Do Growth Options Affect Investment Decision-Making?

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Abstract

Many investments create both fixed assets and options for future investments, or "growth options". It is natural to expect these future growth options to affect the timing of the initial decision to invest. This paper examines the impact of growth options on this initial decision in a capacity expansion context. We show that the impact of the growth options depends on whether the initial investment opportunity is short- or long-lived. When the investment opportunity has no time limit, the growth options have no effect on the investment decision—optimal behavior is to act as if there were no growth options. On the other hand, when the investment opportunity is finite, consideration of growth options leads to earlier action.

1 INTRODUCTION

Many investments result not only in assets in place, but also create options for future investments. For example, consider a firm that has an opportunity to invest to create a critical mass of production and distribution capabilities in a new market. If the firm makes this initial investment, it will then have the ability to expand its network of production and distribution if such expansion is justified by changes in the market. Hence, there are two sequential options inherent in the firm's investment opportunity: (1) the option to initiate operations through an initial capital investment—to put "assets in place"—and (2) the option to expand capacity only after the initial investment has been made. This second option, the "growth option", is relevant to the initial investment decision even though it may not generate any cash flows for sometime after the initial investment. This paper is concerned with how growth options change the timing of the initial decision to invest.

The notion that growth options have the potential to influence investment decision-making is not new. For example, Myers (1977, 1987), Kester (1984), Trigeorgis and Mason (1987), and Amran and Kulatilaka (1999), among others, have noted that the value of an investment should include not only the assets in place from the investment, but also the growth options it creates. Taking into account the growth options may significantly increase the value of the investment. A change in the value of the investment, however, does not necessarily translate into a change in initial investment behavior. This leaves open the question of whether growth options can change the timing of the initial decision to invest. To date, most researchers exploring the impacts of growth options on initial investment decisions have made direct assumptions about the cash flows arising from growth options rather than explicitly modeling their optimal exercise over time. [1] For example, Willner (1995) assumes a jump process for start-up venture growth options, and Hevert, McLaughlin and Taggart (1998) define the value of the growth options as the discounted cash flows evolving according to a binomial process. Under this approach, the impact of growth options on investment decisions is directly presumed.

At the same time, a number of researchers have explicitly explored one type of growth options: capacity expansion options. These researches have examined the value of capacity expansion options and associated optimal capacity expansion strategy *after* firms have established themselves in mar-

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[1]It is understood that without further investments, there should be no growth options.

kets. Pindyck (1988) and Dixit and Pindyck (1994) considered the value of capacity expansion options and the optimal expansion threshold for a monopolist assuming stochastically varying product demand. Grenadier (2002) extended and modified this approach to consider capacity expansion option value and optimal expansion threshold in a strategic setting. An important contribution of Pindyck (1988) has been to explicitly break out and calculate the value of assets in place, on the one hand, and growth options, on the other. However, the focus of these researchers has been on the optimal exercise of the capacity growth options and not the impact of the growth options on the initial investment decision.

The presence of capacity expansion options is an important feature of many initial investments. In this paper, we extend the work of Dixit and Pindyck (1994) and Grenadier (2002) to consider the impact of capacity expansion options on the initial decision to invest (e.g., to enter the market). We first follow their work to derive the value of assets in place and the value of growth options, assuming stochastically varying product demand, and then extend their work to trace the growth options back to the initial investment decision. Throughout, the focus of this paper is on the impact of the growth options on the initial investment rather than on their value. While a range of factors ultimately influence any decision to invest, and growth options in different contexts will have different fundamental characteristics, our goal has been to isolate the capacity expansion options to better understand its impact on initial investment decision-making by abstracting away from other factors.[2] We undertake this exercise considering both a case when the initial investment opportunity is infinite (i.e., it will never expire) and a case when it is finite.

This paper shows that the presence of growth options *alone* does not always imply a change in initial investment decision-making. When capacity expansions are the only growth options available and when the initial investment option will never expire, firms are equally well off making the initial decision assuming an absence of growth options. In contrast, when the initial opportunity to invest has a finite life, the presence of growth options will always call for a more aggressive initial investment policy. We provide an analytical proof to show that this is true regardless of model parameters.[3]

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[2]Other factors may be critical in weighing the role of the growth options. For example, some factors may affect the firm's ability to learn about its environment.

[3]A numerical example can be used to show that this result *can* be true, and intuition suggests that it is always true. The analytical proof confirms this intuition.

The remainder of this paper is organized as follows. Section 2 presents the model of production and capacity expansion under stochastically varying product demand. Section 3 specifies the optimal expansion strategy, decomposes project value into the value of the assets in place and the value of the growth options, and discusses the dynamic characteristics of the two sources of value. Section 4 then studies investment decisions in an infinite-time horizon, and section 5 studies investment decision in a finite-time horizon. Section 6 concludes the paper.

2   Model and Assumptions

We begin by considering a firm that has an option to invest in a production facility that will allow it to produce a homogeneous good. The initial investment is indivisible: a fixed investment I is required to initiate operations. Based on this initial investment, the firms will be capable of producing q_0 units of product.[4]

At any point in time after the firm has established its position in the new market, it can invest in additional capacity to increase its output in response to changing market conditions. This expansion potential represents the growth options. We assume that output can be increased by an infinitesimal increment at any time after the initial investment.[5] The cost of increasing output is constant, at a cost of k per unit of output.[6]

At any time t, the profit flow is the function of firm's output, q_t and an exogenous demand shock parameter Y_t:[7]

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[4]The initial investment could include the cost of market research, licenses and permits, or capital facilities. It could also represent the cost of research and development activities to prepare the product for the new market (e.g., the cost of drug development).

[5]Note that this incremental expansion possibility contrasts with the indivisible character of the initial investment. The fixed initial investment could not be optimal if it were feasible to initiate production in infinitesimal units. Where the initial investment might capture a set of one-time costs of "critical mass" investments, the capacity investments might represent such smaller scale investments as increased numbers of workers or incremental increases in production equipment.

[6]In general k is less than I/q due to some additional costs for initial investment, see footnote (4). Our later assumptions on profit function ensure that optimal expansion policy exists. See Grenadier (2002).

[7]A similar assumption on the function form of profit flow can be found in Dixit and Pindyck (1994) for a monopoly and in Grenadier (2002) for an oligopoly.

$$\Pi(q_t, Y_t) = Y_t \pi(q_t) \tag{1}$$

We assume that the continuous function $\pi(q_t)$ is an increasing function of firm's output q_t and that the condition $\partial^2 \pi / \partial q^2 < 0$ holds. The demand shock parameter varies stochastically with time, t. Specifically, following Dixit and Pindyck (1994), Baldursson (1998), Grenadier (2002), and many other authors, we assume that Y_t follows the geometric Brownian motion

$$dY_t = \mu Y_t dt + \sigma Y_t dw_t \tag{2}$$

where μ and σ are constants, and $\mu < r$ is the risk-free rate of interest. We assume that the risk-free rate of interest is constant and cash flows are valued in a risk-neutral framework.

3 The Two Components of Project Value

To provide a basis for exploring the initial investment decision in the coming sections, we proceed by decomposing the value of the investment into two components: the value of the assets in place (the production capacity from the initial investment) and the value of the growth options (the opportunity to expand production).[8] Our decomposition, in essence, follows Pindyck (1988) and Dixit and Pindyck (1994), but with a view of the initial investment as a going concern—its value reflects an expectation of future discretionary investments made possible by the current investment project.[9] Our formulation states that the value of the initial investment hinges on the value of the future growth opportunities it creates.[10]

The value of the growth options are based on optimal capacity expansion strategy. Because the value depends on this future behavior, solving for the value of an investment is formulated as a stochastic control problem. For convenience, assume that at time $t = 0$ the firm exercises the option to invest in initial production capacity and begins producing an amount of output q in the market. The increase in capacity can only occur for $t > 0$. At any time $t > 0$, the value of the firm in the new market can be written as

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[8]Since the focus of this paper is on the growth options, other options resulted from the initial investment, such as, disinvestment options, will not be considered here.

[9]See Myers (1977) for the discussion of such view.

[10]Our formulation results in an almost identical derivation to Dixit and Pindyck (1994), we therefore skip most of it.

$$W(q_t, Y_t) = \max_{\{q_\tau : \tau > t\}} \left[E \int_t^\infty e^{-r(\tau - t)} \{\Pi(q_\tau, Y_\tau) d\tau - k dq_\tau\} \right] \qquad (3)$$

Equation 3 says that the value of the firm, having invested in the market, is equal to the discounted sum of all future profits, assuming an optimal expansion path going forward.[11]

Assuming that $W(q_t, Y_t)$ is continuous on $\Re_+ \times \Re_+$ and is $C^{1,2}(\Re_+ \times \Re_+)$ (continuous differentiability, once with respect to q and twice with respect to Y on $\Re_+ \times \Re_+$), one can show that the value function W satisfies[12]

$$\max\{1/2\sigma^2 Y^2 W_{YY} + \mu Y W_Y - rW + \Pi, \quad W_q - k\} = 0 \qquad (4)$$

This dynamic programming equation says that at every point in the state space, one of the terms on the left must be equal to zero. Hence, the state space splits into two regions, "no expansion" region corresponding to the first term on the left side, and an "expansion region" corresponding to the second term on the left side. In other words, there will be combinations of demand and capacity for which expansion is optimal and combinations for which it is not. We assume that the initial investment places the firm in the no expansion regime. Only through increased demand will it be optimal to expand capacity. It follows that the solution to (4) can be obtained as[13]

$$W(q, Y) = \frac{\pi(q)}{r - \mu} Y + B(q) Y^\beta \qquad (5)$$

where $\beta > 1$ is the positive root of the fundamental quadratic equation $1/2\sigma^2\beta(\beta - 1) + \mu\beta - r = 0$, and $B(q)$ is[14]

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[11]Note that since the profit is an increasing function of the output, the firm always wants to operate at full capacity.

[12]For convenience we suppress t. See Fleming and Soner (1993) for the discussion.

[13]It needs one more optimality condition that is the "super contact" condition derived in Dumas (1991). The value of the initial investment found here is technically the same as the firm's value in Dixit and Pindyck (1994) due to the fact that the value of the investment is the value of the "new" firm once the initial investment is made.

[14]As shown in Dixit and Pindyck (1994), $B(q)$ is solved together with the optimal capacity expansion strategy, which is defined by the investment threshold curve for the capacity expansion: $Y_{exp}^*(q) = \frac{\beta}{\beta-1} \frac{k(r-\mu)}{\pi'(q)}$. The capacity expansion strategy is to invest whenever Y is equal to or larger than Y_{exp}^*. Since the function $\pi(q)$ is concave in q and upward-sloping, the optimal capacity expansion threshold curve is upward-sloping: Y_{exp}^* increases as q increases

$$B(q) = \left(\frac{\beta - 1}{k}\right)^{\beta - 1} \int\limits_{q}^{\infty} \left(\frac{\pi'(x)}{\beta(r - \mu)}\right)^{\beta} dx \qquad (6)$$

We assume the convergence of the integral in Equation 6.

Equation 5 shows that the value of the project can be broken into two terms characterized by different stochastic processes. The first term represents the value of the assets in place, V, and the second term represents the value of the growth options, G.[15] It follows that we can write

$$W = V + G = aY + bY^{\beta} \qquad (7)$$

where $a = \frac{\pi(q)}{(r - \mu)}$ and $b = B(q)$.

The dynamics of the assets in place and of the growth options are not a concern in Pindyck (1988) and Dixit and Pindyck (1994) since these dynamics are irrelevant for determining optimal capacity expansion strategy. However, it is a concern for the initial investment decision, which is the focus of this paper. As is well known, the dynamics of the value of an investment can affect the investment decision when the decision is not a now or never proposition. For example, to invest early, the firm requires that the expected rate of capital gain on the project is less than the expected rate of return from owning a completed project. Otherwise, the firm would always be better off keeping the option to invest alive. Hence, the final task in this section is to characterize the stochastic processes underlying the two components of the project value. Applying Ito's Lemma, the dynamics of the value of the assets in place, V, can be obtained as

$$dV_t = \mu V_t dt + \sigma V_t dw_t \qquad (8)$$

Equation 8 shows that the expected growth rate of the value of the assets in place is less than r (because $\mu < r$). The difference $r - \mu$ represents an opportunity cost of delaying the establishment in the market. Therefore, the optimal investment policy has to be trade off between the cost and the risk of uncertainty in the future.

Similarly, the dynamics of the value of the growth options, G, can be found as

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[15]To see that the first term is the value of the assets in place, we could obtain it by directly computing the expected present value of the assets in place, which is straightforward.

$$dG_t = rG_t dt + \beta\sigma G_t dw_t \tag{9}$$

Equation 9 indicates that, unlike the value of the assets in place, the growth options would yield their appropriate risk-adjusted returns from their expected value gains alone.[16] There is no cost to keeping the options alive: delaying investment in the project incurs no opportunity cost in terms of the growth options.[17] Because $\beta > 1$, Equation 9 also shows that the growth options have a higher risk than the assets in place. In other words, $\beta\sigma > \sigma$.

That the entire return on the growth options is captured in its value movements is consistent with our observation that the growth options yield no cash flows. However, the converse is not always true. In other cases, the growth options could yield no cash flows, but there could be an opportunity cost of delaying acquisition of the growth options. The opportunity cost associated with delay could reflect entry and expansion by competitors, learning curve effects, and so forth.

4 The Infinite-Horizon Case

Now we turn to our primary concern: the impact of growth options on the initial investment. In this section we consider the impact assuming that the initial opportunity to invest will be long-lived, which we characterize in the limit as an infinite-horizon setting.

4.1 Ignoring the Growth Options

To begin, assume that the firm ignores the growth options, so, for the purposes of decision-making $W = V$. We will then compare decision-making under this scenarios with decision-making when the growth options are explicitly considered.

Irrespective of the growth options, the initial decision to invest is a standard option problem, and therefore not amenable to NPV analysis. The decision to invest must take into account the option of delaying the investment, and therefore must be addressed by consideration of the investment's

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[16]Note, in a risk-neutral framework, the total expected rate of return equals the risk-free rate of interest, r.

[17]Note that such dynamic characteristics are a necessary condition for the growth options to be irrelevant to the initial decision-making, but not a sufficient condition.

option value. Let Y^* be the firm's investment threshold: the firm will invest in the facility if $Y \geq Y^*$. Let $f(Y)$ denote the value of the option to make the initial investment. Recall that the current value of the assets in place, were the option to be exercised, is $V = aY$. The option pricing approach yields a valuation equation relating $f(Y)$ to Y when Y is less than the optimal investment threshold, Y^*. That is, $f(Y)$ must satisfy the following familiar differential equation

$$\frac{1}{2}\sigma^2 Y^2 f''(Y) + \mu Y f'(Y) - rf = 0; \qquad Y < Y^* \tag{10}$$

subject to the boundary conditions

$$f = aY - I; \qquad Y = Y^* \tag{11}$$
$$f'(Y) = a; \qquad Y = Y^* \tag{12}$$
$$f(Y) = 0; \qquad Y = 0 \tag{13}$$

Equations 11 and 12 are standard value-matching and smooth-pasting conditions for real options. Equation 13 comes from the assumption that the asset in place would be worth nothing if there is no demand. As shown by Equation 8, the value of the asset in place follows a geometric Brownian motion. Thus, if the value of the asset in place goes to zero, it will stay at zero and the investment option will be of no value.

It is straightforward to show that the solution to the above differential equation is given by

$$f = (\frac{a}{\beta})^\beta (\frac{\beta - 1}{I})^{\beta - 1} Y^\beta \tag{14}$$

where β is the positive root of the quadratic equation $\frac{1}{2}\sigma^2 x(x-1)+\mu x-r = 0$. The optimal investment threshold Y^* can be found as

$$Y^* = \frac{\beta}{a(\beta - 1)} I \tag{15}$$

Expressing the decision rule in terms of the stochastic demand parameter, Y, eliminates potential comparisons with decisions based on the NPV rule. Therefore, we would like to express the investment threshold in terms of the value of the assets in place. Since $a > 0$, V is a strictly increasing function of

Y. The optimal investment threshold Y^* in (15) corresponds to a unique V^*, that is, $V^* = aY^*$. Substituting it into Equation 15, we obtain a standard result in real options literature,

$$V^* = I + \frac{1}{\beta - 1}I \tag{16}$$

Equation (16) shows that the firm will invest whenever the value of the asset in place is larger than or equal to the cost of the investment I, plus the opportunity cost of exercising the investment option, $\frac{1}{(\beta-1)}I$.

4.2 Considering Growth Options

Now assume that the firm considers the value of the growth options—assumed to be exercised optimally—in making the initial decision on whether or not to invest. Hence, $W = V + G$.

Let $F(Y)$ denote the value of the initial investment option when we consider the growth options (to be contrasted with f in the last subsection where we did not consider the growth options). $F(Y)$ must satisfy the following differential equation

$$1/2\sigma^2 Y^2 F''(Y) + \mu Y F'(Y) - rF = 0; \qquad Y < Y^* \tag{17}$$

subject to the boundary conditions

$$F(Y) = aY + bY^\beta - I; \qquad Y = Y^* \tag{18}$$
$$F'(Y) = a + b\beta Y^{\beta-1}; \qquad Y = Y^* \tag{19}$$
$$F(Y) = 0; \qquad Y = 0 \tag{20}$$

Note that the dynamics of F are similar to that of f except their boundary conditions.

The solution to the above differential equation is given by

$$F(Y) = bY^\beta + (\frac{a}{\beta})^\beta(\frac{\beta-1}{I})^{\beta-1}Y^\beta \tag{21}$$

Comparison with Equation (14) shows that the value of the option to invest in the production facility is simply the sum of the value of the option to

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invest in the assets in place (the second term on the right side of Equation 21) and the value of the growth options ($G = bY^\beta$).

The optimal investment threshold Y^* is

$$Y^* = \frac{\beta I}{a(\beta - 1)} \tag{22}$$

This is exactly the result as for the case without growth options, which leads to the following proposition:

Proposition 1 *The investment decision-making in an infinite-time horizon is independent of the firm's capacity growth options. Investment timing is completely determined by the dynamics of the value of the assets in place.*

Proposition 1 says that, under the circumstances in this theoretical study, it is feasible to make investment decisions involving growth options as if there were no growth options, simplifying decision-making. As shown by the dynamics of the growth options in equation (9), there is no opportunity cost to keep the growth options alive. It seems to suggest that the value of growth options with such characteristics would be irrelevant to investment decision-making. However, it is worth pointing out that it is not always true even when an initial investment opportunity is infinite. Although such dynamic characteristics of growth options are a necessary condition for its irrelevance to initial investment decision-making, it is not a sufficient condition.[18] As we will discuss later, this also does not prove to be the case when the initial investment is finite. It is therefore useful to explore what it is about the infinite case that leads to this result.

Growth options can affect initial investment decision-making only if there is a link between the timing of the initial investment and the potential profits available from the growth options. By construction, the infinite case essentially eliminates any such link. Assume, as we have, that the initial investment will place the firm in a "no-expansion" zone: after the initial investment, the firm will not be inclined to immediately exercise any growth options unless market conditions improve. Investing earlier (i.e., at a lower Y) would certainly allow the firm to exercise the growth options earlier, but why? If the firm has no reason to exercise growth options at Y^*, it certainly will not have reason to exercise them at $Y < Y^*$. Exercising earlier will not

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[18]One such example would be abandonment option. See Dixit (1989).

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in any way enhance the potential profits from the growth options. This point will be made more clear when we consider the finite case: the finite case will differ because there will come a point at which a failure to invest will forestall any ability to exercise the growth options.[19]

As a final point, it is important to note that decision-making and a value are two different matters. Growth options may not change the initial investment timing, but they do increase the value of the firm because the payoff from the investment comes from both the assets in place and the growth options.

5 The Finite-Horizon Case

Now let us turn to a situation in which the initial opportunity to invest is short-lived. Note that once the investment is made, the option to expand is infinite; it is only the initial opportunity to invest that is short-lived. This section provides a proof (rather than simply showing counter-examples) to confirm that growth options will *always* cause the firm to invest earlier in the finite horizon case, in contrast to the infinite horizon case. We discuss this intuitive result at the end of this section.

Assume that the investment opportunity expires at time T and the firm explicitly considers the growth options in its investment decision-making. Let J be the value of the investment option. Given our assumptions, this value must satisfy the following partial differential equation:

$$\frac{\partial J}{\partial t} + \mu Y \frac{\partial J}{\partial Y} + \frac{1}{2}\sigma^2 Y^2 \frac{\partial^2 J}{\partial Y^2} - rJ = 0; \quad Y < Y^* \ \& \ t < T \qquad (23)$$

subject to the boundary conditions

$$J = \max(aY + bY^\beta - I, 0); \quad t = T \qquad (24)$$
$$J = aY + bY^\beta - I; \quad Y = Y^* \ \& \ t < T \qquad (25)$$
$$\partial J/\partial Y = a + b\beta Y^{\beta-1}; \quad Y = Y^* \ \& \ t < T \qquad (26)$$
$$J = 0; \quad Y = 0 \qquad (27)$$

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[19]Note that the assumption of starting in the "no-expansion" zone is essentially irrelevant. If we begin the problem in the expansion zone, then the firm invests immediately growth options or not. This is a trivial case.

Condition (24) says that, at expiration, the investment option will be exercised if the value of the investment is larger than the cost I. Conditions (25), (26) and (27) follow those for the infinite horizon case with the growth options.

To understand the impact of growth options on decision-making in this finite-horizon context, is necessary to determine $J(t, Y)$ and the corresponding decision rule. Unfortunately, it is not feasible to solve analytically the partial differential Equation 23 subject to the boundary conditions in Equations 24 - 27. The difficulty is due to the fact that the equation has to be solved differently for the two regions which are divided by the optimal investment threshold Y^*, and this time-dependent optimal investment threshold must be found as part of the solution.

It would certainly be possible to solve the problem numerically using a specific set of parameters. While this approach might prove informative, it would not be a general result. Hence, if a general result is desired a different approach is required. Here, we provide such an approach. We first follow Carr, Jarrow and Myneni (1992) to develop an alternative characterization for the value of investment option, and then use this characterization to derive a general result. Proposition 2 provides this characterization.[20]

Proposition 2 *At any time $t \in [0, T]$, if the value of the option to invest in the production facility is not equal to $max(0, V_t + G_t - I)$, then it must satisfy the following stochastic representation:*

$$J = E_t[e^{-r(T-t)}(V_T + G_T - I)^+] + \int_t^T e^{-r(\tau-t)} E_t \left[((r-\mu)V_\tau - rI) 1_{\{V_\tau \geq V_\tau^*\}} \right] d\tau$$

(28)

where E_t is the conditional expectation at time t, and V_τ^* is the optimal investment threshold at time τ in terms of V.

Proposition 2 states that the value of the option to invest can be expressed as the value of the option to invest at the expiration date T plus an expectation of a stochastic integral that may be considered as a gain from a feasible early investment. Equation 28 shows that this early investment premium does not include the growth options. It might appear that the growth

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[20]For brevity here, a proof is provided in the appendix.

options have no impact on the optimal investment timing rule, except at the expiration date T. However, that is not the case here. The following proposition states the impact of the capacity growth options on the optimal investment timing when the investment opportunity is not infinitely lived.

Proposition 3 *When an investment opportunity is not infinitely lived, the capacity growth options not only affect the investment decision at the expiration date, but also encourage firm to invest early.*

To find out if the V^* in Equation 28 would be the same as the one without the growth options, we can similarly express the value of the option to invest in the assets in place as

$$f(t,V) = E_t[e^{-r(T-t)}(V_T - I)^+] + \int_t^T e^{-r(\tau-t)} E_t \left[((r-\mu)V_\tau - rI)1_{\{V_\tau \geq V_\tau^*\}} \right] d\tau \tag{29}$$

Our proposition 1 implies that if V^* in Equations 28 and 29 are the same, then $J = G + f$. Now substituting the optimal investment threshold determined by Equation 28 into Equation 29, the second term in both equations, early investment premium, should be the same. And then, combining Equations 28 and 29, we obtain

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\begin{aligned}
J &= E_t[e^{-r(T-t)}(V_T + G_T - I)^+] + f - E_t[e^{-r(T-t)}(V_T - I)^+] \\
&= f + e^{-r(T-t)}E_t[(V_T + G_T - I)^+ - (V_T - I)^+] \\
&< f + G
\end{aligned}
\tag{30}
$$

Since $J < f + G$, the early investment thresholds, V^*, in Equations 28 and 29 are different. Furthermore, Equation 30 indicates that the investment threshold is lower when the firm considers the growth options. To see this, let V_G^* and V_O^* be the investment thresholds at time t with and without growth options. From Equations 25 and 30[21], we obtain $J = V_G^* + G^* - I < f + G^*$. That is, at $V = V_G^*$, we would have $f > V_G^* - I$. At the investment threshold for the firm *with* growth options, it would not be appropriate to invest *without* growth options. The investment threshold with growth options is lower than without growth options (i.e., $V_G^* < V_O^*$).

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[21]Note f in Equation 30 is calculated by using the optimal investment threshold implicitly defined in Equation 28.
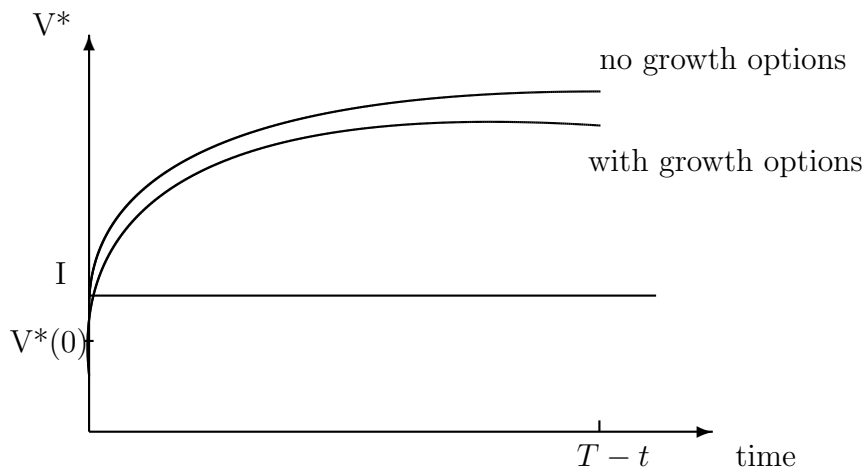
Figure 1: Investment Threshold as a Function of Time to Expiration

Figure 1 illustrates the dependence of the optimal investment threshold on the time to expiration and the impact of growth options on investment timing. The optimal investment thresholds are nondecreasing functions of time to expiration—the closer we get, the more pressing the need to take advantage of the investment. With the growth options, the optimal investment threshold is lower than if the firm were only considering the value from the assets in place. Hence, the firm is more likely to invest in the project if it takes the growth options into consideration.

Now let us turn to the intuition behind the finite horizon case and, by extension, the infinite horizon case. Again, growth options will impact initial investment decision-making only if the approach to the initial investment has an impact on the ability to exercise the growth options. The finite case provides this opportunity. If the firm does not exercise the initial investment option at the expiration date (assuming it has not already done so), the opportunity to exercise the growth options and therefore obtain their attendant profits will be lost. This obligates the firm to invest at a lower Y than would otherwise be the case at expiration; the firm is more aggressive because it doesn't want to miss the opportunity to gain growth option profits. Working backwards from the expiration date, the firm must consider that, if it is going to invest at a lower Y anyway, perhaps it would be useful to do so earlier as well to take advantage of any potential profits in the interim. This calculus

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operates all the way back to the present, with the impact of the early exercise attenuating as we work backwards in time. The growth options help offset the volatility effect. The infinite case differs simply in that there exists no such finality; there is no point at which the firm must consider that if it does not act quickly it might forego profits from the growth options.

6   Conclusion

Growth options are an important characteristic of a multitude of investments; understanding their impacts, and the circumstance under which they might or might not be largely ignored, is therefore valuable for firm decision-making. This paper has used one type of growth options—capacity expansion growth options—as a vehicle to better understand the implications of growth options on decision making. In this case, an initial investment buys not only a set of assets in place, but also the potential to expand capacity as appropriate in the future. The initial investment decision in these capacity expansion cases might therefore hinge not only on the expected cash flows generated from the assets in place, but also on the capacity expansion options they create. This paper has explored the impact of capacity expansion growth options on the timing of the decision to put initial assets in place when product demand varies stochastically over time. The fundamental question of the paper is: do these growth options alter the initial decision to invest?

The results of the analysis indicate that growth options need not always alter the initial decision to invest. When the option to invest is infinite, the capacity expansion options are irrelevant to the initial investment decision, although they do increase the potential benefits of investment. While this result is specific to the capacity expansion case, it reinforces the notion that a variety of factors influence the role of growth options; the lifetime of the initial investment option is one of them. As a counterpoint to the infinite case, we have also confirmed through proof (rather than through numerical example) that capacity expansion growth options *do* always alter the initial decision to invest when the initial opportunity is finite. In this case, consistent with intuition, they call for an early investment trigger.

Taken in total, the analysis suggests that the lifetime of the initial opportunity to invest—for example, to enter a new market—has important implications for the relevance of growth options to decision-making. All other things being equal, the longer is this initial opportunity, the less important

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will be growth options. The infinite case represents the theoretical extreme of this intuition. Large potential profits from growth options may not, by themselves, be enough to justify early action.

A   Appendix: Proof of Proposition 2

Proof. For simplicity, we assume the current time is $t = 0$. Notice that $V = aY^{\frac{1}{1-\nu}}$ and $G = bY^{\beta}$, and both are continuous and monotonic increasing functions on $[0, +\infty)$. Therefore, there is a one-to-one relationship between V and Y, and a one-to-one relationship between G and Y. J can be expressed as $J(t, Y_t)$ on $[0, T] \times [0, +\infty)$. Let $e^{-rt}J(t, Y_t)$ be the discounted value of the investment option, defined in the region $D \equiv \{(t, Y) : t \in [0, T], Y \in [0, +\infty)\}$. An application of Ito's rule to the process $e^{-rt}J(t, Y_t)$ yields:

$$e^{-r\tau}J(T, Y_T) - J(0, Y_0) - \int_0^T A(e^{-rt}J(t, Y_t))dt = \sigma \int_0^T e^{-rt}Y_t J_Y'(t, Y_t)dw(t) \tag{31}$$

where A is a linear operator, defined as

$$A = \frac{\partial}{\partial t} + (r - \delta)Y\frac{\partial}{\partial Y} + \frac{1}{2}\sigma^2 Y^2 \frac{\partial}{\partial Y^2}$$

Since $e^{-rt}Y_t J_Y'(t, Y_t)$ satisfies a polynomial growth condition on the region D, the right hand side of (31) is a martingale. By taking expectations we obtain

$$J(0, Y_0) = E\left[e^{-rT}J(T, Y_T)\right] - E\left[\int_0^T A(e^{-rt}J(t, Y_t))dt\right]$$

To find the value for the second term on the right side of the above equation, we separate the value of the investment option into two regions, $J(t, Y_t) = J(t, Y_t)1_{\{Y_t < Y_t^*\}} + J(t, Y_t)1_{\{Y_t \geq Y_t^*\}}$, and applying the operator A to the process $e^{-rt}J(t, Y_t)$, obtaining

$$A(e^{-rt}J(t, Y_t)) = A(e^{-rt}J(t, Y_t)1_{\{Y_t < Y_t^*\}}) + A(e^{-rt}J(t, Y_t)1_{\{Y_t \geq Y_t^*\}})$$

From (23), we know that when $Y_t < Y_t^*$

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$$A(e^{-rt}J(t, Y_t)) = 0$$

and when $Y_t > Y_t^*$, from (25) we have

$$
\begin{aligned}
&A(e^{-rt}J(t, Y_t)) \\
&= e^{-rt}\left\{[\frac{1}{2}\sigma^2\beta(\beta-1) + (r-\delta)\beta - r]bY^\beta + [\frac{1}{2}\sigma^2\frac{\nu}{(1-\nu)^2} + \frac{r-\delta}{1-\nu} - r]aY^{\frac{1}{1-\nu}} + rI\right\} \\
&= e^{-rt}\left\{rI - \eta a Y^{\frac{1}{1-\nu}}\right\}
\end{aligned}
\tag{32}
$$

Where the last line (32) uses the definition of η and the fact that β satisfies the quadratic equation 7. Finally, the proof is completed by recognizing the fact that $V = aY^{\frac{1}{1-nu}}$ and $V^* = a(Y^*)^{\frac{1}{1-\nu}}$, in conjunction with condition 24.

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