrate short-term debts and callable debts allow firms to minimize the agency costs of debt that result from information asymmetry, managerial risk incentives and foregone growth opportunities resulting from Myers' under-investment problems. John and Nachman (1985) find that a "reputation" effect plays an important role to mitigate conflict between shareholders and debt holders in a dynamic setting.

This paper starts the investigation of agency problems under credit risk under a multi-period setting. Such kind of agency problem occurs when the firm should have been default but the shareholders still have the control of the firm. The reason is that a difference exists between the definition of default in the real-world structure form model and that in the Geske (1977) model. The definition of default in the Geske (1977) model is assumed the “correct” one in this paper because it fits the spirit of fairness of security design: the party that holds residual rights should bear its own investment/operation risks. This is why this kind of agency problem is named “the structural agency problem under credit risk”. Different from the single-period agency problems, even under no information asymmetry, the structural agency problem under credit risk still exists. Moreover, application of Geske’s (1977) compound option model allows us to calculate the magnitude of the agency cost and quantify the agency cost in a meaningful way. Several possible solutions are also proposed to alleviate this agency problem. Finally, a case study of Lucent Technologies Inc. is provided to demonstrate the existence of the structural agency problem under credit risk.

This paper is organized as following: Section 2 is the literature review about credit risk models and agency problems. Explanation of the structural agency problem under credit risk, the model to quantify the agency cost, and resolution for the agency problem are in section 3. Section 4 provides a case study of Lucent Technologies Inc. as an empirical demonstration of our model. Section 5 is the conclusion and the plan for future research.

2 Literature Review

2.1 Credit Risk Models

2.1.1 Structural Models vs. Reduced Form Models

Models for credit risk can be roughly divided into two very different categories: reduced form[2] and structural models. The differences can be seen in the following table:

	structural models		reduced form models
	compound option model	barrier option model	
default condition	equity value below coupon	asset value below an arbitrary boundary	jump
recovery	asset value (random)	arbitrarily given	arbitrarily given
data used	equity/bond	equity/bond	bond
data type	fundamental/market	fundamental/market	market
application	default prediction	default prediction	derivative pricing
default as a surprise	no	no	yes

Generally speaking, structural models are used for default prediction that focuses on equity prices while reduced-form models are used for credit derivative pricing that focuses on debt values. In structural models, default is like a stopping time for a continuous process. However, default in reduced-form models is more like a jump process for a continuous process. Moreover, structural models of bond pricing are equilibrium models that derive this property of default as part of the model. On the other hand, reduced-form models assign probabilities of default and recovery rates

[2] Since reduced form models are not the focus in this paper, more information can be obtained from Jarrow and Turnbull (1995),Das and Tufano (1996), Jarrow, Lando, and Turnbull (1997), Lando (1997,1998), Madan and Unal (1998), Duffie and Singleton (1999), and Jarrow (2001).

exogenously. Due to these exogenous properties which are important for pricing credit derivatives, reduced-form models are more computationally efficient. Nevertheless, structural models give us a complete sight when the credit risk is seen from the interaction of distinct securities of a firm. More specifically, since default results from the firm value being too low, the structural models must specify the exact conditions defining "too low". Hence the default boundary can be decided either exogenously or endogenously. Models of default in the reduced form setting tend to differ on how they specify the process for default time and how they model the recovery rate. Due to the computation-oriented character, default boundaries in reduced form models are always determined exogenously. Since the agency problem in this paper is an incentive problem results from the distinct structures of default, it is the structural models of credit risk that we should put all focuses on.

2.1.2 The Original Structural Model: The Black-Scholes-Merton Model

The structure of the structural models for default is the recognition of the share holders of the company as call option holders who have the residual claim of the company. As a result, any call option (written on the asset value) model is a structural model. This revolutionary idea (as opposed to shareholders are owners of the company; shareholders are now the agents for debt holders) was first given by Black and Scholes (1973) and extended by Merton (1974). In the Black-Scholes-Merton framework, at the maturity of the debt, the debt owners either receive the full redemption (under which the company survives and is handed over to the equity owners) or receive the assets as the recovery (under which the company defaults and the equity owners receive nothing). The whole configuration can be seen more clearly in the following derivation.

Assume the firm only have a zero coupon debt with face value K outstanding and the asset value of the firm, V, follows the process

$$\frac{dV}{V} = \mu dt + \sigma dw$$

where μ is the instantaneous expected rate of return, σ is the instantaneous standard deviation of the asset value, and dw is the increment of a standard Wiener process. At the maturity day of the debt, the value of the debt, D, is either K if the firm is solvent or V if the firm is default. That is,

$$D = Min(V, K).$$

Since asset value of the firm is the sum of equity value and debt value, equity value, E, can be simply derived as:

$$E = A - D = A - Min(A, K) = Max(0, A - K)$$

which is exactly a call option formation. Therefore, using the no-arbitrage method, the partial differential equation (PDE) of the original Black-Scholes-Merton call option can be obtained,

$$\frac{1}{2}\sigma^2 V^2 E_{vv} + rVE_v + E_t - rE = 0,$$

and a unique solution can also be obtained under specific boundary conditions. This is the originality of Black-Scholes-Merton that applies the option pricing method to the default prediction work.

While theoretically desirable, the original Black-Scholes-Merton model is limited in two ways:

(1) It permits only a single debt that does not pay any coupon.

(2) Default can occur only at the maturity of the debt.

As a result of the limitation of the model, researchers extend the model to account for more flexible default.

Actually, Merton (1974) does modify the original model to overcome the first limitation above. The model starts from assuming the value of the firm, V, follows the process

$$dV = (\mu V - C)dt + \sigma V dw$$

where C is the dollar payout by the firm per unit time to either its shareholders (dividend) or debt holders (coupon). Taking both cash inflows and outflows of all securities into consideration, Merton (1974) derives a well known PDE for security F,

$$\frac{1}{2}\sigma^2 V^2 F_{vv} + (rV - C)F_v + F_t - rF + C_y = 0,$$

where C_y is the dollar payout per unit of time to this security. Merton (1974) claims that this PDE applies to the valuation of any security which is based on the asset value of a firm. Because of the extensiveness of this PDE, it is widely used in academy thereafter.

2.1.3 Barrier Option Models

The barrier option models of credit risk are raised by researchers to overcome the second limitation of the original Black-Sholes-Merton model, i.e. default can occur only at the maturity of the debt. Actually, default may happen before the maturity of the debt in many cases in reality. A good example is that instead of letting shareholders totally exhaust the value of the firm asset, debt holders may make safe covenants to protect themselves. These safe covenants can prevent the asset value from dropping out of some critical level by making an early default. In this case, valuation of equity is just like valuation of a barrier option (a down-and out call option). Since the barrier option models describe many observed patterns in reality well, the literature in this area is voluminous.

I. The Exponential Barrier Model: Black and Cox Model

The first paper in this area was by Black and Cox (1976) who propose a barrier option model for default. Instead of only one point at maturity that divides the state space to default and survival, Black and Cox (1976) assume a continuous barrier function over time. Moreover, for the ease of implementation, they further propose the barrier to be an exponentially rising function over time, $C_b e^{-r(T-t)}$, where C_b is an arbitrary number decided exogenously in the safe covenant. Default is defined as the asset value falling below the defined barrier. This model also considers the dividend payout by assuming the dividend payout is proportional to the value of the firm and the value of asset follows the process

$$dV = (\mu - C_d)Vdt + \sigma V dw$$

where $C_d V$ is the total dividend payout. Applying the general PDE of securities from Merton (1974), the PDE of the debt D is

$$\frac{1}{2}\sigma^2 V^2 D_{vv} + (r - C_d)VD_v + D_t - rD = 0$$

and the PDE for the equity E is

$$\frac{1}{2}\sigma^2 V^2 E_{vv} + (r - C_d)VE_v + E_t - rE + C_d V = 0\ .$$

With the boundary conditions under their assumptions, Black and Cox (1976) are able to derive a closed-form valuation formula for the bond with safe covenants. In this model, the valuation formula in their paper holds when

$$C_b e^{-r(T-t)} \leq P e^{-r(T-t)}$$

where P is the face value of the debt. This constraint is much reasonable for an exogenous default barrier setup because debt holders are always not recovered more than 100% (if the barrier is set to be or above the risk free present value of the face value of the debt) when default occurs in reality. In their same paper, Black and Cox (1976) further discuss the subordinated bonds and the effect of restrictions on the financing of interest and dividend payment.

II. Stochastic Interest Rate Models

In addition to the Black-Cox exponential barrier model (1976), Kim, Ramaswamy, and Sundaresan (1993) and Longstaff and Schwartz (1995) propose flat default barrier models with random interest rates. These models are an improvement over the Black-Cox model because it is possible to include interest rate risk in bonds.

Kim, Ramaswamy, and Sundaresan (1993) assume the asset value follows the process

$$dV = (\mu - \gamma)Vdt + \sigma_1 V dw_1$$

and use the Cox, Ingersoll, and Ross (1985) term structure model (CIR mode) for the interest rate process

$$dr = \kappa(\vartheta - r)dt + \sigma_2 \sqrt{r} dw_2$$

where γV is the net cash outflow, σ_1 is the instantaneous standard deviation of the asset value, ϑ is the long-run mean rate of interest, κ is the speed with which the interest rate r approaches the long-run mean rate and the instantaneous variance of change in r is proportional to its level. w_1 and w_2 are standard Wiener processes. The instantaneous correlation between w_1 and w_2 is ρ. Kim, Ramaswamy, and Sundaresan (1993) also assume that shareholders are not allowed to sell the assets of the firm and the debt holders have priority and must be paid the continuous coupon c. Therefore, this model has a flat default barrier $\frac{c}{\gamma}$. Given all the assumptions in their paper, Kim, Ramaswamy, and Sundaresan (1993) derive the following PDE for the value of the coupon bond W:

$$\frac{1}{2}\sigma_1^2 V^2 W_{vv} + \rho\sigma_1\sigma_2\sqrt{r}VW_{rv} + \frac{1}{2}\sigma_2^2 rW_{rr} + \kappa(\vartheta - r)W_r + (r-\gamma)VW_v + W_t - rW + c = 0 .$$[3]

Since there is no closed-form solution for the PDE, Kim, Ramaswamy, and Sundaresan (1993) use numerical methods to provide solutions for noncallable bonds and callable bonds.

Longstaff and Schwartz (1995) also develop a barrier option model which incorporates both default risk and interest rate risk. They first assume the asset value of the firm follows the process

$$\frac{dV}{V} = \mu dt + \sigma_1 dw_1$$

and the interest rate follows the process of Vasicek (1977) model

$$dr = (\zeta - \beta r)dt + \sigma_2 dw_2 = \beta(\frac{\zeta}{\beta} - r)dt + \sigma_2 dw_2$$

[3] Kim, Ramaswamy, and Sundaresan (1993) set the market price of interest rate risk to zero.

where $\frac{\zeta}{\beta}$ is the long-run mean rate of interest and β is the speed with which the interest rate r approaches the long-run mean rate. Then they define a constant, K, as the default barrier in the model. Longstaff and Schwartz (1995) also allow for violation of absolute priority in the bankruptcy process. Based on their assumptions, a PDE for a risky zero coupon bond H can be obtained as the following feature:

$$\frac{1}{2}\sigma_1^2 V^2 H_{vv} + \rho\sigma_1\sigma_2 V H_{rv} + \frac{1}{2}\sigma_2^2 H_{rr} + (\alpha - \beta r)H_r + rVH_v + H_t - rH = 0$$

where α represents the sum of the parameter ζ and a constant representing the market price of interest rate risk. Solving this PDE under certain boundary conditions, Longstaff and Schwartz (1995) obtain closed-form solutions for risky fixed rate and floating rate debts.

Although it is a success to incorporate the interest rate risk into a credit risk model, Kim, Ramaswamy, and Sundaresan (1993) and Longstaff and Schwartz (1995) have a common shortcoming: a flat exogenous default barrier (i.e., the recovery remains constant) which is less desirable than the exponential default barrier in Black and Cox (1976). In order to avoid this shortcoming, researchers explored the structural model of credit risk from two directions: deriving an endogenous barrier from the capital structure optimization decision and relaxing the flat default barrier to be stochastic.

III. Endogenous Default Barrier Models (Optimal Capital Structure Models):

Extending the result of Black and Cox (1976), Leland (1994) derives a closed-form solution for a perpetual coupon debt. The reason for using a perpetual coupon debt setup is to construct a time independent stationary debt structure and for ease of solving the optimal leverage problem. Moreover, Leland (1994) assumes that the cash outflows for paying continuous coupons must be financed by raising equity. Because of the character of time independence of stationary debt structure, Leland (1994) is able to assume ex ante a flat default boundary V_B. Then the author modifies the general PDE of Merton (1974) to an ODE for all the factors which affect the decision of optimal capital structure

$$\frac{1}{2}\sigma^2 V^2 F_{vv} + rVF_v - rF + c = 0 .$$

Since this ODE has a closed-form solution, solving this ODE with different boundary conditions for the debt $D(V)$, tax benefits $TB(V)$, and bankrupt costs $BC(V)$, equity value $E(V)$ can be obtained from the following equation

$$E(V) = V + TB(V) - BC(V) - D(V) .$$

Under a smooth-pasting condition, $dE / dV\big|_{V=V_B} = 0$, the endogenous default condition V_B is solved. Substituting V_B into the valuation equations of the factors, the optimal equity value and debt value for the firm can be known. We should notice that although the default boundary in Leland (1994) is also flat, its character is totally different from an exogenously defined flat barrier. As shown in the derivation process, the barrier is derived endogenously from an optimal leverage decision. Its flatness results from the specifically designed stationary debt structure.

Leland and Toft (1996) further extend the Leland (1994) model and include more decision factors into their model, such as dividend and maturity of debt. They start from a finite maturity debt with continuous coupon and a flat default barrier, V_B, which is presumed ex ante. Then they set up a stationary debt structure again by issuing and retiring the same amount of debt simultaneously and integrate all the outstanding debts in a certain period. By using the same problem solving technique as that of Leland (1994), Leland and Toft (1996) obtain an endogenous default barrier V_B which has a more complicated feature than that of Leland (1994) due to the increase of decision factors. Later on, Leland (1998) shows what kind of role an endogenous default plays in a dynamic model that incorporates both optimal leverage and investment risk (asset substitution problem of Jensen and Meckling (1976)) issues. Lately, Huang, Ju, and Ou-Yang (2003) consider the Leland and Toft (1996) debt structure in the Longsatff and Schwartz (1995) setting, i.e. the Leland and Toft (1996) with a stochastic interest rate of Vasicek (1977) model.

Different from other barrier option models, the models in this area emphasize more on the optimal capital structure decision than on the default prediction problem. However, since these models apply the option valuation method of Black-Sholes-Merton, the default condition can be determined endogenously as part of the optimal capital structure decision making model.

IV. Stochastic Default Barrier Models

In order to overcome the limitation of an exogenous flat barrier, several papers, such as Nielsen, Saa-Requejo, and Santa-Clara (1993), Briys and de Varenne (1997), Schobel (1999), and Collin-Dufresne and Goldstein (2001), relax their default barriers to be stochastic. In most of these models, the stochastic default boundaries result from the assumption of stochastic interest rate. For example, Briys and de Varenne (1997) assume the interest rate follows the process of Vasicek (1977) model

$$dr = a(t)\left[b(t) - r\right]dt + \sigma(t)dw .$$

This assumption leads the return of the risk free zero coupon bond $P(t,T)$ which matures at T follows a geometric Brownian motion

$$\frac{dP(t,T)}{P(t,T)} = rdt - \sigma_P(t,T)dw$$

where $\sigma_P(t,T) = \sigma(t) \cdot \int_t^T \exp\left(-\int_t^u a(s)ds\right)du$.

Then Briys and de Varenne (1997) exogenously define their default boundary

$$v(t) = \alpha \cdot F \cdot P(t,T)$$

where F is the face value of the corporate bond and $0 \le \alpha \le 1$. Obviously, $v(t)$ is stochastic because of $P(t,T)$. Briys and de Varenne (1997) argue that the design of a stochastic default barrier like theirs can overcome the defect arises from the flat default barrier in Longstaff and Schwartz (1995) model.

Later on, Collin-Dufresne and Goldstein (2001) also argue that the Longstaff and Schwartz (1995) model is an approximation because Longstaff and Schwartz (1995) use a univariate barrier as opposed to a bivariate barrier. The barrier in Longstaff and Schwartz (1995) should be bivariate because both asset value and the interest rate are assumed stochastic. In addition to providing the exact valuation algorithm, Collin-Dufresne and Goldstein (2001) also propose a more realistic default barrier that is stochastic consistent with stationary leverage ratios. Furthermore, to overcome the perfectly predicted default, Zhou (2001) adds jumps to the asset price process.

2.1.4 Compound Option Models

Geske (1977) takes a completely different approach. Geske (1977) argues that default cannot occur continuously. Without the cash payment pressure, companies need not worry about default. Hence, companies will only face default when they have to pay coupons to their debt owners. This realistic assumption matches with the compound option model Geske (1979) developed. Based on the argument of Black-Scholes-Merton that equity is a call option, Geske (1977) extends the argument to argue that equity is a compound call. Each time when the company faces a coupon payment, its shareholders consider if it is worthwhile to pay the coupon. The coupon is paid only if the company has a positive value to the shareholders; or the shareholders will not pay the coupon and default. The no-arbitrage default condition is at the point where the company can raise new equity. If the company cannot raise equity (i.e. the company has negative equity value) to pay of its coupon, then the company is in default. As a result, there is an implied default barrier for the asset value. Geske (1977) proves that this barrier is identical to the market value of all debts.[4] As a result, the Geske (1977) model inherits the spirit of the Black-Scholes-Merton model in that both recovery and default barrier are endogenously determined, a model that maintains the most structure. Chen (2003) recently extends the Geske (1977) model to incorporate random interest rates that lead to the Vasicek (1977) model of the term structure.

2.1.5 Implementations of Structural Models

Several papers have implemented the structural models on risky debts pricing. Wei and Guo (1997) make an empirical comparison of Merton (1974) and Longstaff and

[4] This is equivalent to the present value (properly discounted) of all future cash flows.

Schwartz (1995) models by using Eurodollars as risky debt and U.S. Treasury Bills as riskfree debt. Their surprising result shows that Merton (1974) model performs better than Longstaff and Schwartz (1995) model does. Using aggregate time series data for the U.S. corporate bond market, Anderson and Sundaresan (2000) find that the performance of endogenous default barrier models is superior to the original Merton (1974) model. Instead of using aggregate data, Lyden and Saraniti (2000) use the noncallable bond prices of 56 firms to compare Merton (1974) and Longstaff and Schwartz (1995) models find that both models underestimate yield spreads. Recently, Eom, Helwege, and Huang (2004) implement five structural models, Merton (1974), Geske (1977), Longstaff and Schwartz (1995), Leland and Toft, and Collin-Dufresne and Goldstein (2001), and test them against bond data from the Fixed Income database on the last trading day of each December from 1986 to 1997. The result of this paper indicates that all of these models make significant errors in predicting credit spreads: Merton (1974) and Geske (1977) models both underestimate the spreads, while Longstaff and Schwartz (1995), Leland and Toft, and Collin-Dufresne and Goldstein (2001) overestimate the spreads. Different from previous studies which have suggested that structural models generally predict spreads too low, the result of Eom, Helwege, and Huang (2004) exhibits more complicated phenomena.

2.2 Agency Problem

2.2.1 Agency Problems and Optimal Capital Structure under Credit Risk

As a pioneer in capital structure theory, Modigliani and Miller (1958) claim that the market value of any firm is independent of its capital structure under the perfect market assumptions. Later on, Modigliani and Miller (1963) imply an optimal capital structure model with 100% debt by incorporating corporate income taxes. However, apparently, neither of these models is able to properly explain the observed capital structure patterns in the real world. Therefore, several theories are developed to bridge this gap, such as the Agency Cost/Tax Shield Trade-off Models, the Pecking Order Hypothesis Models, and the Signaling Models.[5] Among these theories, the agency cost theory put forward by Jensen and Meckling (1976) is a splendid discovery in the finance literature.

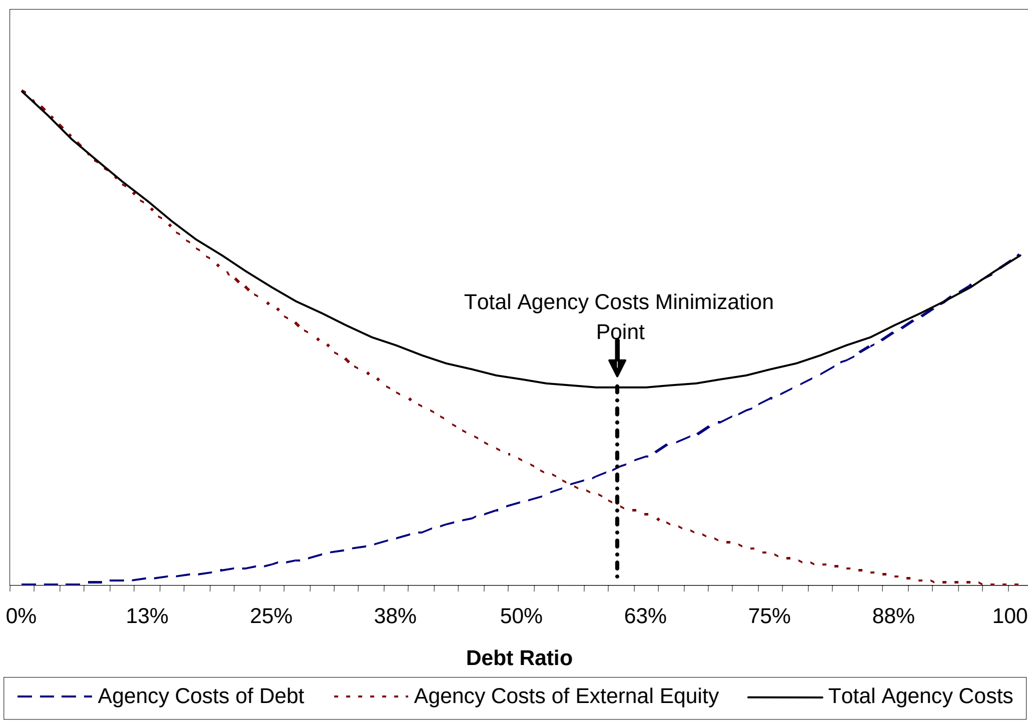
The agency problem is first raised by Jensen and Meckling (1976) to relax the perfect market assumptions in Modigliani and Miller (1958). Seeing current corporate structures as "nexuses of contracts", Jensen and Meckling (1976) observe "incentive problems" behind these nexuses. More precisely, because of the character of modern corporate structures, separation of management (the agent) and ownership (the principal), it is possible that the managers act in their own interest instead of the owners'. Therefore, since these incentive problems result from the agent/principal relationship, they are generally called "agency problems". Costs result from these problems are called "agency costs" , which are more specifically defined as the difference between the value of an actual firm and that of a firm exists in a perfect world where agents' and principals' incentives are aligned.

Occurrences of agency problems are not simply limited to the management/shareholder relationship. They also exist in the shareholder/debt-holder relationship and co-ownership (existing shareholders and outside shareholders). Jensen

[5] About the Peking Order Models, see Myers (1984), Myers & Majiluf (1984), Bruner (1988), Shyam-Sunder and Myers, and Smith and Kim (1994). About the Signaling Models, see Ross (1977), Kim and Maksimovic (1990), Maksimovic and Titman (1991), and Harris and Raviv (1991).

and Meckling (1976) name these agency costs as agency costs of debt and agency costs of external equity respectively in their model. For a firm, the higher proportions of debt (external equity), the higher the agency costs of debt (external equity). However, since relatively higher proportions of debt means relatively lower proportions of equity, an optimal capital structure can be obtained by minimizing total agency costs. The decision making configuration can be seen clearly in the following figure:

Figure1: Agency Costs Minimization Model



This is the original agency cost model introduced by Jensen and Meckling (1976). Afterwards, agency problems are widely discussed and applied in the literature. Generally, agency costs are put together with tax shield and bankruptcy costs to build the famous Agency Cost/Tax Shield Trade-off Theory of corporate capital structure. This theory has the following configuration:

$$
\begin{aligned}
\text{Value of a Levered Firm} = {} & \text{Value of an Unlevered Firm} \\
& + \text{Present Value of Tax Shield} \\
& - \text{Present Value of Bankruptcy Costs} \\
& + \text{Present Value of Agency Costs of Outside Equity} \\
& - \text{Present Value of Agency Costs of Outside Debt}
\end{aligned}
$$

From this structure, we can easily observe that this theory is an integration of Modigliani and Miller (1958, 1963) and Jensen and Meckling (1976) models. Because of its extensiveness on modern corporate structure patterns, this theory thereafter has become the mainstream in the capital structure related literature and is widely applied in structural form of credit risk models.[6]

The optimal capital structure problem under credit risk is a fast growing area. Brennan and Schwartz (1978) are the first to study the capital structure problem under the credit risk framework set up by Black-Scholes-Merton. Different from the general assumptions in the standard structural models, Brennan and Schwartz (1978) assume that the value of a levered firm may be written as a function of the value of an otherwise identical unlevered firm. This paper is concerned mainly with relaxing the assumption that the tax savings due to debt financing constitute a "sure stream" (due to the possibility of default). Under specification of appropriate boundary conditions, Brennan and Schwartz (1978) present a numerical solution yielding the value of the unlevered firm in terms of the value of levered firm, the par value of the outstanding debt (and the interest rate on the debt), and the maturity of the debt.

As mentioned in the earlier section, Leland (1994) introduces a model of optimal capital structure under credit risk with a single perpetual coupon debt. Leland and Toft (1996) consider the Leland (1994) problem under finite debt maturity. In the stationary

[6] Rajan and Zingales (1994) empirically examine the model using the United States and International data and find that the result is consistent with the model prediction.

case, an ex ante, endogenous default boundary can be determined as a function of principal, coupon rate, and maturity by maximizing the equity value. Substituting the boundary back into the bond valuation formula, a closed form expression for the bond value can be obtained. Arguing that Leland (1994) and Leland and Toft (1996) models are limited by the assumption of static capital structure, Leland (1998) derives a dynamic optimal capital structural model which reflects both the tax advantages of debt less default costs (Modigliani and Miller (1963)), and the agency costs resulting from asset substitution problem (Jensen and Meckling (1976)). To fully present the spirit of asset substitution problem, the investment activity of the firm is designed to be able to shift the risk level from low to high. In this extensive model, Leland (1998) shows that although agency costs restrict leverage and debt maturity and increase yield spreads, their importance is small for a range of environment. Recently, Titman and Tsyplakov (2002) also develop a model in which a firm can dynamically adjust its capital structure and investment choice. Huang, Ju, and Ou-Yang (2003) incorporate random interest rates of Vasicek (1977) model in the Leland and Toft (1996) model.

2.2.2 Agency Problems with Financial Distress: Asset Substitution Problem and Underinvestment Problem

Since this paper focuses on the topic of agency problems under credit risk, we are going to further investigate two well-known types of agency problems with financial distress: underinvestment problem and asset substitute problem. In fact, we should notice that these two agency problems occur even when the financial distress is not considered.

However, it is these two agency problems which describe the situation well under the financial distress case.

Before discussing these two agency problems of debts, several assumptions should be made for the firm, FIRM, in the following example: First, assume it is a one period model and there are only two securities in FIRM, equity and debt. Second, there is no agency problem between management/shareholders relationship in FIRM. That is, managers and shareholders have the identical incentive and managers act in shareholders' best interest. Third, managers realize that FIRM is very likely to default, but they are still in control of the company.

(1) Asset substitution problem:

Assume there is a debt payment $60 outstanding but is not mature yet. Managers have to make a choice to invest between the two distinct projects, A and B, which have the same expected payoff. Moreover, assume that cash required for the two investment projects are equivalent and will exhaust total cash of FIRM. Thus, no cash should be reserved after choosing one of the projects. At the end of the period, the payoff of the project is the final amount that shareholders and debt-holders obtain. The payoffs and probabilities under two conditions (successful and unsuccessful) of Project A and Project B are listed in the following table:

	Project A		Project B	
	Unsuccessful	Successful	Unsuccessful	Successful
	Payoff (Probability)	Payoff (Probability)	Payoff (Probability)	Payoff (Probability)
Total Cash Flow	\$0 (80%)	\$250 (20%)	\$50 (50%)	\$50 (50%)
Cash Flow to Shareholders	\$0 (80%)	\$190 (20%)	\$0 (50%)	\$0 (50%)
Cash Flow to Debt-holders	\$0 (80%)	\$60 (20%)	\$50 (50%)	\$50 (50%)

From the table above, since the expected payoff of choosing project A for shareholders is \$38 (\$0 × 80% + \$190 × 20%) which is higher than that of choosing project B, \$0 (\$0 × 50% + \$0 × 50%), managers will definitely choose project A to maximize shareholders' wealth. For shareholders, even if project B is successful, the payoff is still not enough to pay out the debt and they still have to hand the firm over to debt holders eventually when the debt is mature (default happens). However, for debt holders, project B is their preference because choosing project B gives them \$50 for sure (\$50 × 50% + \$50 × 50%) which is higher than the expected payoff of choosing Project A, \$12 (\$0 × 80% + \$60 × 20%). This interest conflict between shareholders and debt holders is called "asset substitution problem".

The asset substitution problem is first raised by Jensen and Meckling (1976). It mainly describes the situation that when default is very likely to happen, shareholders will have nothing to lose and will tend to pursue extremely risky but not necessarily positive NPV investment projects. In this condition, shareholders gamble with debt holders' money.

(2) Under-investment problem:

Now assume \$60 of debt repayments is outstanding and FIRM only has Cash \$30 in hand for investment. A profitable project, C, is available which requires \$31 for investment and will have payoff \$60 for sure at the end of the period. Accepting project C will not only increase total wealth of FIRM but also fully repay its debt. However, in order to access project C, managers will need \$1 contribution from shareholders. In this case, managers will definitely forgo project C since accepting it will accrue all proceeds to debt holders and leave nothing to shareholders. Therefore, interest conflict problem happens again between shareholders and debt holders. This is an under-investment problem first described by Myers (1977).

2.2.3 Resolution for Agency Problems

In order to mitigate the agency problems under credit risk, various methods are suggested in literature.

Secured debt, debt collateralized by tangible assets of the firm, is proposed very early as a resolution for agency problems of debts. Scott (1976) proposes that the optimal leverage may be related to collateral value of tangible assets held by a firm. When default happens, debt holders will lose less from receiving the compensation by liquidating the collaterals. Under this circumstance, monitoring costs of the firm can be considerably reduced. Stulz and Johnson (1985) suggest the same reason for proving that adopting secured debts increases the real value of the firm. Another important reason is that secured debts make shareholders to take positive NPV projects more advantageously. Since secured debts reduce agency costs of debts and increase the real value of the firm, Myers-Majluf (1984) show that secured debts are generally preferred to unsecured debts empirically.

Protective bond covenants are also widely proposed as an effective resolution for the interest conflict problem between shareholders and debt holders. Smith and Warner (1979) empirically examine an random example of 87 public debts between January 1974 and December 1975 and classify the effect of bond covenants into five categories: restrictions on the firm's production/investment policy, restrictions on the payment of dividends, restrictions on the subsequent financing policies, restrictions on modifying the pattern of payoffs to bondholders, and restrictions specifying bonding activities by the firm. They claim that bond covenants mitigate the agency problems of debts. Afterwards, Asquish and Wizman (1990), Crabbe (1991), and Bae, Klein, and Padmaraj (1994) all support that making protective bond covenants can be an efficient method to avoid some strategic actions from managers/shareholders.

Myers (1977) is the first author who indicates that debt maturity choice may be able to alleviate the under-investment problems. If default possibility is concerned, shareholders may not be able to fully capture the benefits from investing a positive NPV project because debt holders will take part of them as compensation for default risk. Myers (1977) show that shareholders can pay off the debt holders' fixed claim and obtain all of the benefits of the project by funding it themselves with debt that matures before investment opportunities expire. Therefore, for firms with many investment opportunities, short-term debt is suggested. Barnea, Haugen, and Senbet (1980) also demonstrate short-term debts and callable debts allow firms to minimize the agency costs of debt that result from information asymmetry, managerial risk incentives and foregone growth opportunities resulting from Myers' under-investment problems.

John and Nachman (1985) study the agency problems between corporate insiders (managers/shareholders) and debt holders in a dynamic setting with asymmetric information. Based on the fact that debt financing is not a one time event for most of firms, the authors examine investment incentives of firms with risky debt outstanding. They find that as part of equilibrium, an endogenous effect, “reputation”, also plays an important role in moderating the underinvestment problem. A more intuitive explanation is that managers/shareholders need to keep their reputation in order to have easier subsequent debt financing. It is reputation that alleviates the agency problems of debt.

3 THE MODEL

The agency problem we study is a situation where the agent of the equity holders (managers of the company) continues to operate the firm when the equity value is negative. Under such a situation, the firm cannot raise new equity capital since its equity cannot be sold for a positive value. To be seen shortly, this is the condition equivalent to call value of the firm being larger than the debt payment (coupon or principal) at the payment date. However, in reality, companies continue to operate as long as the asset value being greater than the payment. Hence, the company can no longer issue new equity when the debt payment is in between the call option value and asset value.[7] The main intuition can be seen in a simple 2-period model. We also derive the n-period model.

3.1 A Numerical Demonstration of Agency Problem: The Geske Model at a Glance

Suppose a company has two zero coupon debts, one and two years to maturity and each has \$100 face value. Also suppose currently the asset is worth \$400 and the debts are together worth \$170. This is graphically represented by the following balance sheet:[8]

[7] The call option value is the Black-Scholes value of the firm. If the company does not default, then this is the equity value. If this call value is less than the coupon amount, the equity value is negative.
[8] We assume the risk free rate to be about 10%. Since the company is extremely solvent, both debts are roughly priced at the risk free rate.

Balance Sheet
as of year 0

assets	400	maturity $t = 1$ debt	90
		maturity $t = 2$ debt	80
		equity	130
total	400	total	400

note: both debts have face values of $100

Assume that one year later, the asset grows to $450 and the firm faces the first debt payment of $100. Geske (1977) argues that the firm at this time should raise equity to pay for the first debt so that the asset value can avoid a sudden drop. So the asset value after paying off the first debt is still $450. Assume that at this time ($t = 1$) the second debt has a value of $90. As a result, the equity should be $360 (= $450 – $90) that includes $100 new equity and $260 old equity. The balance sheet becomes:

Balance Sheet
as of year 1 before payment of first debt

assets	450	maturity $t = 1$ debt	100
		maturity $t = 2$ debt	90
		equity	260
total	450	total	450

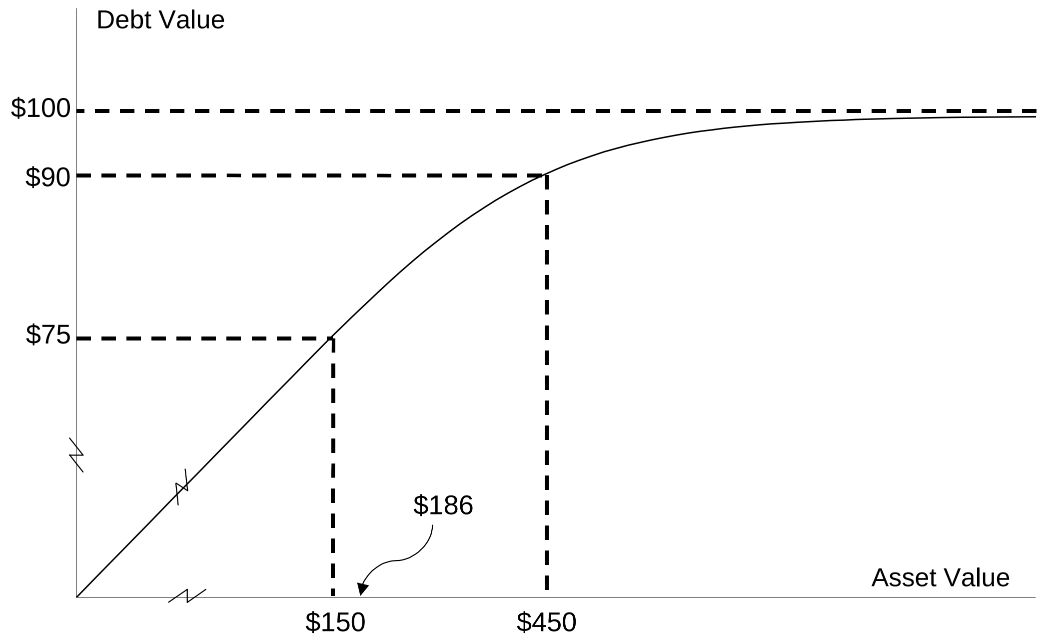
Balance Sheet
as of year 1 after payment of first debt

assets	450	maturity $t = 2$ debt	90
		old equity	260
		new equity	100
total	450	total	450

note: issue new equity to pay for the first debt

Now, instead of $450, suppose that the economy is bad and the asset value drops to $150. Bad economy and lower asset value imposes higher default risk on the second debt so it is priced at $75 (lower than $90 due to higher risk). Hence, the equity (value of old equity and the “should be raised” equity) is $75 (= $100 – $25 = $150 – $75)[9], as the following picture depicts.

Figure 2: Market Value of Debt vs. Market Value of Asset



The firm, as in the previous case, would like to raise equity to pay off the first debt. But the new equity value needs to be $100 – a clear contradiction. This means that the new equity owner pays $100 in cash but in return receives a portion of $75. Therefore, any rational investor would not invest equity in this firm. Since the firm cannot raise equity

[9] Actually , the balance sheet before the payment of first debt should be:

Balance Sheet
as of year 1 before payment of first debt

assets	150	maturity $t = 1$ debt	100
		maturity $t = 2$ debt	75
		old equity	-25
total	150	total	150

capital to continue its operation, it must declare bankruptcy. There is point where the new equity owner is indifferent and this is the default point of the company. Suppose the (break-even) asset value in one year is hypothetically set at $186.01. Also suppose the second debt is $86. Consequently, the new equity owner has $100 and the old equity has $0.01. And we know that the default point is $86.[10]

Balance Sheet
as of year 1 before payment of first debt

assets	186.01	one-year debt	100
		two-year debt	86
		equity	0.01
total	186.01	total	186.01

Balance Sheet
as of year 1 after payment of first debt

assets	186.01	two-year debt	86
		old equity	0.01
		new equity	100
total	186.01	total	186.01

note: issue new equity to pay for the first debt

Anything lowers than $186 will cause default. However, with $186 of assets, the company can pay the first debt and continue to operate. But under that circumstance (selling asset to pay off the first debt without raising any new equity), the second debt will drop significantly in value as the following chart demonstrates:

[10] This value is precisely the "implied strike price" in the Geske model.

Balance Sheet
as of year 1 before payment of first debt

assets	186	one-year debt	100
		two-year debt	86
		equity	0
total	186	total	186

Balance Sheet
as of year 1 after payment of first debt

assets	86	two-year debt	76
		old equity	10
total	86	total	86

note: selling asset to pay for the first debt

The reason is that the equity immediately has value at the cost of the debt. In the above hypothetical tables, \$10 is transferred from debt to equity. At $t=0$, the debt holders know about this if there is no information asymmetry and shall pay less for the debt.

Usually, the company will roll over old debt to new debt. In the case of extreme solvency the problem is not severe. But in the case of near default, as described above, we have

Balance Sheet
as of year 1 before payment of first debt

assets	186.01	one-year debt	100
		two-year debt	86
		equity	0.01
total	186.01	total	186.01

Balance Sheet

as of year 1 after payment of first debt			
assets	186.01	two-year debt	86
		new debt	100
		old equity	0.01
total	186.01	total	186.01

note: issue new debt to pay for the first debt

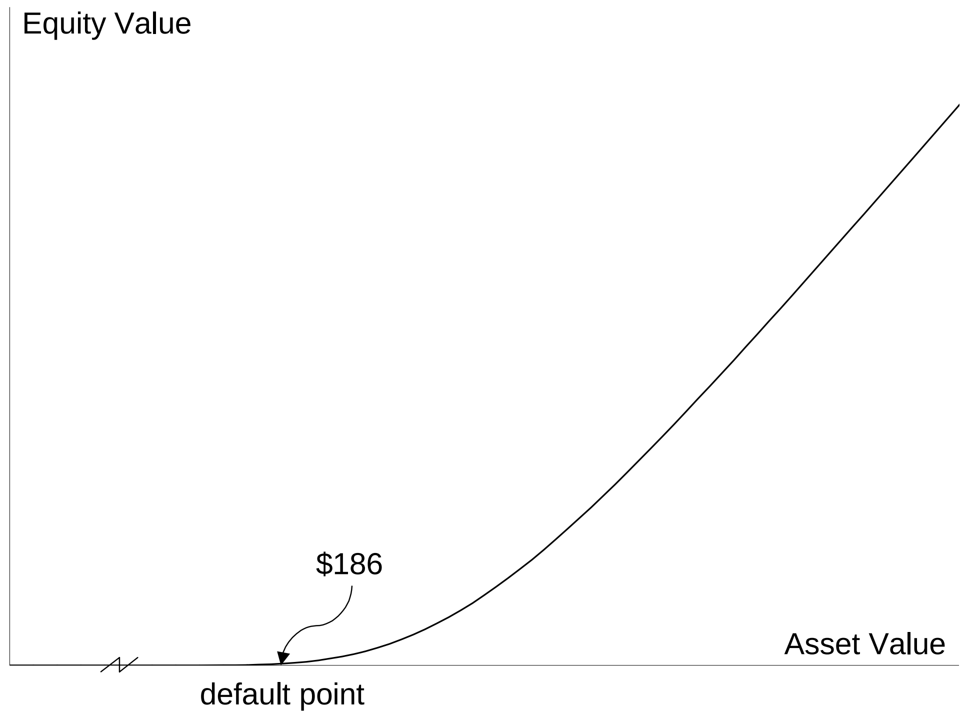
Note that the principal of the new debt can be extremely high to reflect the very risky situation. But since the existing debt matures earlier (and hence has a higher seniority), its value should not be affected by the new debt. On the other hand, the equity will be less valued because there is one more claim with higher priority. In the equilibrium, the equity value should return to 0.01, as in the Geske (1977) case. In summary, to pay off the current debt, it should not matter if the fund comes from new equity or new debt (at the break-even point). The Geske (1977) result holds.

In reality, companies do not default at \$186 but default at \$100. In other words, even if the asset value is \$150, the company continues to survive and operate. In that case, clearly the company is not able to raise capital, but it is certainly able to pay the debt with its assets and leaves the second debt with \$50. Under this condition, the debt will be worth less than \$50, possibly very little, especially if the equity owners adopt high volatility projects. When the volatility is infinity in the extreme case, the debt value is \$0. If the firm is forced default, the second debt is entitled to \$50, but now it is worth only so much as a consequence of the shareowners activities. The transfer of wealth from debt owner to equity owner is what we define as the agency problem.

As long as the company spends assets to pay for the debt, the existing debt holders will be hurt and shareholders should benefit. Since there is no asymmetric information, it is not the equity holders' interest to use assets to pay for the debt. Agency problem only occurs when the value of assets is so low that neither new equity nor new debt is possible.

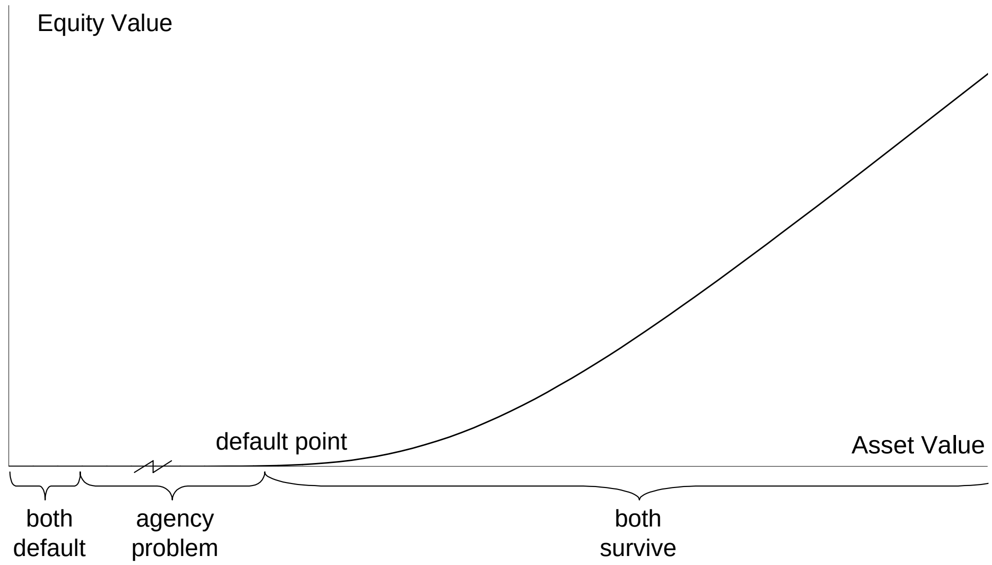
As to be shown later, the Geske (1977) model can be solved within the Black-Scholes-Merton framework. A general plot of the equity value is shown below:

Figure 3: Equity Value vs. Asset Value in Geske Model (1977)



The default point is $186 in the previous example. The agency problem can be depicted by the following graph:

Figure 4: Default Difference and the Agency Problem



The Geske (1977) model argues that at the due date of the first debt, the company faces a decision whether to pay the coupon. This is a compound option question in that if the company decides to pay, the company continues to survive much like exercising the compound option to keep the option alive. The company's survival criterion relies upon if the company can raise new equity capital. In other words, the technical condition in the Geske (1977) model for staying solvent (paying the coupon) is that the company must use new equity to pay for the coupon. If such new equity cannot be raised, then the company should go bankrupt. Interestingly, this condition translates into another equivalent condition that the market value of the assets of the company must stay above the market value of the liabilities at the moment of the coupon. This condition is regarded as the no-arbitrage condition.

Since the agency problem investigated in this paper results from the difference between the definitions of default in distinct structural credit risk models (the real-world structure form model and the Geske (1977) model), this kind of agency problem is named "*the structural agency problem under credit risk*". Different from the single-period agency problem in literature, the character of the structural agency problem under credit risk, which will be described in details in the following sections, will give us a new perspective on investigating the incentive conflicts between shareholders and debt holders in a multi-period credit risk model.

3.2 The Two-Period Geske Model

To see that, take a two-period setting: $t = 0,1,2$ where $t = 0$ is the current time. The company owes a coupon bond where K_1 is the coupon at $t = 1$ and K_2 is the bond redemption value at $t = 2$. The total asset value at both times is represented by A_1 and A_2 respectively.

At $t = 1$, the company faces an exercise decision. The company will pay the coupon to stay alive only if new equity can be raised. In other words, the technical condition is:

$$\begin{cases} C_1 > K_1 & \text{company can survive} \\ C_1 \le K_1 & \text{company is in default} \end{cases}$$

where C_1 is the call option value (equity value) at time $t = 1$ which is a Black-Scholes result:

$$\begin{aligned} C_1 &= C(A_1, K_2, r, \sigma, h) \\ &= e^{-rh}\mathbb{E}_1[(A_2 - K_2)^+] \\ &= A_1 N(d + \sigma\sqrt{h}) - e^{-rh} K_2 N(d) \end{aligned}$$

where

$$d = \frac{\ln A_1 - \ln K_2 + (r - \frac{1}{2}\sigma^2)h}{\sigma\sqrt{h}},$$

$\mathbb{E}_1[\cdot]$ is the risk neutral expectation conditional on information available at time $t=1$, A_1 is the asset price at time $t=1$, h is the time distance between time $t=1$ and $t=2$ (which is also assumed to be the same time distance between $t=1$ and $t=0$), and σ is the asset volatility. If the new equity is raised to pay for the coupon, then there is no reduction in asset value. In the balance sheet, it is simply a transfer from debt to equity by the amount of coupon. The total asset value should not be changed. The following table helps to understand the before/after- coupon condition more clearly:

	Before Coupon	After Coupon
Equity	$C_1 - K_1$	C_1
Debt	$D_1 + K_1 = A_1 - C_1 + K_1$	$D_1 = A_1 - C_1$
Total Asset	A_1	A_1

Hence, the before payment equity value is $E_1 = C_1 - K_1$ which must be greater than 0 to avoid bankruptcy. The debt value after coupon is:

$$\begin{aligned} D_1 &= A_1 - C_1 \\ &= A_1[1 - N(d+\sigma\sqrt{h})] + e^{-rh}K_2N(d) \end{aligned}$$

and before coupon is $D_1 + K_1$. Note that the default condition $C_1 - K_1 > 0$ can be re-written as:

$$\begin{cases} C_1 - K_1 > 0 \\ A_1 - D_1 - K_1 > 0 \\ A_1 - K_1 > D_1 \end{cases}$$

which is that the asset value, after paying the coupon, should be greater than the debt value. However, interestingly, note the debt value is a function of the asset value. Hence it is never the case that the asset value can be ever lower than the debt after coupon. The only way to allow for this condition is to pay the coupon with new equity so that the asset value is unchanged. This brilliant equivalent condition used in Geske naturally provides the no-arbitrage condition for the default.

Note that the value of assets at time $t = 1$, A_1, decides the value of the existing equity. If A_1 is large enough to avoid default, then the equity value must equal to the call value C_1. If A_1 is too small[11], so small that the call option value is less than the coupon amount, i.e. $C_1 < K_1$, then the company must declare bankruptcy. The reason is that under such a situation, the company cannot raise new equity. If the company cannot raise new equity, it must be the case that the old equity is worth nothing (since the old and new equity must be valued on the same basis).

In conclusion, the equity value under the no-arbitrage condition of default[12], set by Geske (1977) is

$$E_1 = \max\{C_1 - K_1, 0\}$$

3.3 The Structural Agency Cost under Credit Risk

In reality a firm only defaults when it lacks enough assets[13] to pay for the coupon. In other words, a firm defaults when $A_1 < K_1$. Since $A_1 > C_1$ by definition, it is perceivable that a firm can continue to operate when $C_1 < K_1 < A_1$. Under such a situation, the equity value is negative. Then it is the equity holders' interest to take on highly risky

[11] Due to bad investments.
[12] Before coupon.
[13] We assume perfect liquidity so all assets can be regarded as cash.

projects to increase the value of C_1 until it is greater than K_1. This is the agency problem we study.

Not being able to be studied under the single period framework of Barnea, Haugen, and Senbet (1980), the multi-period default promotes the agency problem of debt in that the shareholders will shift from a normal σ in a non-default situation to an extremely high σ when $K_1 < A_1 < \bar{A}_1$, where $\bar{A}_1 = D_1 + K_1$ [14]. The equity value under this situation can be computed as

$$
\begin{aligned}
E_1^* &= C((A_1 - K_1)^+, K_2, r, \sigma, h) \\
&= \begin{cases} e^{-rh}\mathbb{E}_1[((A_1 - K_1)e^{(r-0.5\sigma^2)h+\sigma\sqrt{h}z} - K_2)^+] & A_1 > K_1 \\ 0 & A_1 \le K_1 \end{cases} \\
&= \begin{cases} (A_1 - K_1)N(d^* + \sigma\sqrt{h}) - e^{-rh}K_2 N(d^*) & A_1 > K_1 \\ 0 & A_1 \le K_1 \end{cases}
\end{aligned}
$$

where

$$
d^* = \frac{\ln(A_1 - K_1) - \ln K_2 + (r - \frac{1}{2}\sigma^2)h}{\sigma\sqrt{h}}
$$

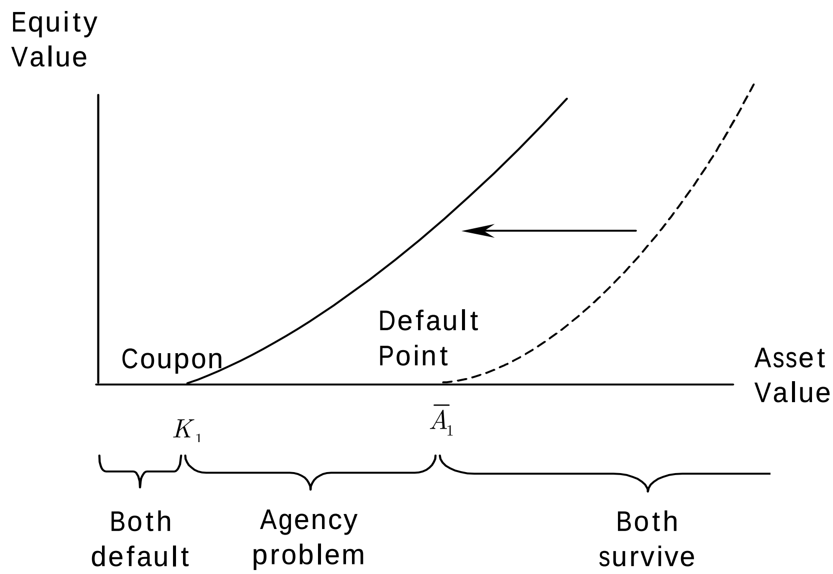
When $C_1 > K_1$ (or equivalently $A_1 > \bar{A}_1$) and under no asymmetric information, Barnea, Haugen and Senbet (1980) show that it is not the equity holders' interest to take on risky projects.[15] Hence, the volatility of assets remains the same throughout. When $A_1 < K_1$,

[14] When $C_1 = K_1$.

[15] Different from the meaning of 'informational asymmetry' in Barnea, Haugen and Senbet (1980) , 'no asymmetric information' we indicate here means that debt holders are fully aware of shareholders' incentive problem under the assumption of the option valuing method, i.e. shareholders always prefer riskier investment projects. In Barnea, Haugen and Senbet (1980), "If bondholders are fully aware of the incentive problem, its effects are discounted in the price at which they buy the debt….Thus, in this case, the problem of wealth transfer ceases to exist. (pp. 1228)"

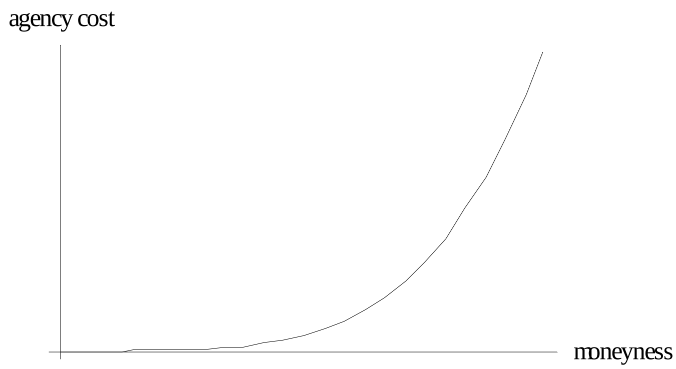
the company actually defaults which is also suggested by the Geske model. Hence, the equity holders cannot adopt high volatility projects.

Figure 5: Structural Agency Problem under Credit Risk



Note that $E_1^* > E_1$. The agency cost is measured as $E_1^* - E_1$. When the company is extremely solvent there is no incentive to take on risky projects (the point made by Barnea, Haugen and Senbet). When the company is just under default boundary, there is large incentive for the equity holders to raise volatility. But when the company is deeply under default (very low asset value), it is hard for the equity to gain from high volatility. As we know in the Black-Scholes model, the highest volatility sensitivity (so called vega) is when an option is at the money. As a result, it causes the largest agency cost there.

Figure 6: Agency Cost vs. Moneyness



We also look at the agency cost at other dimensions.[16]

Figure 7: Agency Cost vs. Volatility

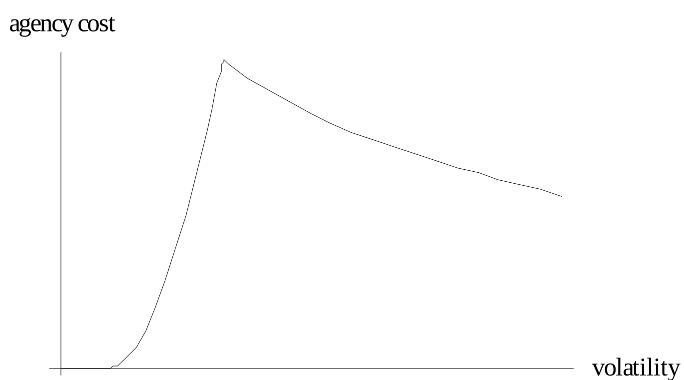
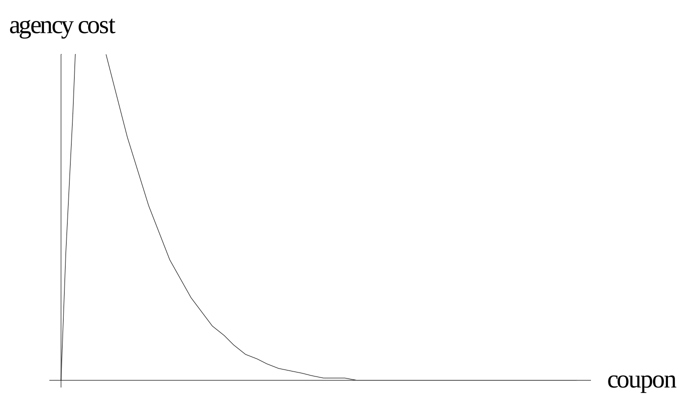
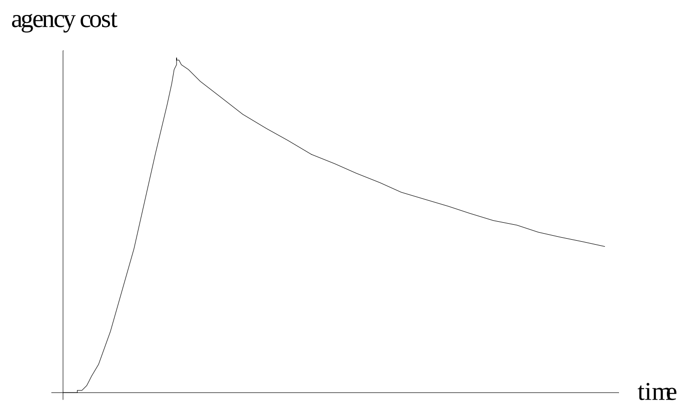


Figure 8: Agency Cost vs. Coupon



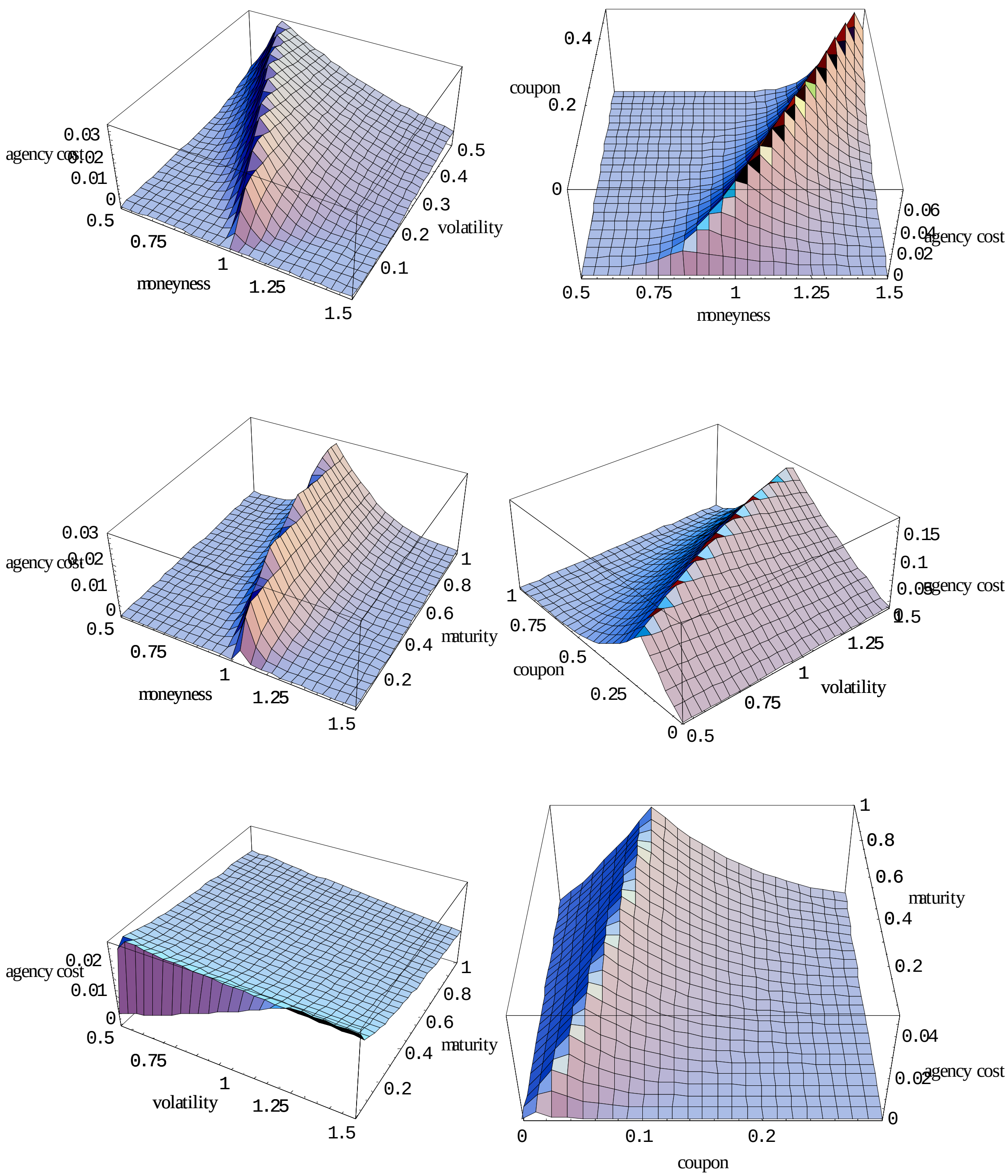
[16] Analytical partial derivatives are shown in Appendix.

Figure 9: Agency Cost vs. Time



Interestingly, when the equity is “at the money” the volatility doesn’t always benefit the equity. Also, contrast to Barnea Haugen and Senbet, low coupon doesn’t always benefit the equity either. However, further investigation shows that these function forms are not stable. They change shapes as different values of the other factors are held constant. To examine fully how each factor affects, we generate a series of three dimensional charts.

Figure 10: Serial 3-D Graphs of Agency Cost



3.4 The *n*-period Geske Model

The *n*-period Geske model can be best seen by a three-period model. The three period setting is $t = 0,1,2,3$. At $t = 2$, we obtain the same result as the previous sub-section. At $t = 1$, we have,

$$C_1 = A_1 M(d_2 + \sigma\sqrt{h}, d_3 + \sigma\sqrt{2h}; \sqrt{\tfrac{1}{2}}) - e^{-2rh} K_3 M(d_2, d_3; \sqrt{\tfrac{1}{2}}) - e^{-rh} K_2 N(d_2)$$

where $M(d_1, d_2; \rho)$ is a bi-variate normal probability with d_1 and d_2 being the limits and ρ being the correlation coefficient, and:

$$d_j = \frac{\ln A_1 - \ln \bar{A}_j + (r - \sigma^2 / 2)(j-1)h}{\sigma\sqrt{(j-1)h}}$$

where $3 \geq j > 1$, $\bar{A}_3 = K_3$, and $\bar{A}_2$ is the internal solution for equation $E_2 = K_2$. The equity value is:

$$E_1 = \max\{C_1 - K_1, 0\}$$

and the debt value after coupon is:

$$\begin{aligned} D_1 &= A_1 - C_1 \\ &= A_1[1 - M(d_2 + \sigma\sqrt{h}, d_3 + \sigma\sqrt{2h}; \sqrt{\tfrac{1}{2}})] + e^{-2rh} K_3 M(d_2, d_3; \sqrt{\tfrac{1}{2}}) + e^{-rh} K_2 N(d_2) \end{aligned}$$

This equation is very intuitive – the second and last terms represent the coupon value of K_1 and K_2, each weighted by the corresponding survival probability $N(d_1)$ and $M(d_1, d_2; \rho)$.[17] Under lack of information asymmetry, bond holders know about the agency problem, and hence will adjust the bond price lower. As a result, bond holders will price according to the reality bankruptcy rule. When $A_1 > K_1$,

[17] Note that to survival till the second period, it must first survive the first period. Hence, the probability is for the normalized asset price to stay above both d_1 (at period 1) and d_2 (at period 2), while to survive only the first period regardless of the second period is for the asset price to stay above d_1.

$$E_1^* = (A_1 - K_1)M(x_2 + \sigma\sqrt{h}, x_3 + \sigma\sqrt{2h}; \sqrt{\tfrac{1}{2}}) - e^{-2rh}K_3M(x_2, x_3; \sqrt{\tfrac{1}{2}}) - e^{-rh}K_2N(x_2)$$

where

$$x_j = \frac{\ln(A_1 - K_1) - \ln K_j + (r - \sigma^2/2)(j-1)h}{\sigma\sqrt{(j-1)h}}$$

and when $A_1 \le K_1$, $E_1^* = 0$. The difference between the two values, $E_1^* - E_1$, is the suffering born by the equity holders due to perfect information symmetry. However, this is to say that the equity holders will fix the volatility throughout the life of the bonds. Under this situation, like the single period model as in Barnea, Haugen, and Senbet (1980), it is at the equity holders' interest to avoid such cost.

In our two-period model, we must note that the equity holders do not have to maintain the volatility. In fact, the equity holders will increase the volatility once the asset value falls below the implicit default barrier, $\bar{A}_1$, but still stays above the coupon K_1. Once doing so, the equity holders will increase the value of the equity at the expense of the debt holders. Following the same logic of three-period model above, the Geske (1977) model can be easily extended to *n* periods (see Geske, 1977).

3.5 Characters of Structural Agency Problem under Credit Risk

In sum, the *structural agency problem under credit risk* examined above is that the shareholders still have the control of the firm even when the firm should have been default. In reality the firm defaults when the asset value is unable to pay off the current debt payment. However, this paper defines that the default barrier of Geske (1977), the firm defaults when the current debt payment cannot be paid off by raising enough equity (the asset value is under the present value of all outstanding debts), should be the “correct” one. The reason for defining the default barrier of Geske (1977) as the “correct” one is that it fits the spirit of fairness of security design: the party that holds residual rights should bear its own investment/operation risks. Therefore, the occurrence of structural agency problem under credit risk is defined as when the asset value is under the default barrier of Geske (1977) and above the current payment because under such a condition shareholders are taking advantage of debt holders. As shown in the analysis, if the default structure of Geske (1977) is defined as the correct one, the equity value in the reality case, E^*, is overvalued. Thus the structural agency cost under credit risk is defined as the difference of the equity value between a default structure in reality and the Geske (1977) model, $E^* - E$.

The structural agency problem under credit risk bears several interesting characters. A very special character for this kind of agency problem is that it still happens even if there is no asymmetric information. Under a condition with no asymmetric information, debt holders will know the incentive of shareholders and discount the debt price in advance. Hence shareholders bear the structural agency cost under credit risk. Actually, shareholders do not necessarily want to get rid of this agency cost. In fact, for shareholders, bearing this agency cost is more like buying several kinds of rights from debt holders when facing financial distress: the right to survive under the “correct” default barrier, the right to shift risk (until the asset value goes back to the “correct” default barrier), and the right to overvalue the equity. These are all the characters result from the structural agency problem under credit risk investigated in this paper. Therefore, investigating the structural agency problem under credit risk also examines the timing and incentive of the asset substitution problem with financial distress from a multi-period viewpoint. Moreover, lately Jensen (2005) explains the dramatic increase in corporate scandals and value destruction by defining and analyzing agency costs of overvalued equity. Although the main purpose of Jensen (2005) is to investigate the agency problem between the management and shareholders, this paper is able to extend the range of the research (overvalued equity) to the structural agency problem under credit risk between shareholders and debt holders.

3.6 Resolution

Under no information asymmetry, it is conceivable that the debt holders should realize such a structural agency problem under credit risk and discount the debt value at the very beginning when the debt is issued. As shown in the model, the Geske model has a closed form solution so that we know the exact amount of the agency cost. Subsequently the company will have difficulty to raise enough capital from debt financing for its investment projects. Therefore, it is at shareholders' best interest to avoid this structural agency cost under credit risk.

In order to mitigate the structural agency problems under credit risk, the resolution can be investigated from two aspects: agents (shareholders/managers) and principals (debt holders). In terms of agents, as in Barnea, Haugen, and Senbet (1980), short maturity debts may be considered as a good method. As for short maturity debts, as shown in our model, very short maturities do decrease the structural agency costs under credit risk. Moreover, short maturity debts do give shareholders less chance for exploit debt holders' wealth.

In terms of debt holders, the intuition is to let debt holders own a tool to keep shareholders from taking advantage of them. In this situation, debt holders will have no incentive to lower the bond price at the very beginning when the debts are issued. Actually, the perfect solution for the structural agency problem under credit risk is to make a protective covenant which regulates the company to issue new equity to pay for the coupons. If this kind of safe covenant is possible, the structural agency costs will be totally eliminated. The intuition for this is just to force the party with residual rights to bear its own investment/operation risks, which fits the spirit of fairness of security design. A put option issued at the same time with the debt is another conceivable solution. The put option owned by the debt holders can guarantee that the debt holders can always get the market value of the debt. In this case, debt holders can exercise the put option to stop the exploration in time when the total asset value is about to below the market value of the debts. A main difficulty for issuing this kind of put option is how to determine the strike price. Convertible bonds can also be another kind of possible resolution. The concept is that when the debts are converted to equity, the agency problems disappear immediately since the agent-principal relation does not exist anymore. This is a more "conceptual" solution for the problem because there is no guarantee that the debt holders will appreciate being shareholders.

4 Case Study: Lucent Technologies Inc.

Lucent Technologies Inc. is the best example for the agency of the kind under study. In late 1999, in an attempt to continue its glorious appreciation in equity in 1997 and 1998, its CEO, Richard McGinn, and its board of directors embarked a series of inappropriate business practices that inflated its equity price even more by trading off its fundamentals for book value revenues. As the scandal broke out in 2000, which coincided with the burst of high-tech bubble, Lucent equity free fell from over $60 per share to near 50 cents. At this time, Lucent engaged in a series of activities to prevent default. In this case study, we shall reveal the structural agency problem under credit risk and compute the cost.

4.1 Background

The trouble of Lucent began in late 1999. The stock price plummeted sharply and the debt mounted. Due to the fact that the scandal of Lucent coincided with the internet boom in 2000, we must de-trend Lucent by the market in general in order to see the agency problem caused by Lucent's management and board of directors. As we can see, in both Figure 11 and Figure 12, Nasdaq and a broader index (S&P 500) lose less than Lucent after the burst of the internet bubble.

Figure 11: Lucent vs. S&P500

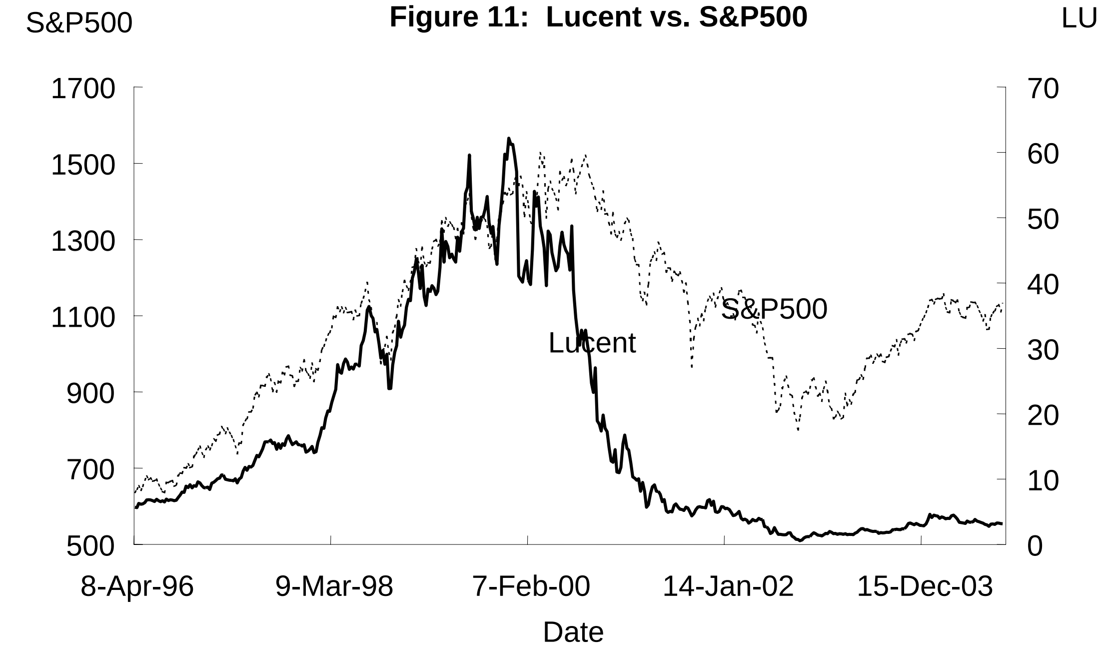


Figure 12: Lucent vs. Nasdaq100

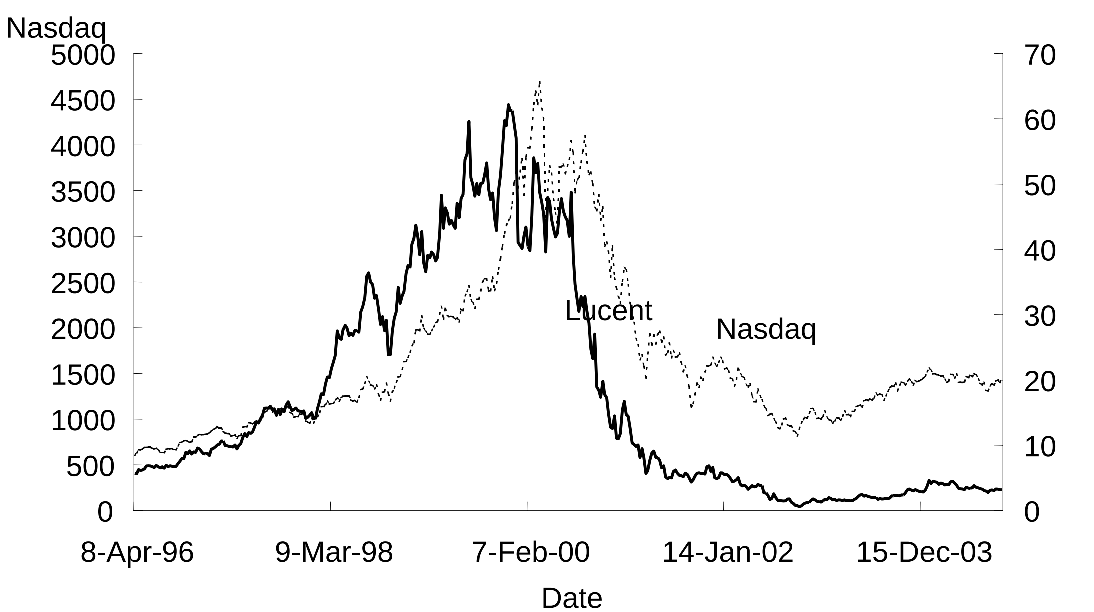


Figure 13: Change of Capital Structure of Lucent Technologies Inc.

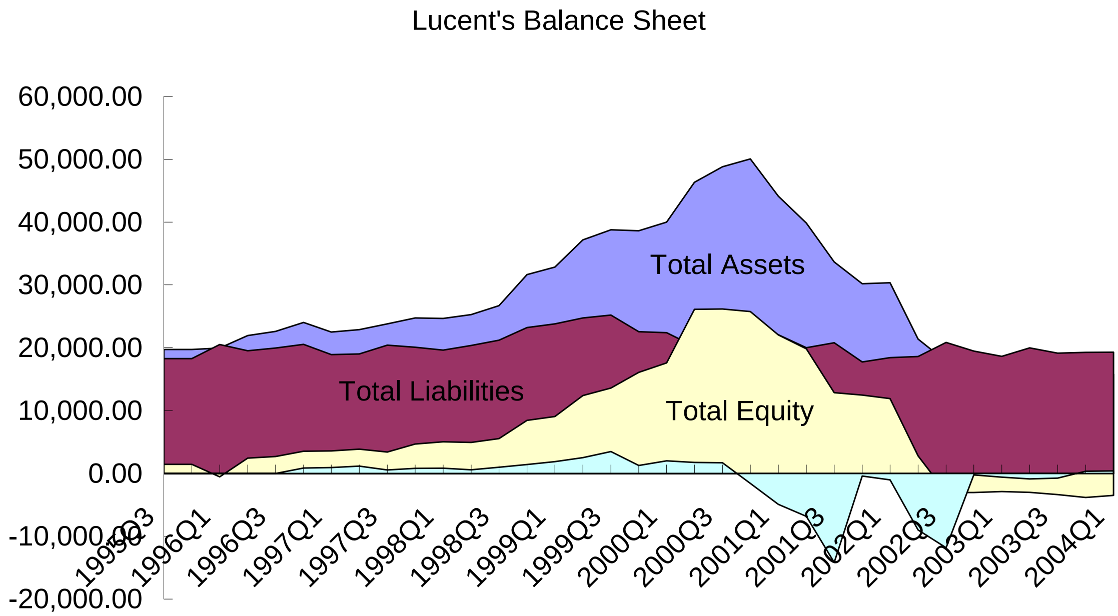
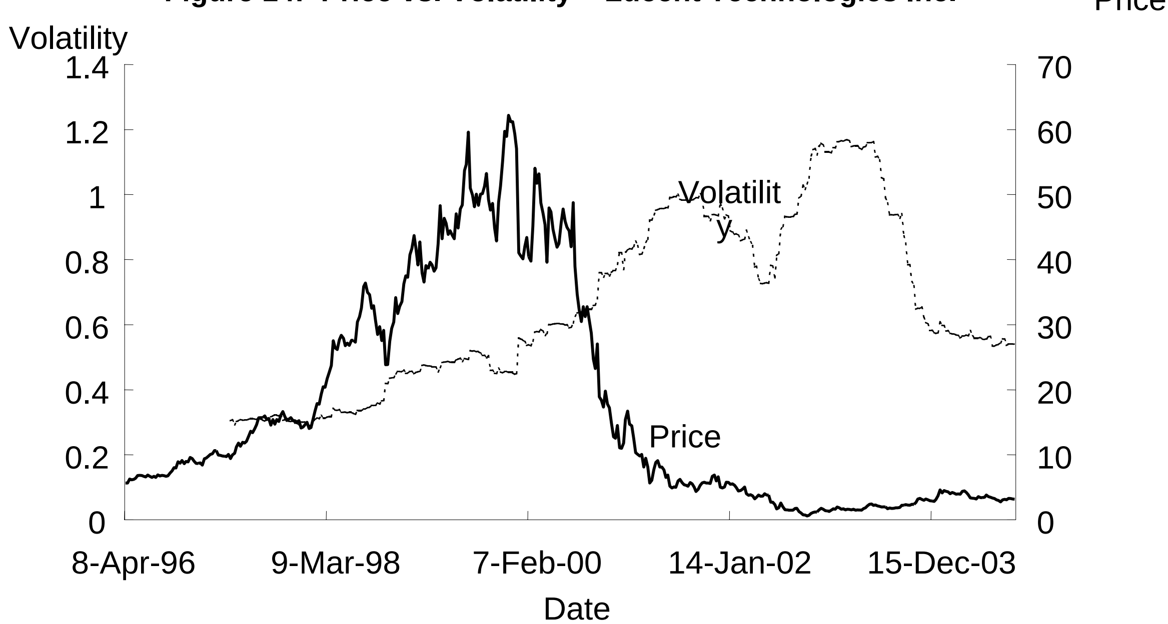


Figure 14: Price vs. Volatility – Lucent Technologies Inc.



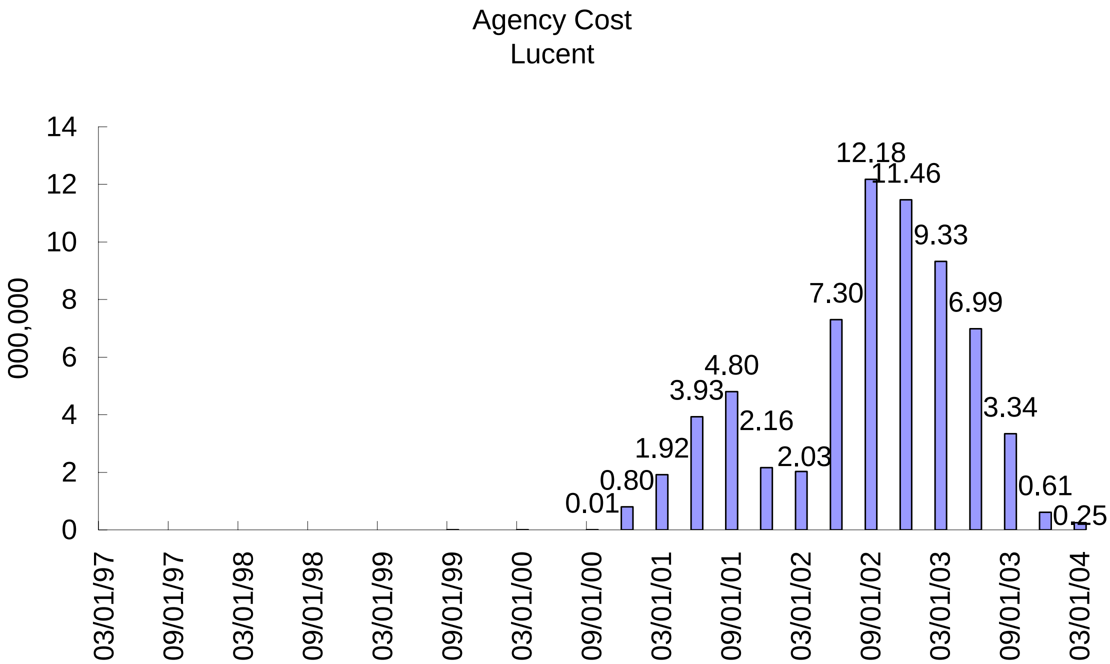
Under pressure to meet revenue goals, Lucent in 1999 began giving large discounts to meet its numbers and began giving more loans to service providers to win their business. As we can see, in Figure 13, the illegal wrongdoing by Lucent's management and the board inflated the revenues and earnings and brings to their peaks at the second quarter of 2000. But then, after the company could no longer artificially inflate its earnings, the company started to crumble. Though the board took action and fired CEO Richard McGinn in October 2000, it gave him a golden parachute of more than $12 million as a parting gift.

The price-volatility picture in Figure 14 demonstrates that Lucent is taking on risky projects and agency problems deteriorated as Lucent's equity is trying to get out of the position. The fact that current Lucent's price came back while the book value equity is still negative explains that the agency cost is high.

4.2 Data and Results

In order to look into Lucent's agency problem, we collect weekly equity prices from Yahoo Finance website. Quarterly financial reports from December 1995 to March 2004 are obtained from Compustat. For the risk free rate, we use CMT (Constant Maturity Treasury) 1-year rates that are obtained from the Federal Reserve Bank of St. Louis web site.

Figure 15: Agency Costs: Lucent Technologies Inc.



We see from the above diagram that Lucent's agency problem started to appear at the end of 2000, roughly one year after the scandal broke out. This is when the company should already be under default and yet the company continues to operate at the debt holders' expense. As the company's situation continued to deteriorate the agency cost is higher. Note that in quarter 3 2002, the book value of Lucent is negative for its equity.

5 Conclusion and Further Research

This paper examines the structural agency problem under credit risk. Such kind of agency problem occurs when the firm should have been default but the shareholders still have the control of the firm. The reason is that there is a difference between the definition of default in the real-world structure form model and that in the Geske (1977) model. The definition of default in the Geske (1977) model is assumed the “should-be-correct” one in this paper because it fits the spirit of fairness of security design: the party that holds residual rights should bear its own investment/operation risks.

In sum, this paper contributes to the literature in four ways. First, it provides a multi-period view of an agency problem. As opposed to the existing literature whose discussions are mostly based upon the single period model (i.e. single maturity debt so that the Black-Scholes-Merton model can be applied), the multi-debt model of the Geske (1977) is adopted in this paper. Application of Geske (1977) model allows us to examine the agency problem in a new perspective: under no information asymmetry, the structural agency problem under credit risk still exists. Second, investigating the characters of structural agency problem under credit risk offers explanations for asset substitution problem and equity overvalued problem in a multi-period perspective. Third, because the multi-debt structure represents the reality closely, we can actually calculate the magnitude of the agency cost and quantify the agency cost in a meaningful way. Finally, the investigation of the definition of the “should-be-correct” default and the resolution for the structural agency problem under credit risk give us a new thought in a security design aspect.

In the future, the continuous work of this paper will focus on doing more empirical cases based on the model. With the burst of high-tech bubble in recent years, there should be more interesting cases such as Lucent Technologies Inc. worth to be investigated.

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Appendix Analytical partial derivatives of Agency Costs

Define

$$\Theta = -\frac{Az(d)\sigma}{2\sqrt{h}} - rKe^{-rh}N(d-\sigma\sqrt{h})$$
$$\mathcal{V} = A\sqrt{h}z(d)$$
$$\Delta = N(d)$$

To understand the agency cost, we take partial derivatives. Note for $A_1 < K_1$ and $E_1 > K_1$, the difference is:

$$D = (A_1 - K_1)N(d_1^* + \sigma\sqrt{h}) - e^{-rh}K_2N(d_1^*) - \{A_1N(d_1 + \sigma\sqrt{h}) - e^{-rh}K_2N(d_1) - K_1\}$$

partial w.r.t. h,

$$\frac{\partial D}{\partial h} = \Theta^* - \Theta$$

partial w.r.t. K_1

$$\frac{\partial d}{\partial K_1} = 0$$
$$\frac{\partial d^*}{\partial K_1} = \frac{-1}{\sigma\sqrt{h}(A_1 - K_1)}$$

hence,

$$\frac{\partial D}{\partial K_1} = -N(d_1^* + \sigma\sqrt{h}) + z(d_1^* + \sigma\sqrt{h})\frac{-1}{\sigma\sqrt{h}} - e^{-rh}K_2z(d_1^*)\frac{-1}{\sigma\sqrt{h}(A_1 - K_1)} + 1$$

partial w.r.t. σ

$$\frac{\partial D}{\partial \sigma} = \mathcal{V}^* - \mathcal{V}$$

partial w.r.t. A_1

$$\frac{\partial D}{\partial A_1} = \Delta^* - \Delta$$