The Autoregressive Conditional Duration Model under Price and Duration Momenta

Min-Hsien Chiang
Institute of International Business
National Cheng Kung University
Tainan, TAIWAN

February 2005

Correspondence:
Min-Hsien Chiang
Associate Professor
Institute of International Business
National Cheng Kung University
Tainan, 70101
TAIWAN
TEL: +886-6-2757575 ext. 53511
FAX: +886-6-2766459
Email: mchiang@mail.ncku.edu.tw

ABSTRACT

This paper presents a model for the analysis of duration data, called momentum autoregressive conditional duration (MACD) model, which considers price and duration changes. The model allows the process of the conditional expected duration to switch in a smooth transition process, which broadens the autoregressive conditional duration (ACD) model in Engle and Russell (1998). The model is applied to empirical data and estimation results indicate that the process of the conditional expected durations is nonlinear and positively affected by the unexpected trade durations in the downward market. The negative shocks generate a larger impact on trade durations than positive shocks.

JEL: C51, G14

Keywords: Autoregressive Conditional Duration Model, Smooth Transition, Trade Duration, Momentum

1 Introduction

The information residing in the asset prices is revealed to the market through investors' trades. Thus, retrieving and extracting information from the order sequence can help us understand the information assimilation process of asset prices. Easley and O'Hara (1992) build up a model demonstrating that time between trades may contain information due to event uncertainty. Accordingly, the correlation between time between trades and information occurs as the order sequence can be connected with time between trades.

The seminal paper by Engle and Russell (1998) establishes the autoregressive conditional duration model (ACD) to capture the dynamic behavior of trade durations (time between trades). The ACD model considers the intensity of trades and expresses the conditional expectation of trade durations as an autoregressive relationship of past trade durations. Such modeling for high-frequency data has been proven useful in examining empirical implications of the market microstructure theory. Engle and Russell (1997) apply the ACD model to investigate foreign exchange quotes and support the asymmetric information model of price setting. Hamelink (1998) discovers significant correlation between durations and returns of French CAC 40 using the ACD model. Consequently, durations are a channel of information of asset prices.

Following Engle and Russell (1998), there are several duration models put forth. Jasiak (1998) considers the long range of time dependence in trade durations and introduces a fractionally integrated ACD model. Bauwens and Giot (2000) consider the logarithmic ACD model which is able to avoid the positivity constrain in parameters. Gramming and Maurer (2000) apply the Burr distribution to make the conditional hazard function more flexible. Zhang et al. (2001) propose a threshold ACD (TACD) model in which the nonlinearity in the ACD model depends on the past trade duration. Bauwens and Giot (2003) propose an asymmetric ACD model in which the asymmetry in the ACD model relies on price change states. Bauwens and Veredas (2004) introduce a stochastic conditional duration (SCD) model with a mixture of distributions which allows the conditional expected duration to be random. Ghysels et al. (2004) also develop an ACD model with a mixture of distributions, called the stochastic volatility duration (SVD) model, in which the conditional mean and the conditional overdispersion are driven by two dynamic factors. Considering the relationship between price movements and trade durations, Russell and Engle (2004) propose a joint model of the autoregressive conditional multinomial model (ACM) and the autoregressive conditional duration (ACD) model, called ACM-ACD model, in which the discrete price movements are modeled as a multinomial model. In general, the variants of the ACD model above rest on features of nonlinearity, past price changes, past durations, or distribution assumptions. However, neither of them considers the combined features of nonlinearity, past price changes and past durations.

Previous empirical studies have indicated that past price changes and past trade durations are important factors influencing the trade duration process. Hamelink (1998) studies the French CAC 40 by clustering duration regimes based on returns and durations. He finds that the duration processes are distinct among different regimes. Zhang et al. (2001) apply the threshold ACD (TACD) model to the IBM stock in the Trades, Orders, Reports and Quotes (TORQ) data set and find that nonlinearity in the ACD model identified by past trade durations delineates the trade duration process much better. On the other hand, Bauwens and Giot (2003) apply the asymmetric ACD model to IBM and Disney stocks and find that there exist asymmetries in the trade duration process during the upward and downward price changes. Examining Airgas stock (ticker symbol ARG) using the ACM-ACD model, Russell and Engle (2004) find that past price returns influence trade durations. Chiang et al. (2005) study the trade durations of the futures markets using the ACD model and find that the trade duration dynamics vary according to the size of price changes. Consequently, the trade duration process is not a simple linear function of past trade durations but can be affected by sizes of past price changes and past trade durations.

This paper proposes a nonlinear type of the autoregressive conditional duration model, called momentum autoregressive conditional duration (MACD) model, in which the previous price change and the previous duration change are taken into consideration. The nonlinearity in the MACD model allows the process of the conditional expected trade duration to follow a regime switching behavior. In the meantime, the regime switches in a smooth transition process where the smooth transition probabilities vary according to momenta of previous price and duration (i.e., trends of price change and duration change). Originated by Bacon and Watts (1971) and made popular and in-depth by Teräsvirta (1994) and Granger and Teräsvirta (1993), the smooth transition model characterizes the regime switching process as gradualness and continuity instead of instancy in the Markov switching model or of discontinuity in the threshold model. The gradualness and continuity properties make the smooth transition model more suitable for high-frequency data since intraday trades are so intensive and are likely to be persistent.[1] (Lin et al. (1995), Hasbrouck (1991), and Choi et al. (1988)) Consequently, the MACD model is able to capture the process of the conditional expected trade duration subject to variations in price and duration.

---

[1] The order persistence is referred to an order submission phenomenon that buy (sell) orders tend to follow buy (sell) order.

The MACD model is employed to investigate intraday transaction data of the IBM stock traded on the New York Stock Exchange (NYSE). Empirical results show that the process of the conditional expected trade durations is nonlinear. The conditional expected trade duration is significantly affected by the unexpected trade duration in the downward market while the persistence of trade durations has relatively larger impact on the conditional expected trade duration in the upward market. Simulated moments point out that means of trade durations in the upward market is larger than in the downward market. Thus, the trading activity is more intensive in the downward market. In addition, results of the generalized impulse response function indicate that negative shocks have a larger impact on the trade durations than positive shocks.

The remainder of this paper is organized as follows. Section 2 reviews the ACD model and logarithmic ACD model. The specification and the likelihood function of the momentum ACD model are constructed. Section 3 develops the specification tests of the momentum ACD model. Section 4 presents and discusses the empirical results. Section 5 concludes.

2 The Specification of the Momentum ACD

The autoregressive conditional duration (ACD) model by Engle and Russell (1998) is an intraday duration model which accommodates the clustering property of durations. Fundamentally, Engle and Russell (1998) consider a sequence of arrival times and assume that the actual duration can be expressed as a random process in which the conditional expected duration is an autoregressive process of past conditional and actual durations. Let $x_i = t_i - t_{i-1}$ be the duration between two successive arrival times, t_i and t_{i-1}. The observed trade duration in the ACD model can be represented as follows:

$$x_i = \Theta_i \, \varepsilon_i \tag{1}$$

where $\{\varepsilon_i\}$ are positive *i.i.d.* random variables with a certain distribution over $(0, \infty)$, and $E(\varepsilon_i) = u$. Therefore, the conditional distribution of x_i given a certain distribution of $\{\varepsilon_i\}$ and an information set of Ω_{i-1} up to the transaction $i-1$, $E(x_i \mid \Omega_{i-1})$, is ψ_i equal to $\Theta_i u$ which is specified as follows:[2]

[2] Basically, the ACD model can have more lags of x_i and ψ_i. We use ACD (1,1) throughout the paper since empirical results in most previous studies show one lag is enough (e.g., Hamelink (1998), Bauwens and Giot (2003), and Zhang et al. (2001)).

$$\psi_i = w + \alpha x_{i-1} + \beta \psi_{i-1} \tag{2}$$

where the root of the difference equation lies outside the unit circle and w is larger than zero. Those two conditions guarantee that the unconditional mean of the duration exists and is positive. On the other hand, Bauwens and Giot (2000) propose a logarithmic ACD model, called log-ACD, in which the positivity constraint in the ACD model can be relaxed. The observed duration in the log-ACD model is expressed as follows:

$$x_i = \Theta_i \varepsilon_i = \exp(\phi_i)\varepsilon_i \tag{3}$$

where the logarithm of the conditional mean of x_i given a certain distribution of $\{\varepsilon_i\}$ and an information set of Ω_{i-1} up to the transaction $i-1$, $\ln[E(x_i \mid \Omega_{i-1})]$, is ψ_i which is equal to $\phi_i + \ln u$. In addition, the logarithm of the conditional mean, ψ_i, can be specified as follows:

$$\psi_i = w + \alpha \varepsilon_{i-1} + \beta \psi_{i-1} \tag{4}$$

where $|\beta|$ is less than 1 in order to make sure the stationarity of ψ_i (Geweke (1986) and Bauwens et al. (2004)). Actually, the specification of the log-ACD model is close to exponential GARCH setting in Nelson (1991) and assures the non-negative conditional expectation of the trade duration.

We consider a new nonlinear ACD model which includes the feature of log-ACD and makes use of information contained in the price and duration movements, i.e., momenta of price and duration. The observed trade duration in this new model takes the form of the log-ACD model as in the equation (3). In addition, the logarithm of the conditional trade duration is modeled as a smooth transition model. Consider a transaction i with a price change $\delta_i = p_i - p_{i-1}$ where p_i and p_{i-1} are two successive transaction prices and with a trade duration $x_i = t_i - t_{i-1}$ where t_i and t_{i-1} are two successive arrival times, t_i and t_{i-1}. The standardized durations, $\varepsilon_i = \frac{x_i}{\Theta_i}$, are assumed to be *i.i.d.* with some distribution over $(0, \infty)$ as in Engle and Russell (1998). Then, the conditional duration which is affected by the price change can be modeled as the following specification with a smooth transition probability, $G_1(\delta_{i-1}; \gamma_{i-1}, c_1)$:

$$\psi^{\delta}_{i(1)} = (\Psi_1(1 - G_1(\delta_{i-1};\gamma_1, c_1)) + \Psi_2 G_1(\delta_{i-1};\gamma_1, c_1)) \tag{5}$$

$$\Psi_1 = w_1 + \alpha_1 \varepsilon_{i-1} + \beta_1 \psi_{i-1}$$
$$\Psi_2 = w_2 + \alpha_2 \varepsilon_{i-1} + \beta_2 \psi_{i-1}$$

where $G_1(\delta_{i-1};\gamma_{i-1}, c_1)$ is a transition probability function with a state variable of price change, δ_{i-1}, which is assumed to be at least twice differentiable and range from 0 to 1, γ_1 is the smoothness parameter, and c_1 is a threshold value. The specification in equation (5) models the conditional duration process as a regime switching process in which the conditional duration process is governed by the price changes. This kind of specification accommodates the features of allowing the conditional duration process to alternate when the price changes. The popular choice of the transition function is the logistic function:

$$G_1(\delta_{i-1};\gamma_1, c_1) = \frac{1}{1 + \exp(-\gamma_1(\delta_{i-1} - c_1))} \tag{6}$$

If the $\gamma_1 \to 0$, the transition probability becomes 0.5 and then equation (5) becomes linear. On the other hand, if $\gamma_1 \to \infty$, the transition probability function becomes a Heaviside function with a value of 1 when $\delta_{i-1} > c_1$ or with a value of 0 when $\delta_{i-1} > c_1$. In addition to the price change effects, the conditional duration process may be affected by the duration itself which causes the conditional duration process to take changes. Therefore, we expand equation (5) to the following specification:

$$\psi_{i(1)} = \psi^{\delta}_{i(1)}(1 - G_2(x_{i-1};\gamma_2, c_2)) + \psi^{\delta}_{i(2)} G_2(x_{i-1};\gamma_2, c_2) \tag{7}$$

$$\psi^{\delta}_{i(1)} = \Psi_1(1 - G_1(\delta_{i-1};\gamma_1, c_1)) + \Psi_2 G_1(\delta_{i-1};\gamma_1, c_1)$$

$$\psi^{\delta}_{i(2)} = \Psi_3(1 - G_1(\delta_{i-1};\gamma_1, c_1)) + \Psi_4 G_1(\delta_{i-1};\gamma_1, c_1)$$

where $G_2(x_{i-1};\gamma_2, c_2)$ is a transition probability function with a state variable of the lagged duration, x_{i-1}, γ_2 is the smoothness parameter, and c_2 is the threshold value associated with G_2. Therefore, G_2 can be written as follows:

$$G_2(x_{i-1};\gamma_2, c_2) = \frac{1}{1 + \exp(-\gamma_2(x_{i-1} - c_2))} \tag{8}$$

Like G_1, G_2 is assumed to be at least twice differentiable and ranges from 0 to 1 as well. Within equation (7) governed by the transition probability function, G_2, the movement of the conditional duration process transits between two conditional duration processes, $\psi_{i(1)}^{\delta}$ and $\psi_{i(2)}^{\delta}$, which are also duration processes governed by price changes. Accordingly, the specification in equation (7) contains the features which allow not only price changes to have impacts on the duration process but also past duration changes to affect the duration process. The model setting above adopts the framework of the multiple-regime smooth transition autoregressive model by Dijk and Franses (1999) which is an extension of basic smooth-transition autoregressive model (STAR) made popular by Chan and Tong (1986), Granger and Teräsvirta (1993) and Teräsvirta (1994). Since the conditional duration process above takes price changes and past duration effects into consideration, the model is called the momentum autoregressive conditional duration model (MACD).

As for the estimation of the MACD model, we adopt the maximum likelihood estimation approach. The distribution assumption of standardized durations is the Weibull distribution.[3] Accordingly, the associated conditional intensity process, λ, can be represented as follows (Engle and Russell (1998)):

$$\lambda(t \mid x_1, x_2, \ldots, x_i) = (\Gamma(1+\frac{1}{r})\ \Theta_i^{-1})^r (t-t_i)^{r-1} r \tag{9}$$

where $\Gamma(\cdot)$ is the gamma function and r is the Weibull distribution parameter which governs the shape of the conditional intensity process. Consequently, the density function under the Weibull distribution can be expressed as follows:

$$f(x_i \mid \Omega_{i-1}, \theta) = \frac{r}{x_i}\left(\frac{x_i\ \Gamma(1+\frac{1}{r})}{\Theta_i}\right)^r \exp\left(-\left[\frac{x_i\ \Gamma(1+\frac{1}{r})}{\Theta_i}\right]^r\right) \tag{10}$$

Furthermore, if the location parameter of the Weibull distribution is set to zero and the scale parameter is set to equal to Θ_i, then the density function becomes (Hamelink (1998) and Lee (1992)):

[3] The reason why we use the Weibull distribution assumption is that the Weibull distribution has been widely used in the literature, such as Engle and Russell (1997, 1998), Hamelink (1998), Bauwens and Giot (2000, 2003), and embeds a feature of non-constant conditional trading intensity function.

$$f(x_i \mid \Omega_{i-1}, \theta) = \frac{r}{\Theta_i}\left(\frac{x_i}{\Theta_i}\right)^{r-1} \exp\left(-\left[\frac{x_i}{\Theta_i}\right]^r\right) \tag{11}$$

Hence, the conditional intensity process will become:

$$\lambda(t \mid x_1, x_2, \ldots, x_i) = \Theta_i^{-r}(t - t_i)^{r-1} r \tag{12}$$

Following equation (11), the log-likelihood function for the MACD model can be written as follows:

$$\ln L = \sum_{i=1}^{N}\left[\log r - \log \Theta_i + (r-1)\log\left(\frac{x_i}{\Theta_i}\right) - \left(\frac{x_i}{\Theta_i}\right)^r\right] \tag{13}$$

3 The Specification Test for the Momentum ACD Model

To test for whether the MACD is much more suitable than a regular ACD, we adopt the following approach in which the test for the price change effect is diagnosed first and then the duration effect is examined subsequently. We notice that the trade duration process Ψ_j for a given regime *j* can be expressed as follows:

$$\Psi_j = w_j + \alpha_j \varepsilon_{i-1} + \beta_j \psi_{i-1} = \Pi_j' Y_{i-1}, \quad j = 1,2,3,4 \tag{14}$$

$$\Pi_j = [w_j, \alpha_j, \beta_j]'$$

$$Y_{i-1} = [1, \varepsilon_{i-1}, \psi_{i-1}]'$$

Therefore, in order to test whether the price changes affect the trade duration process, we first rearrange the equation (5) into the following form:

$$\psi_i^{\delta} = \frac{1}{2}(\Pi_1' + \Pi_2')Y_{i-1} + (\Pi_2' - \Pi_1')Y_{i-1}G_1^*(\delta_i; \gamma_1, c_1) \tag{15}$$

where $G_1^*(\delta_{i-1}; \gamma_1, c_1) = G_1(\delta_{i-1}; \gamma_1, c_1) - \frac{1}{2}$. Replacing the transition function

$G_1^*(\delta_{i-1};\gamma_1,c_1)$ with a third order Taylor expansion at $\gamma_1=0$ for equation (15), we can express equation (15) as follows:

$$\psi_{i(1)}^{\delta}=\lambda_0^{\delta}+\Phi_{\delta}'Y_{i-1}+\varsigma_1 Y_{i-1}\delta_{i-1}+\varsigma_2 Y_{i-1}\delta_{i-1}^2+\varsigma_3 Y_{i-1}\delta_{i-1}^3 \tag{16}$$

$$\Phi_{\delta}'=\frac{1}{2}(\Pi_1'+\Pi_2')-\frac{1}{4}\gamma_1 c(\Pi_2'-\Pi_1')-\frac{1}{48}\gamma_1^3c^3(\Pi_2'-\Pi_1')$$

$$\varsigma_1=\frac{1}{4}\gamma_1(\Pi_2'-\Pi_1')+\frac{1}{16}\gamma_1^3c^2(\Pi_2'-\Pi_1')$$

$$\varsigma_2=-\frac{1}{16}\gamma_1^3c^2(\Pi_2'-\Pi_1')$$

$$\varsigma_3=\frac{1}{48}\gamma_1^3(\Pi_2'-\Pi_1')$$

where λ_0^{δ} is the intercept. This kind of transformation can avoid the Davies problem. That is, there are some unidentified nuisance parameters under the null hypothesis. This method is proposed by Luukkonen et al. (1988). After the transformation, the null hypothesis $H_0:\gamma_1=0$ can be restated as $H_0:\varsigma_1=\varsigma_2=\varsigma_3=0$. Consequently, using equation (16) above and plugging it into the log likelihood function of equation (13), we have a test statistics of the lagrangian multiplier (LM) type, LM_{pc}, for no price change effects stated in the following theorem:

Theorem 1 *The LM-type test for no price change effect on the conditional duration model can be expressed as:*

$$LM_{pc}=S_{\delta}'H_{\delta}^{-1}S_{\delta}\Rightarrow\chi^2(9)$$

where Ξ is the parameter set and S_{δ} and H_{δ} are the followings:

$$S_{\delta}=\frac{\partial\ln L}{\delta\Xi_{\delta}}\Big|_{H_0}$$

$$H_{\delta}=\frac{\partial^2\ln L}{\partial\Xi_{\delta}\partial\Xi_{\delta}'}\Big|_{H_0}$$

Remark 1: *If the MACD model is extended to incorporate p lags of* ε_i *and q lags of* ψ_i*, then the degree of freedom is 3(p+q+1)*

The LM_{pc} test above involves the first and second derivatives, which may not be applicable when data expose some irregular behaviors. On the other hand, if we keep the parameter r of the Weibull distribution is fixed, the first derivative of the log likelihood function for each observation i under the null hypothesis can be calculated as follows:

$$\frac{\partial l_i}{\partial \Phi_\delta}\Big|_{H_0} = \zeta_i^\delta Y_{i-1} \tag{17}$$

$$\frac{\partial l_i}{\partial \varsigma_j}\Big|_{H_0} = \zeta_i^\delta Y_{i-1}\delta_{i-1}^j,\ j = 1,2,3 \tag{18}$$

$$\zeta_i^\delta = -r + r\left(\frac{x_i}{\Theta_i}\right)^r \tag{19}$$

As suggested by Luukkonen et al (1988) and Dijk and Franses (1999), the LM-type test statistic to test null hypothesis of no price change effects can be performed in the following steps:

Theorem 2 *The following procedure can be taken to form the* LM'_{pc} *:*

$$LM'_{pc} = \frac{(SSR_0^\delta - SSR_1^\delta)/9}{SSR_1^\delta/(T-12)} \Rightarrow F(9,(T-12))$$

where SSR_0 and SSR_1 are computed from the following steps:

1. The MACD model is estimated using equation (4) as the conditional expectation model and the associated $\hat{\zeta}_i^\delta$ in equation (19) are calculated.
2. Regress $\hat{\zeta}_i^\delta$ on Y_{i-1} to obtain residuals $\tilde{\zeta}_i^\delta$ and calculate the $SSR_0^\delta = \sum_{i=1}^N \tilde{\zeta}_i^{\delta^2}$
3. Regress $\tilde{\zeta}_i^\delta$ on $Y_{i-1}\delta_{i-1}^j,\ j = 1,2,3$ to obtain residuals $\bar{\zeta}_i^\delta$ and calculate $SSR_1^\delta = \sum_{i=1}^N \bar{\zeta}_i^{\delta^2}$

In order to test whether the duration effect exists, we follow the same procedure as we test the price change effect. First of all, equation (7) is rewritten as follows:

$$\psi_i^x = \Psi_1^* + \Psi_2^* G_1(\delta_{i-1};\gamma_1,c_1) + \Psi_3^* G_2(x_{i-1};\gamma_2,c_2) + \Psi_4^* G_1(\delta_{i-1};\gamma_1,c_1)G_2(x_{i-1};\gamma_2,c_2) \tag{20}$$

$$\Psi_1^* = \Psi_1, \Psi_2^* = \Psi_2 - \Psi_1$$

$$\Psi_3^* = \Psi_3 - \Psi_1, \Psi_4^* = \Psi_1 - \Psi_2 - \Psi_3 - \Psi_4$$

After replacing $G_2(x_{i-1};\gamma_2,c_2)$ with a third-order Taylor expansion around the point $\gamma_2(x_{i-1}-c_2)=0$, then the model becomes

$$\begin{aligned}\psi_i^x &= \lambda_0^x + \Phi_1' Y_{i-1} + \Phi_2' Y_{i-1} G_1(\delta_{i-1};\gamma_1,c_1) + \kappa_1 Y_{i-1} x_{i-1} + \kappa_2 Y_{i-1} x_{i-1}^2 + \kappa_3 Y_{i-1} x_{i-1}^3 + \\ &\quad (\kappa_4 Y_{i-1} x_{i-1} + \kappa_5 Y_{i-1} x_{i-1}^2 + \kappa_6 Y_{i-1} x_{i-1}^3) G_1(\delta_{i-1};\gamma_1,c_1)\end{aligned} \tag{21}$$

$$\Phi_1' = \Pi_1', \Phi_2' = (\Pi_2' - \Pi_1')$$

$$\begin{aligned}\kappa_1 &= \eta_1 \times (\Pi_3' - \Pi_1'),\\ \kappa_2 &= \eta_2 \times (\Pi_3' - \Pi_1'),\\ \kappa_3 &= \eta_3 \times (\Pi_3' - \Pi_1'),\\ \kappa_4 &= \eta_1 \times (\Pi_1' - \Pi_2' - \Pi_3' + \Pi_4'),\\ \kappa_5 &= \eta_2 \times (\Pi_1' - \Pi_2' - \Pi_3' + \Pi_4'),\\ \kappa_6 &= \eta_3 \times (\Pi_1' - \Pi_2' - \Pi_3' + \Pi_4')\end{aligned}$$

where λ_0^x is the intercept and η_1, η_2 ,and η_3 are correspondent multipliers. Therefore, the original null hypothesis of $H_0:\gamma_2=0$ can be restated as $H_0:\kappa_j=0$, for $j=1,\ldots,6$. Hence, the *LM-type* test statistic for no duration effect can be formed as follows:

Theorem 3 *The LM-type test for no duration effects on the conditional duration model can be expressed as:*

$$LM_D = S_D' H^{-1} S_D \Rightarrow \chi^2(18)$$

where Ξ is the parameter set and S_D and H_D are the followings:

$$S_D = \frac{\partial \ln L}{\partial \Xi}\Big|_{H_0}$$

$$H_D = \frac{\partial^2 \ln L}{\partial \Xi \partial \Xi'}\Big|_{H_0}$$

Remark 2: *If the MACD model is extended to incorporate p lags of* ε_i *and q lags of* ψ_i*, then the degree of freedom is 6(p+q+1)*

Like LM_{pc} test above, the first and second derivatives in LM_D may not be applicable when data expose some irregular behaviors. Hence, if we keep the parameter *r* of the Weibull distribution is fixed, the first derivative of the log likelihood function for each observation i under the null hypothesis of no duration effect can be obtained as follows:

$$\frac{\partial l_i}{\partial \Phi_1}\Big|_{H_0} = \zeta_i^D \mathrm{Y}_{i-1} \tag{22}$$

$$\frac{\partial l_i}{\partial \Phi_1}\Big|_{H_0} = \zeta_i^D \mathrm{Y}_{i-1} \tag{23}$$

$$\frac{\partial l_i}{\partial \Phi_2}\Big|_{H_0} = \zeta_i^D \mathrm{Y}_{i-1}\, G_1(\delta_{i-1};\gamma_1, c_1) \tag{24}$$

$$\frac{\partial l_i}{\partial \kappa_j}\Big|_{H_0} = \zeta_i^D \mathrm{Y}_{i-1}\, x_{i-1}^j,\ j = 1,2,3 \tag{25}$$

$$\frac{\partial l_i}{\partial \kappa_j}\Big|_{H_0} = \zeta_i^D \mathrm{Y}_{i-1}\, G_1(\delta_{i-1};\gamma_1, c_1) x_{i-1}^{j-3},\ j = 4,5,6 \tag{26}$$

$$\frac{\partial l_i}{\partial \gamma_1}\Bigg|_{H_0} = \zeta_i^D \Phi_2' Y_{i-1} \frac{\partial G_1(\delta_{i-1};\gamma_1, c_1)}{\partial \gamma_1} \tag{27}$$

$$\frac{\partial l_i}{\partial c_1}\Bigg|_{H_0} = \zeta_i^D \Phi_2' Y_{i-1} \frac{\partial G_1(\delta_{i-1};\gamma_1, c_1)}{\partial c_1} \tag{28}$$

$$\zeta_i^D = -r + r\left(\frac{x_i}{\Theta_i}\right)^r \tag{29}$$

$$\frac{\partial G_{i1}}{\partial \gamma_1} = G_{i1\gamma_1} = G_1[1-G_1](\delta_{i-1} - \widehat{c}_1) \tag{30}$$

$$\frac{\partial G_{i1}}{\partial c_1} = G_{i1c_1} = \widehat{\gamma}_1 G_1[1-G_1] \tag{31}$$

Consequently, the *LM*-type test statistic to test null hypothesis of no duration effect can be performed in the following steps:

Theorem 4 *The following procedure can be taken to form the* LM_D':

$$LM_D' = \frac{(SSR_0^D - SSR_1^D)/(18)}{SSR_1^D/(T-24)} \Rightarrow F(18,(T-24))$$

where SSR_0^D and SSR_1^D are computed from the following steps:

1. The momentum ACD model is estimated using equation (5) as the conditional expectation model and the associated $\hat{\zeta}_i^D$ in equation (29) are calculated. Then, we calculate the $SSR_0 = \sum_{i=1}^N \hat{\zeta}_i^{D^2}$
2. Regress $\hat{\zeta}_i^D$ on Y_{i-1}, $Y_{i-1}G_{i1}$, $\Phi_2 Y_{i-1} G_{i1\gamma_1}$, $\Phi_2 Y_{i-1} G_{i1c_1}$, $Y_{i-1}x_{i-1}^j (j=1,2,3)$, and $Y_{i-1}G_{i1}x_{i-1}^{j-3} (j=4,5,6)$ to obtain residuals $\tilde{\zeta}_i^D$ and calculate the $SSR_1^D = \sum_{i=1}^N \tilde{\zeta}_i^{D^2}$

4 Empirical Estimation of IBM Stock

In this section, we apply the MACD model to trade durations of the IBM stock in order to illustrate empirical implementation of the MACD model. The data description is shown first and then results of estimation and specification tests are presented. Next, the data are grouped into four different regimes according to price and duration changes. Finally, the moments, impulse responses and conditional trading intensity of four regimes are discussed.

4.1 Data Description

The empirical data are comprised of trades and quotes of the IBM stock traded on the New York Stock Exchange (NYSE). The data are collected from the Trades and Quotes (TAQ) database which is complied and made available by the NYSE. The transaction data are retrieved from the consolidated trade file while the quote data are retrieved from the consolidated quote file. In the meantime, the sample period covers the whole month of May 2001. The reason why we choose the month of May is that financial quarterly reports of companies are mostly announced and made public in April. This may impact the regular trading behavior of investors. Following Engle and Russell (1998) and Zhang et al. (2001), we discard the data during opening hours from 9:30 to 10:00 AM for reasons of opening delay and extreme short durations around the open.

Table 1 shows the summary statistics of trade durations in the sample. There are 68,801 transactions with distinct trading time. As in Engle and Russell (1998), Bauwens and Giot (2003), and Zhang et al. (2001), it is important to deseasonalize raw trade durations before estimating the model due to the trading structure of the exchange and the trading behavior of traders and market makers. The seasonally adjusted trade durations are constructed as a multiplicative form: $x_i^* = x_i / \phi(t_i)$, where x_i is the raw trade duration, $\phi(t_i)$ is the seasonal component, and x_i^* is the seasonally adjusted trade duration. As in Zhang et al. (2001), the seasonal component, $\phi(t_i)$, are obtained using super smoother method proposed by Friedman (1984) in which the local cross-validation is used to find the span. The mean and standard deviation of seasonally adjusted trade durations are smaller than of raw trade durations while the skewness and kurtosis of seasonally adjusted trade durations are similar to those of raw trade durations. Moreover, the autocorrelation in seasonally adjusted trade durations is still present as in raw trade durations.

4.2 The Estimation Results of the MACD model

Table 2 reports the estimation results for three different MACD models. Model 1 presents estimation results of the MACD model without price and trade duration changes. As expected, the parameters for the lagged unexpected trade duration, α_2, and the lagged expected trade duration, β_2, are all significant. The positive α_2 coefficient associated with the lagged unexpected trade duration indicates that the current expected trade duration becomes elongated when there exists an unexpected longer lagged duration. The β_2 coefficient is significantly less than one, which ensures that the estimated model is stationary. This also means that the current expected duration is positively affected by the accumulated information of past expected trade durations since the lagged expected trade durations represent the persistence of past trade durations. Meanwhile, the Weibull parameter is significantly different from one, which shows that the constant conditional trading intensity is not suitable for the model. The specification test shows that there still exists nonlinearity in Model 1. Ljung-Box tests point out the autocorrelation of standardized residuals.

The estimation results of the MACD model with price changes but without trade duration changes are presented in Model 2. From equation (6), we know that the transition probability will be larger than 0.5 if the price change tends to be greater than the threshold value, c_1, of the price change. Therefore, we call this regime as the upward price change regime with α_2 and β_2 coefficients once the lagged price change is greater than the threshold value while the downward price change regime with α_2 and β_2 coefficients is for the lagged price change to be less than the threshold value. It is found that the parameter, α_2, regarding the unexpected lagged trade duration in the upward price change regime is insignificant while the parameters, α_1, in the downward price change regime is significantly positive. On the other hand, the impact of the persistent trade duration on the conditional trade duration is smaller in the downward price change regime (β_1=0.8915) than in the upward price change regime (β_2=0.9982). Thus, this finding indicates that the upward and downward trends have distinct impacts on the trade duration behavior. This means that, following the upward price change, information of most recent past trades has been impounded in the asset price and then expected trade duration of the next trade becomes affected by the persistence of past trades.

The asymmetric findings during the upward and downward markets are consistent with a stylized fact in the finance literature. In general, the negative returns generate higher unexpected volatility than positive returns. (For example, French et al. (1987), Schwert (1989)) Engle and Ng (1993) demonstrate an asymmetric pattern of a news impact curve in which negative return shocks increase predictable volatility than positive return shocks. The reason for this asymmetric pattern during the upward and downward trends could be that investors are highly risk averse during the downward market. This leads to faster information assimilation to the asset price during the downward trend, which makes significant α_1 and smaller β_1 coefficient. Ljung-Box tests show no autocorrelation in the standardized residuals but the specification test still indicates existence of nonlinearity.

The MACD model considering both price and duration changes are presented in Model 3 of Table 2. Estimation results show a similar pattern to that in Model 2. It is found that coefficients, α_1 and α_3, associated with the lagged unexpected trade duration in the downward price change regime are significantly positive while coefficients, α_2 and α_4, associated with the lagged unexpected trade duration in the upward price change regime are insignificant. In the meantime, coefficients, β_1 and β_3, associated with lagged expected trade duration in the downward price change regime are smaller than coefficients, β_2 and β_4, associated with the expected trade duration in the upward price change regime. Ljung-Box tests show no autocorrelation in the standardized residuals.

4.3 Moments and Characteristics of four Regimes

In order to explore implications of the MACD model more, the data are grouped into four different regimes according to lagged price changes and lagged trade duration changes: (1) the PGDG regime in which the lagged price change is larger than or equal to zero and the lagged trade duration change is larger than or equal to the mean of trade durations, (2) the PLDG regime in which the lagged price change is less than zero and the lagged trade duration change is greater than or equal to the mean of trade durations, (3) the PGDL regime in which the lagged price change is larger than or equal to zero and the lagged trade duration change is less than the mean of trade durations, and (4) the PLDL regime in which the lagged price change is less than zero and the lagged trade duration change is less than the mean of trade durations. The PGDG and PGDL regimes stand for the upward price change regimes while the PLDG and PLDL regimes stand for the downward price change regimes. On the other hand, the PGDG and PLDG regimes represent the longer trade duration regime while the PGDL and PLDL represent the shorter trade duration regime. Panel A in Table 3 summarizes the market microstructure characteristics of 4 regimes. Information asymmetry theory in the market microstructure predicts that wider spreads, higher trading volume, and larger volatility are indicators of informed trading. It is found that the spread and trading volume per second in the PGDG regime are significantly larger than in the whole data set. Shown in Panel B of Table 3, the estimation results of a Logit Model with the PGDG regime as the choice regime also confirm that wider spreads and larger trading volume per second significantly and positively increase occurrence probability of the PGDG regime. We also notice that significantly higher volatility per second is present in the PLDG and PLDL regimes. This supports the asymmetric effect of higher volatility in the downward trend.

We also simulate 100 samples with a sample size of 50,000 for each regime to compute related moments. The simulated moments of four regimes are shown in Table 4. It is found that the means of trade durations under the upward price change regime are generally larger than under the downward price change regime. Meanwhile, the skewness, variance, and leptokurtosis are relatively larger in the PGDG regime than rests of regimes. This shows that the expected trade durations become larger when the lagged price change is in the upward price change regime. In addition, when the asset price is in the downward market, the expected trade duration will be shorter (longer) under the longer (shorter) previous expected trade duration. This is in contrast to the case of the upward market in which the longer expected trade duration follows the longer lagged expected trade duration. These findings may show that the investors trade more actively during the downward market than during the upward market. This is not surprising since investors react more aggressively in the downward market as stated in the literature.

4.4 Impulse Responses and Conditional Trading Intensity

The generalized impulse response functions (GIRF) by Koop et al. (1996) are calculated for four different regimes. The merit of GIRF over the traditional impulse response function is that the GIRF considers three factors in computing the impulse response function of a shock ε_i on the future duration x_{i+n}: (1) the history of the data generating process up to transaction *i*, (2) the size of shock at transaction *i*, and (3) the shocks between transaction *i* and transaction *i+n*. (Koop et al. (1996) and Dijk and Franses (1999)). Consequently, the GIRF for the current shock $\varepsilon_i = \eta_i$ and history $\Omega_{i-1} = \{\delta_{i-1}, x_{i-1}\} = w_{i-1}$ can be defined as follows:

$$GIRF(\eta_i, w_{i-1}, n) = E\left(x_{i+n} \mid \varepsilon_i = \eta_i, \Omega_{i-1} = w_{i-1}\right) - E\left(x_{i+n} \mid \Omega_{i-1} = w_{i-1}\right) \quad (32)$$

for n=1, 2, …., 40. That is, the GIRF is difference between expectation of x_{i+n} conditional on the current shock and history and expectation of x_{i+n} conditional only on the history. The simulation procedure by Koop et al. (1996) is performed by constructing 100 samples with a sample size of 1,000 for each shock size.

Of four regimes, Figures 1 to 4 illustrate generalized impulse responses for next 40 transactions of shock sizes of $\pm 1\sigma$, $\pm 2\sigma$, $\pm 3\sigma$, and $\pm 4\sigma$, where σ is the standard deviation of estimated residuals. The GIRF patterns of four regimes are very similar to each other. Negative shocks generally intrigue a larger impact of trade durations than positive shocks. In addition, the decrease of a negative shock is almost twice the increase of a corresponding positive shock. Consequently, the asymmetric impact patterns of negative shocks are present.

The conditional trading intensity, or hazard function, means the transaction arrival intensity given past price and duration changes. We find that the conditional trading intensity is higher for middle trade durations around 1.5 to 2.5 seconds but lower for smaller trade durations for four regimes. This indicates that transactions most likely occur when past transactions did not take too long or too short to be matched. This is consistent with findings in Zhang et al. (2001). Therefore, investors who are eager to have their orders matched are able to submit orders by observing past trade durations.

5 Conclusions

This paper proposes a momentum autoregressive conditional duration (MACD) model in which price changes and duration changes are taken into consideration. The MACD model expresses a nonlinear type of the autoregressive conditional duration (ACD) model with a regime-switching feature. The regime switches following a smooth transition function where the price changes and duration changes are state variables. Hence, the MACD model is able to capture a relationship between trade durations and momenta of price and trade duration. In the meantime, the specification tests of the MACD model are also constructed to test linearity against the smooth transition nonlinearity.

We apply the MACD model to intraday transaction data of IBM stock. Empirical results show that the nonlinearity exits in the conditional expected trade durations. It is found that the conditional expected trade duration is significantly and positively affected by the unexpected trade duration in the downward market. In addition, the impact of the persistence of trade durations on the conditional expected trade duration is larger during the upward market. Moreover, there exist asymmetric effects of negative shocks on trade durations. Negative shocks generate a larger impact on the trade durations than positive shocks. Consequently, the MACD model is able to provide more insights on the trade duration behavior.

References

Bacon, D. W. and D. C. Watts (1971), “Estimating the Transition between Two Interesting Intersecting Straight Lines,” Biometrika, 58, 525-534.

Bauwens, L. and P. Giot (2000), “The Logarithmic ACD Model: An Application to the Bid-Ask Quote Process of Three NYSE Stocks,” Annales d’Economie et de Statisique, 60, 117-149.

Bauwens, L. and P. Giot (2003), “Asymmetric ACD Models: Introducing Price Information in ACD Models with a Two State Transition Model,” Empirical Economics, 28, 709-731.

Bauwens, L. and D. Veredas (2004), “The Stochastic Conditional Duration Model: A Latent Variable Model for the Analysis of Financial Durations,” Journal of Econometrics, 119, 381-412.

Bauwens, L., P. Giot, J. Gramming and D. Veredas (2004), “A comparison of financial duration models through density forecasts,” International Journal of Forecasting, 20, 589-609.

Chan, K. S. and H. Tong (1986), “On Estimating Thresholds in Autoregressive Models,” Journal of Time Series Analysis, 7, 179-190.

Chiang, M.-H., L.-H. Fan and H.-L. Chen (2005), “On the Intradaily Relationship between Information Revelation and Trade Duration: The Evidence of MSCI Taiwan Stock Index Futures,” working paper, Institute of International Business, National Cheng Kung University.

Choi, J. Y., D. Salandro, and K. Shastri (1988), “On the Estimation of Bid-Ask Spreads: Theory and Evidence,” Journal of Financial and Quantitative Analysis, 23, 219-230.

Dijk, D. V. and P. H. Franses (1999), “Modeling Multiple Regimes in the Business Cycle,” Macroeconomic Dynamics, 3, 311-340.

Easley, D. and M. O’Hara (1992), “Time and the Process of Security Price Adjustment,” Journal of Finance, 47, 577-605.

Engle, R. F. and V. K. Ng (1993), “Measuring and Testing the Impact of News on Volatility,” Journal of Finance, 48, 1749-1778.

Engle, R. F. and J. R. Russell (1997), “Forecasting the Frequency of A Change in Quoted Foreign Exchange Prices with the Autoregressive Conditional Duration model,” Journal of Empirical Finance, 4, 87-212.

Engle, R. F. and J. R. Russell (1998), “Autoregressive Conditional Duration: A New Model for Irregularly Spaced Transaction Data,” Econometrica, 66, 1127-1162.

French, K. R., G. W. Schwert and R. F. Stambaugh (1987), “Expected Stock Returns and Volatility,” Journal of Financial Economics, 19, 3-29.

Friedman, J. H. (1984), “A Variable Span Smoother,” Technical Report No. 5, Laboratory for Computational Statistics, Department of Statistics, Stanford University.

Geweke, J. (1986), “Modelling the Persistence of Conditional Variances: A Comment,” Econometrics Review, 5, 57-61.

Ghysels, E., C. Gouriéroux and J. Jasiak (2004), “Stochastic Volatility Duration Models,” Journal of Econometrics, 119, 413-433.

Gramming, J. and K. O. Maurer (2000), “Non-Monotonic Hazard Functions and the Autoregressive Conditional Duration Model,” The Econometrics Journal, 3, 16-38.

Granger, C. W. J. and T. Teräsvirta (1993), Modelling Nonlinear Economic Relationships, Oxford University Press.

Hamelink, F. (1998), “On the Specification of Duration between Price Changes and the Predictability of High Frequency Returns: An Application to the French CAC 40,” Finance, 19, 139-163.

Hasbrouck, J. (1991), “Measuring the Information Content of Stock Trades,” Journal of Finance, 46, 179-207.

Koop, G., M. H. Pesaran, and S. M. Potter (1996), “Impulse Response Analysis in Nonlinear Multivariate Models,” Journal of Econometrics, 74, 119-147.

Lin, J.-C., G. Sanger, G. G. Booth (1995), “Trade Size and Components of the Bid-Ask Spread,” Review of Financial Studies, 8, 1153-1183.

Luukkonen, R., P. Saikkonen and T. Teräsvirta (1988), “Testing Linearity Against Smooth Transition Autoregressive Models,” Biometrika, 75, 491-499.

Nelson, D. (1991), “Conditional Heteroskedasticity in Asset Returns: A New Approach,” Econometrica, 59, 347-370.

Russell, J. R. and R. F. Engle (2004), “A Discrete-State Continuous-Time Model of Financial Transactions Prices and Times: The ACM-ACD Model,” forthcoming, Journal of Business and Economic Statistics.

Schwert, G. W. (1989), “Why does Stock Market Volatility Change over Time?,” Journal of Finance, 44, 1115-1153.

Teräsvirta, T. (1994), “Specification, Estimation, and Evaluation of Smooth Transition Autoregressive Models,” Journal of American Statistical Association, 89, 208-218.

Zhang, M. Y., J. R. Russell and R. S. Tsay (2001), “A Nonlinear Autoregressive Conditional Duration Model with Applications to Financial Transaction Data,” Journal of Econometrics, 104, 179-207.

Table 1. Descriptive Statistics of IBM Trade Durations

	Mean	Standard Deviation	Skewness	Kurtosis	LB(15)
Original duration	6.9276	6.6041	2.8397	16.5728	6142.0***
Deseasonalized duration	1.0426	0.9627	2.7030	15.5992	3343.3***

a. The number of observations is 68,801 and the unit is seconds.
b. The Ljung-Box statistics of 15 lags are reported in $LB(15)$.
c. *, **, and *** represent significance levels at 10%, 5%, and 1%, respectively.

Table 2. Estimation Results of the MACD Models for IBM Trade Durations

	Model 1	**Model 2**	**Model 3**
w_1	-0.0248 (0.0007)**	-0.0699 (0.0333)**	-0.0461 (0.1158)**
α_1	0.0280 (0.0009)**	0.0507 (0.0129)**	0.0465 (0.0082)**
β_1	0.9915 (0.0006)**a	0.8915 (0.0097)**	0.8654 (0.0132)**
w_2		-0.0366 (0.0232)	-0.1530 (0.2960)
α_2		0.0417 (0.0260)	0.1705 (0.3298)
β_2		0.9982 (0.0003)**	0.9979 (0.0003)**a
w_3			-0.0347 (0.0142)**
α_3			0.0375 (0.0074)**
β_3			0.8850 (0.0192)**a
w_4			-0.0974 (0.1889)
α_4			0.1082 (0.2092)
β_4			0.9986 (0.0002)**
γ_1		0.8146 (0.4858)*	1.5158 (0.5020)**
γ_2			48.5954 (107.0133)
c_1		1.1550 (1.0998)	1.5466 (0.9904)
c_2			1.5466 (0.0466)**
Weibull r	1.2738 (0.0035)**	1.2763 (0.0035)**	1.2771 (0.0035)**
Specification Test	8.2271***	3.0509***	
Variance Test	44.74677***	44.48648***	44.60633***
$LB(15)$	85.484***	18.546	17.038
$LB^2(15)$	37.862	10.698	12.584

a. The number of observations is 68,801. The standard errors are in the parentheses.

b. The Ljung-Box statistics of 15 lags for standardized residuals and squared standardized residuals are reported in $LB(15)$ and $LB^2(15)$, respectively.

c. *, **, and *** represent significance levels at 10%, 5%, and 1%, respectively.

d. The F statistics are reported in the specification test.

e. The duration model is specified as follows:

$$\Psi_j = w_j + \alpha_j \varepsilon_{i-1} + \beta_j \psi_{i-1}, \quad j = 1,2,3,4$$

$$G_1(\delta_{i-1};\gamma_1,c_1) = \frac{1}{1+\exp(-\gamma_1(\delta_{i-1}-c_1))}$$

$$G_2(x_{i-1};\gamma_2,c_2) = \frac{1}{1+\exp(-\gamma_2(x_{i-1}-c_2))}$$

Table 3. The Characteristics of the Whole Period and Four Regimes and Estimation Results of the Logistic Regression

Panel A: The Characteristics of the Whole Period and Four Regimes

	Spread		Volume/Sec		Volatility/Sec	
	Mean	SE	Mean	SE	Mean	SE
Whole Period	0.0695	0.0538	1.4564	3.0875	0.0167	0.0358
PGDG	0.0736[‡]	0.0563	1.6760[‡]	3.7794	0.0159	0.0374
PLDG	0.0706	0.0548	1.6971‡	3.8411	0.0185[‡]	0.0362
PGDL	0.0688	0.0527	1.3951	2.8092	0.0162	0.0345
PLDL	0.0655	0.0527	1.2011	2.1427	0.0178[†]	0.0366

Panel B: Estimation Results of the Logistic Regression with the PGDG as the Choice Regime

	Intercept	Spread	Volume/Sec	Volatility/Sec
	-1.3793 (0.0155)***	1.8634 (0.1679)***	0.0253 (0.0029)***	-1.6879 (0.2840)***

a. Spread represents the difference between prevailing bid and ask prices. Volume/Sec represents the ratio of the deseasonalized transaction volume divided by the deseasonalized trade duration. Volatility/Sec represents the ratio of the absolute change in the midpoint of prevailing quote divided by the deseasonalized trade duration.

b. Mean represents the average. SE represents the standard error. The standard errors of parameter estimates are in the parentheses.

c. [†] and [‡] indicate that the number in the cell is significantly larger than of the whole period at significance levels of 10% and 5%, respectively, under the Mann-Whitney-Wilcoxon mean test.

d. *, **, and *** represent significance levels at 10%, 5%, and 1%, respectively. once the corresponding mean is larger than of the whole period.

e. (1) PGDG regime in which the lagged price change is larger than or equal to zero and the lagged trade duration change is larger than or equal to the mean of trade durations, (2) the PLDG regime in which the lagged price change is less than zero and the lagged trade duration change is greater than or equal to the mean of trade durations, (3) the PGDL regime in which the lagged price change is larger than or equal to zero and the lagged trade duration change is less than the mean of trade durations, and (4) the PLDL regime in which the lagged price change is less than zero and the lagged trade duration change is less than the mean of trade durations.

f. The numbers of observations are 68,801, 15,483, 8,129, 31,853, and 13,336 for the whole period, PGDG, PLDG, PGDL, and PLDL, respectively.

g. The logistic regression is specified as follows:

$$\log\left(\frac{u_i}{1-u_i}\right) = a + b_1\,Spread_i + b_2\,Volume/Sec_i + b_3\,Volatility/Sec_i$$

where $u_i = E(z_i)$ and $z_i = 1$ if the observation i belongs to the PGDG regime

Table 4. Simulated Moments for the Whole Period and Four Regimes

	1st Moment	2nd Moment	3rd Moment	4th Moment
Whole Period	0.8845	0.6293	1.3136	6.1309
	(0.0047)	(0.0120)	(0.0889)	(1.2526)
PGDG	0.9060	0.6632	1.4815	9.2006
	(0.0053)	(0.0161)	(0.1951)	(3.2699)
PLDG	0.8443	0.6109	1.2292	5.0018
	(0.0047)	(0.0098)	(0.0501)	(0.3874)
PGDL	0.8940	0.6217	1.2633	5.5205
	(0.0042)	(0.0102)	(0.0592)	(0.5174)
PLDL	0.8617	0.6201	1.3869	6.3675
	(0.0051)	(0.0128)	(0.0682)	(0.5732)

a. The simulated moments are based on 100 samples with a sample size of 50,000 for each regime using parameter estimates of the model 3 in Table 2. The standard errors are in the parentheses.

b. (1) PGDG regime in which the lagged price change is larger than or equal to zero and the lagged trade duration change is larger than or equal to the mean of trade durations, (2) the PLDG regime in which the lagged price change is less than zero and the lagged trade duration change is greater than or equal to the mean of trade durations, (3) the PGDL regime in which the lagged price change is larger than or equal to zero and the lagged trade duration change is less than the mean of trade durations, and (4) the PLDL regime in which the lagged price change is less than zero and the lagged trade duration change is less than the mean of trade durations.

c. The numbers of observations are 68,801, 15,483, 8,129, 31,853, and 13,336 for the whole period, PGDG, PLDG, PGDL, and PLDL, respectively.

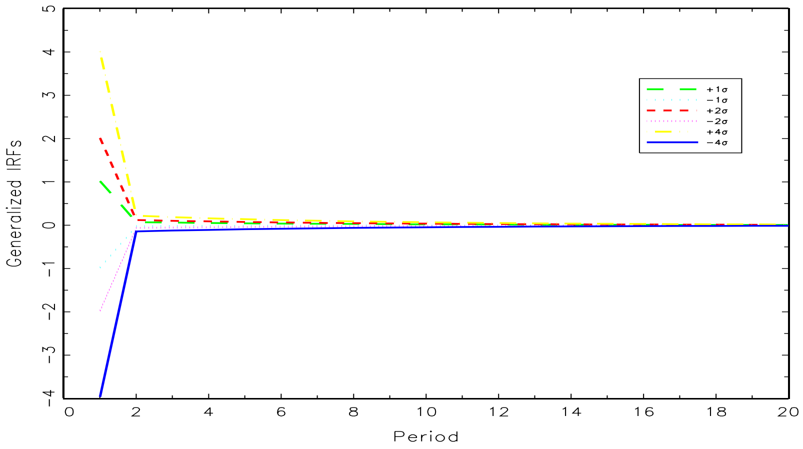

Figure 1. Generalized Impulse Responses of the PGDG Regime

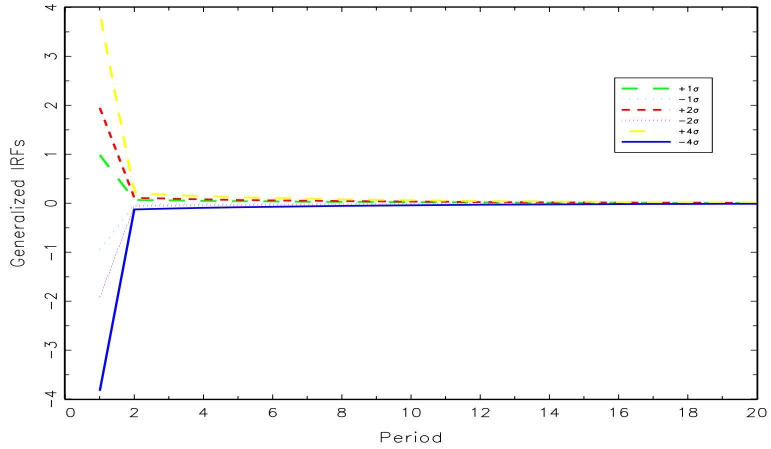

Figure 2. Generalized Impulse Responses of the PLDG Regime

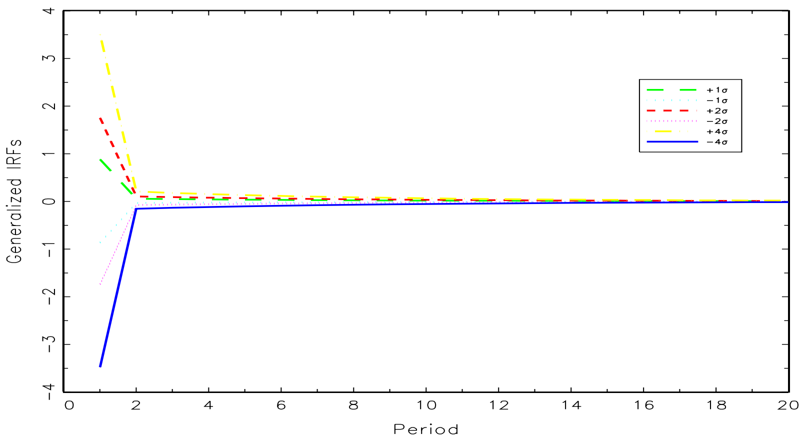

Figure 3. Generalized Impulse Responses of the PGDL Regime

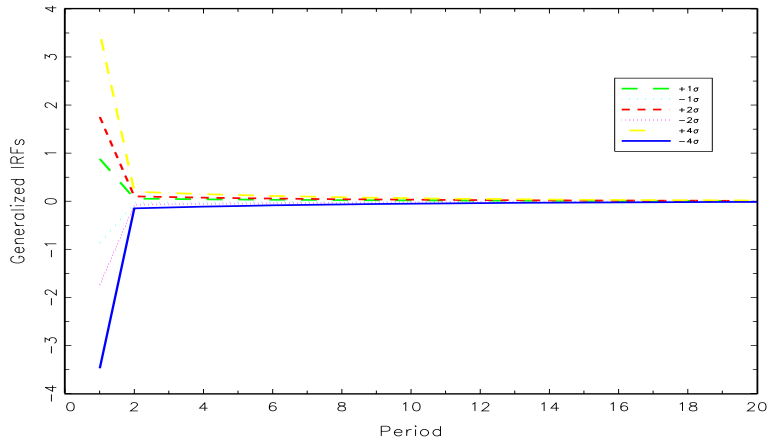

Figure 4. Generalized Impulse Responses of the PLDL Regime

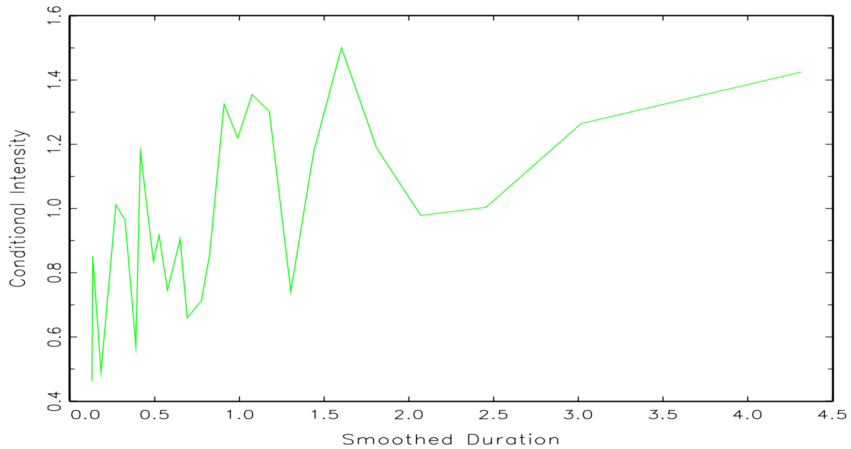

Figure 5. Conditional Intensity Function of the PGDG Regime

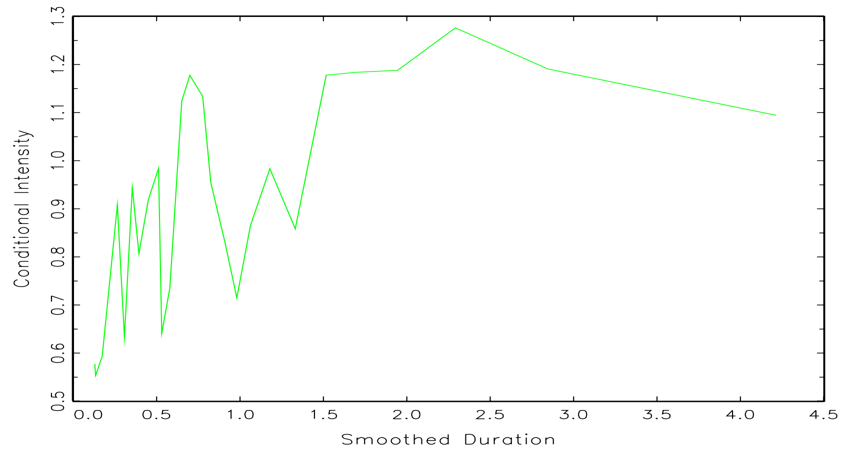

Figure 6. Conditional Intensity Function of the PLDG Regime

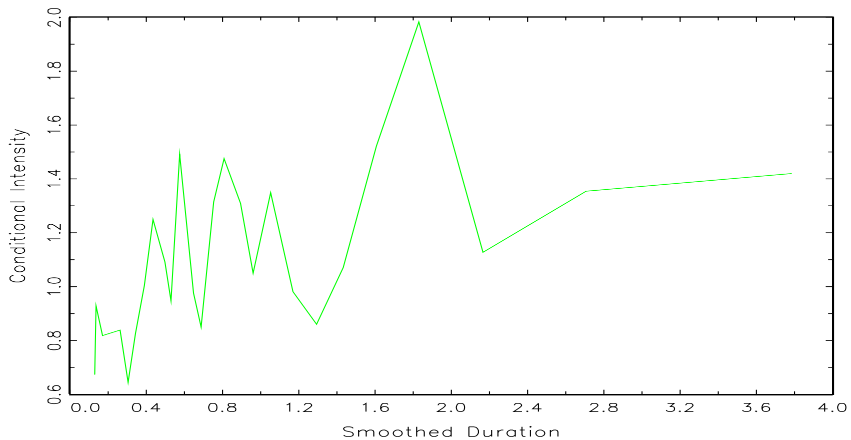

Figure 7. Conditional Intensity Function of the PGDL Regime

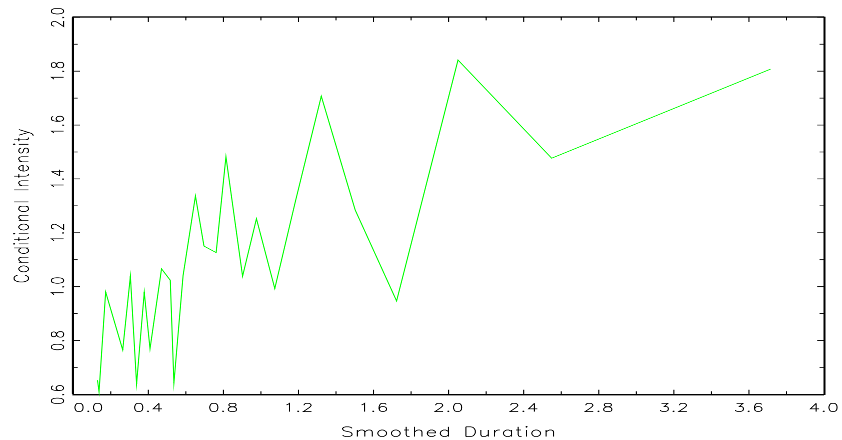

Figure 8. Conditional Intensity Function of the PLDL Regime