The Markovian Dynamics of "Smart Money"

J-H Steffi Yang[1]

Abstract

This paper studies market impact of capital movements driven by fund selection and investor sentiment. It focuses on the resulting effects on downside market behaviour and reward-to-risk efficiency. Updated with net fund profitability, the returns-chasing behaviour of "smart money" and its switch between fund styles are modeled using the Markovian chain. Although the results confirm that liquidity as opposed to capital immobility can indeed be stabilizing, it is however suggested that downside risk and inefficiency are significantly attributable to investor overreaction to profitability. This is particularly disadvantageous in bear markets, where vast investment withdrawal in a hasty fashion can exacerbate the already worsening market condition and be devastating. The finding of this paper provides an alternative perspective to conventional wisdom on capital movements and has its relevance to the recent crises in capital markets.

JEL Classification: G11, G12, G23.
Keywords: Markovian chain, fund managements, drawdown, capital movements.

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[1] Faculty of Economics, University of Cambridge, Research Fellow of British Academy, and Fellow of Wolfson College, Cambridge. Email: s.yang@cantab.net. Permanent contact address: PO Box 104-168, Taipei, Taiwan. Financial support from Bill and Melinda Gates Foundation is acknowledged. The author wishes to thank Stephen Satchell, Thomas Lux and Mark Salmon for their useful comments. Any error remains the responsibility of the author.

1. Introduction

The term “smart money” has been widely used in finance to refer to the investments following winning funds or being able to identify superior fund styles. The movements of smart money are mostly driven by fund performance and investor sentiments. The market condition is also believed to influence fund selection. There are many observations of investment movements, for instance, a shift from active investment management to passive indexing in recent years. In the US equity markets, 14.8% of actively managed domestic equity funds liquidated or merged during 2000-2002, including 6.6% in 2002 alone. At the state level, Connecticut State Trust Fund, in charge of the state’s $12.7 billion pooled pension fund, bumped up their indexed portion from $3.74 billion to $3.99 billion in just two months from July to September 2002, with the long-term goal for the indexed proportion of equities set to be 50%. At the firm’s level, in July 2002 the trustees of Intel’s profit-sharing and pension plans fired their 10 external money managers and decided to switch an additional $300 million of equities to in-house passive management. All of the above are examples of a common phenomenon of moving from active to passive investment management.

There have been abundant but diverse stories on the performance of mutual funds and management styles. Researchers along this line study the cost-benefit comparison across different fund styles and they debate on whether active fund management adds value. Controversy however still remains. Carhart (1997) finds that active fund management tends to have lower benchmark-adjusted net returns than passive indexing funds. He also finds that *net* returns are negatively correlated with expense levels, which are generally much higher for actively managed funds. Gruber (1996) finds that the average US mutual fund underperforms passive market indexes by about 65 basis points per year from 1985 to 1994. On the other hand, Grinblatt and Titman (1989, 1993) and Wermers (1997) both reach the conclusion of higher gross profits by active fund management, especially for growth-oriented funds, which outperform their benchmarks by an average of 2% to 3% per year before expenses. Wermers (2000) supports the value of active mutual fund management based on the dataset of US equity mutual fund market from 1975 – 1994. There are also studies examining whether the evidence of superior performance by active management is due to pure luck or there exist “hot hands” with stock-picking talents. For example, Grinblatt, Titman and Wermers (1995) find that superior fund performance is significantly attributable to the characteristics of the stocks held by funds. They also find that the majority of mutual funds tend to actively invest in stocks with high past returns.

While attempts have been made to answer whether actively managed funds outperform the market, the literature however has not seen much investigation on the market impact of fund selection. It is natural to expect that overall capital flows driven by fund selection will bring about a considerable effect on financial markets. This paper contributes to study what effects may arise from dynamic capital movement, and in particular, its potential impact on downside market behaviour and also reward-to-risk efficiency. The dynamics of capital flows among fund managements is modelled using the Markov chain. The transition probabilities of the Markov chain are formalized as functions of cost-adjusted fund performance. The focus here is on the market impact of investment movement driven by fund selection and investor sentiment. It is in fact not difficult to imagine the link between fund selection and the equilibrium price dynamics. The link can be simply understood by a process where prices determine fund performance; the performance comparison amplified by investor overreaction in turn motivates the movement of “smart money”, whose dynamics then shapes new prices.

In order to understand the long-term behaviour of investment flows, the steady-state closed-form solution is obtained in this study, conditional on the simplified assumption of constant transition probabilities. A change in transition probabilities implies a change in the popularity of different fund styles. The current study applies the analysis of comparative statics to study the impact on prices due to changes in various aspects of transition probabilities. Furthermore, using simulation experiments, it is also investigated the impact of fund selection and capital flows. Simulation of the pricing process is carried out each trading period. The movements of smart money not only depend on fund performance but also investor sentiment. It is thus applied to model varying levels of investor sentiment in response to fund profitability.

The Markovian model is used in this study to capture how investors switch from one investment strategy (or fund style) to another. The focus is placed on their investment switching and the time interval is trading periods. Therefore only *short term* capital movements are considered here. This paper compares varying cases of market sentiment, ranging from the case of capital immobility to the case of overreacting investors drastically moving from one fund management to another. One major finding of this study is that overreaction in chasing winning funds induces significant overall market drawdown. It is also found that a small degree of capital movements as opposed to the static case can surprisingly be a stabilizing force.

The finding of this study therefore supports the recent unconventional view inspired by the Asian Crisis that *short term* speculative capital movements should be controlled if not to be discouraged. Although this perspective has been derived from capital flows in the international scale, it is highly related to the study here. Their relations can be understood by the following two points. Firstly, the rapid integration of capital markets has resulted in an increasing number of international mutual funds as an investment choice for global investors. Global funds have become an important factor, amongst international trade and other foreign investments, that has led to large scale of capital mobility in today's international economy; see Calvo, Leiderman and Reinhart (1996). Investment movements in the global fund market are interconnected with capital flows in the international scale. This can be seen from the following figures. One distinct feature of international capital flows is the replacement of official capital with private capital as the most important component of aggregate flows. The World Bank Global Development Finance (1996) reports that the share of private flows in aggregate net flows to developing countries grew from 44.1% to 85.7% between 1990 and 1996. Furthermore, among all types of private flows, the ratio of portfolio flows grew from 12.4% to 37.7% between 1990 and 1996.

The second point is that the study of investment movements in one capital market can be generalized and explored in the international context. The analysis of market capital movements is analogous in several ways to the study of international capital flows. For example, one striking feature of international capital flows is the increased significance of short term flows, which is also an evident feature that characterizes the modern fund markets. Montiel and Reinhart (1999) found that, in capital-importing Asian countries, short term flows accounted for 39% of total capital inflows over the period of 1990 – 1996, while for Latin America the figure was 32%. In their study, this increase in short term flows is identified with greater volatility in capital flows.

It is noteworthy that the results in this paper are assessed in terms of market drawdown which quantifies uninterrupted falls of prices[2]. Drawdown as a downside measure of market movements has become increasingly popular among researchers and investors largely because of the recent crash of equity markets around the world. Drawdown provides a downside approach different from the conventional risk measures, such as standard deviation, that do not differentiate deviations above and below the mean. Tolerance of drawdown cannot be easily compensated for by the long-term validity of the employed strategy or the attractive expected return characteristic. For example, regardless of the expected future abnormal returns it is unlikely for a consumer/investor to tolerate a drawdown of more than 50% of his account. Another feature of drawdown is that it concerns the duration of loss periods so that consecutive losses are distinguished from intermittent losses. It is highly uncommon that a fund manager can hold a client whose account is in a drawdown for a lengthy period of time even if the drawdown size is small. In this study, the degree of drawdown is examined in three aspects: the number, the duration, and the depth/size of drawdown. These various aspects of drawdown results are compared in different models of dynamic capital movements.

2. The Pricing Model

[2] The statistical properties of drawdown have been studies by Sancetta and Satchell (2003).

Consider a market of S stocks. Let $\mathbf{P}_t$ and $\mathbf{S}_t$ denote the S-dimensional vectors of the prices and the outstanding stock shares. Prices are determined by market equilibrium. Demand is regarded as the time-varying flow of capital into different investment portfolios. Let K be the total size of capital in the economy and for simplicity it is assumed to be fixed. This assumption does not preclude the market capital size from varying; investment entry and exit are modelled as discussed later.

Let θ_t^i denote the capital ratio invested in strategy (or fund) i, and θ_t^X denote the capital ratio staying out of the market, i.e. non-investment. There are N different investment strategies including non-investing, and their capital ratios sum to one, $\sum_{i=1}^{N}\theta_t^i = 1$. $K\theta_t^i$ is the size of the capital flowing into fund i. Let $\mathbf{w}_t^i$ denote the $S\times 1$ vector of the portfolio weights on S stocks by fund type i, and $\mathbf{w}_t^{i}{}'\,\mathbf{1}_S = 1$ except that $\mathbf{w}_t^X = \mathbf{0}_S$. $\mathbf{1}_S$ is an $S\times 1$ vector of ones and $\mathbf{0}_S$ is an $S\times 1$ vector of zeros. Market equilibrium at time t requires

$$\mathbf{S}_t \odot \mathbf{P}_t = K\sum_{i=1}^{N}\theta_t^i \mathbf{w}_t^i, \tag{1}$$

where $\odot$ is the element-by-element multiplication. Notice that although non-market participants, represented by $i = X$, do not invest, i.e. $\mathbf{w}_t^X = \mathbf{0}_S$, they still affect price formation through the constraint $\sum_{i=1}^{N}\theta_t^i = 1$; that is, investment entry and exit can change the capital ratios of different investment strategies and also the total market capital size.

Denote by Θ_t the $N \times 1$ vector of θ_t^i for $i = 1,..,N$. Denote by $\mathbf{W}_t$ the vector of fund i's portfolio weights $\mathbf{w}_t^i$ for $i = 1,..,N$. $\mathbf{W}_t$ thus has a dimension of $NS \times 1$. The equilibrium condition (1) can also be as

$$\mathbf{S}_t \odot \mathbf{P}_t = K\, \Theta_t^{'}\, \mathbf{W}_t. \tag{2}$$

Market equilibrium (2) yields the $S \times 1$ vector of stock prices at time t as

$$\mathbf{P}_t = K\, \Theta_t^{'}\, \mathbf{W}_t \odot \mathbf{S}_t^{-1}. \tag{3}$$

Equation (3) states that the dynamic equilibrium process is intrinsically determined by first, the investment movements among funds (Θ_t), and second, the portfolio allocations of different fund managements ($\mathbf{W}_t$). The dynamics of capital movements is modelled in the next section. We now turn to the discussion of management types and portfolio choices.

3. Portfolio Managements and Fund Styles

In the literature of fund managements, debate has centred on two distinct approaches, namely, passive indexing and active portfolio managements. Indexing refers to passive investments that follow market indexes to form investment portfolios. The portfolios are therefore designed based upon the index weights. Each stock's index weight is a measure of its relative market capitalization, and is calculated as the multiplication of the stock price and the number of shares outstanding, normalized by the market capital size. Therefore, the $S \times 1$ vector of the indexing portfolio weights is given by

$$\mathbf{w}_t^P = \left(\mathbf{S}'_{t-1}\mathbf{P}_{t-1}\right)^{-1}\left(\mathbf{S}_{t-1} \odot \mathbf{P}_{t-1}\right). \tag{4}$$

The logic behind indexing is that since each stock's index weight measures its relative market capitalization, the index weight actually reflects an estimate of the 'relative value' of the company. Indexing is therefore believed to track the 'relative values' of stocks while at the same time to benefit from diversification.

Conversely, instead of passively following market indexes, some fund managers trade actively and strategically. This type of investment management is often referred to as active portfolio management. Active portfolio management can have a wide variety of styles. Fund managers may apply systematic trading rules ranging from simple pattern recognition, such as head-and-shoulders, to sophisticated genetic algorithm. Or they may select a particular class of stocks due to their high expectations on a certain stock attribute such as growth, small cap, global, or emerging markets.

Modern portfolio theory established by the pioneering work of Markowitz (1959) provides a cornerstone in building active portfolios. The key idea of the theory is to maximize the expected reward consistent with the willingness to bear risk, i.e. the mean-variance efficient frontier. A basic form in line with the mean-variance analysis can be given by the $S \times 1$ vector $\mathbf{\Omega}_t^{-1}\mathbf{E}_t$, where $\mathbf{E}_t$ is the $S \times 1$ vector of the expected returns on S stocks, and $\mathbf{\Omega}_t$ is the $S \times S$ covariance matrix at time *t*. The vector of active portfolio weights that satisfies $\mathbf{w}_t^{i}{}'\mathbf{1}_S = 1$ is therefore given by

$$\mathbf{w}_t^A = \left[(\boldsymbol{\Omega}_t^{-1}\mathbf{E}_t)'\mathbf{1}_S\right]^{-1}\left[\boldsymbol{\Omega}_t^{-1}\mathbf{E}_t\right]. \tag{5}$$

Active portfolios allocated according to (5) place more weights on the stocks that are expected to yield higher returns per unit risk. In order to maintain optimality, this form of investment managements involves frequent portfolio revision in response to the information affecting prices. An example of how an active manager forms return conjectures based on the simple moving-average trading rule is given below. $\mathbf{E}_t$ is set as some monotonically increasing function of the moving-average price difference, and is defined as[3]

$$\mathbf{E}_t = f(\mathbf{P}_{t-1} - \frac{1}{m}\sum_{i=1}^{m}\mathbf{P}_{t-i}),\ f \in \Re^+ .$$

m is the moving average length. The expected return is thus based on the comparison of the latest available price and the moving-average price of a chosen length of history. The function f is not required to retain a certain range expect $\Re^+$, since it will undergo normalization, as shown by (5), before the portfolio weights are formed. Throughout this study, we will consider $\mathbf{W}_t$ as a $3S \times 1$ vector given by

$$\mathbf{W}_t = \begin{pmatrix} \mathbf{w}_t^P \\ \mathbf{w}_t^A \\ \mathbf{w}_t^X \end{pmatrix},$$

where $\mathbf{w}_t^P$ is given by (4), $\mathbf{w}_t^A$ by (5), and $\mathbf{w}_t^X = \mathbf{0}_S$.

[3] The definition of E_t essentially follows the feedback function given by Yang and Satchell (2002).

The profitability of fund management style i is assessed by the returns generated by its portfolio choices. Let π_t^i denote the profitability of fund style i for time period t. π_t^i is a scalar defined by

$$\pi_t^i = (\mathbf{w}_{t-1}^i)'\mathbf{R}_t, \tag{6}$$

where $\mathbf{R}_t = \mathbf{P}_t \odot \mathbf{P}_{t-1}^{-1}$ is the $S \times 1$ vector of stock returns from time t-1 to t.

4. The Markov Model of "Smart Money"

As equation (3) shows, one crucial factor in the dynamic equilibrium process is the time-varying capital flows among different investment portfolios. We model the dynamics of investment flows using a Markov chain. In a Markov process, the distribution of next states depends on the transition probabilities and the distribution of current states. Transition probabilities govern the probability of moving from one state to another, which in this study is considered to be time-varying and a function of some explanatory variables. This dependency property of a Markov chain makes it a natural and appealing choice for dynamic modelling.

Denote by θ_t^P and θ_t^A the capital ratios of passive and active portfolio investments, and as before, θ_t^X is the capital ratio staying out of the market. The ecology of capital ratios is thus a 3×1 vector given by

$$\Theta_t = \begin{pmatrix} \theta_t^P \\ \theta_t^A \\ \theta_t^X \end{pmatrix}, \text{ and } \Theta_t^{'} \mathbf{1}_3 = 1. \tag{7}$$

The Markovian dynamics is characterized by

$$\Theta'_{t+1} = \Theta'_t \, \mathbf{M}_t , \tag{8}$$

where $\mathbf{M}_t$ is the transition matrix, and is defined as

$$\mathbf{M}_t = \begin{bmatrix} \Pr_t^{PP} & \Pr_t^{PA} & \Pr_t^{PX} \\ \Pr_t^{AP} & \Pr_t^{AA} & \Pr_t^{AX} \\ \Pr_t^{XP} & \Pr_t^{XA} & \Pr_t^{XX} \end{bmatrix} . \tag{9}$$

$\Pr_t^{ij}$ denotes the transition probability of capital moving from fund style (strategy) i to j. For example, $\Pr_t^{XX}$ denotes the probability of remaining out of market, and $\Pr_t^{AX}$ measures the probability that a client closes his account with active portfolio management and exits the market. The transition matrix is subject to the constraint $\mathbf{M}_t \, \mathbf{1}_3 = \mathbf{1}_3$, i.e.

$$\sum_j \Pr_t^{ij} = 1 . \tag{10}$$

The off-diagonal transition probabilities in (9) can be further expressed in terms of the probability of staying with the original fund and the conditional probability on leaving. Denote by λ_t^{ij} the probability of moving to fund style j conditional on a definite departure from fund i, where $i \neq j$. The off-diagonal transition probabilities are given by

$$\Pr_t^{ij} = \lambda_t^{ij}(1 - \Pr_t^{ii}) \text{, for } i \neq j \text{, and } \sum_j \lambda_t^{ij} = 1 . \tag{11}$$

The transition matrix (9) now becomes

$$\mathbf{M}_t = \begin{bmatrix} \Pr_t^{PP} & (1-\lambda_t^{PX})(1-\Pr_t^{PP}) & \lambda_t^{PX}(1-\Pr_t^{PP}) \\ (1-\lambda_t^{AX})(1-\Pr_t^{AA}) & \Pr_t^{AA} & \lambda_t^{AX}(1-\Pr_t^{AA}) \\ (1-\lambda_t^{XA})(1-\Pr_t^{XX}) & \lambda_t^{XA}(1-\Pr_t^{XX}) & \Pr_t^{XX} \end{bmatrix}. \tag{12}$$

The use of conditional probabilities λ_t^{ij} helps to capture the idea of transition from one state to another in a more hierarchical fashion. Notice that the use of conditional probabilities does not simplify estimation as it involves no parameter reduction; we have six free parameters in (9) and also six in (12). This holds true even when the number of states increases.

The transition probabilities characterize the Markovian dynamics of investment flows. We consider that the probability of capital flowing from one fund management to another is not exogenously prearranged, but instead it depends on the relative fund performance that is regularly updated with new stock prices. That is, the present study endogenizes the transition probabilities to capture how smart money follows the winning fund. Endogenizing transition probabilities in fact completes the investment cycle by linking stock prices, which are shaped by investment flows, with the probabilities that determine the dynamics of investment flows. The following presents how the transition probabilities are endogenized.

We consider that the probability of staying with the original fund style *i*, $\Pr_t^{ii}$, reflects a measure of self efficiency, and that the conditional probability of moving to management style *j* on abandoning *i*, λ_t^{ij}, reflects a comparison across new fund styles other than *i*. An illustrative example is given below. Suppose $\Pr_t^{ii}$ is a logistic function. The transition probabilities are given by

$$\Pr_t^{ii} = \frac{\exp(\alpha\, \pi_t^i)}{1+\exp(\alpha\, \pi_t^i)} = 1 - \frac{1}{1+\exp(\alpha\, \pi_t^i)}; \tag{13}$$

$$\Pr_t^{ij} = \lambda_t^{ij}\left(\frac{1}{1+\exp(\alpha\, \pi_t^i)}\right), \text{ where } \sum_j \lambda_t^{ij} = 1 \text{ and } i \neq j .$$

Fund profitability π_t^i, defined by (6), is chosen to be the explanatory variable but with a slight modification. Here we use log return instead of simple return for $\mathbf{R}_t$. There are two main reasons. First, the sign of π_t^i will now clearly indicate whether or not a loss has occurred. Second, this has the benefit of making the symmetric logistic transition function centre at 0.5 when the profitability is neutral. According to (13), clearly higher profitability leads to a higher probability of staying.

Profitability is one most straightforward measure of investment performance. Other choices include risk-adjusted measures such as efficiency by the Sharpe ratio. Further, the choice of explanatory variables in transition probabilities can go beyond the performance measures to include factors such as market conditions. Although market conditions are not modelled here, the coefficient α is related to investor sentiments that may to some extent reflect market conditions.

We assume $\alpha \geq 0$. The coefficient on profitability, α, measures the smart money's responsiveness to a change in fund profitability. This can be seen by rearranging (13),

$$\alpha = \frac{\partial \ln\left(\dfrac{\Pr^{ii}}{1-\Pr^{ii}}\right)}{\partial \pi^i} .$$

$\frac{\Pr^{ii}}{1-\Pr^{ii}}$ is sometimes called the *odds* of staying with fund style *i*. α is the multiplier on the explanatory variable of the logarithm of the odds of staying. A high α leads to a high probability of staying if the fund management makes positive profits, but also a high probability of changing if a loss occurs. Given a change in the fund profitability, a high α implies a dramatic change in transition probabilities. Therefore, a large α characterizes the "overreacting" smart money. For example, nervous investors change their fund styles or fund managers after one single bad moment. A counterexample is given by pension funds. Pension Funds tend to have a relatively lower α and be sticky to their fund managers.

The conditional probability λ_t^{ij} reflects, given a sure change in the fund management, how smart money picks up a new fund style *j*. λ_t^{ij} can be viewed as a function that compares both the benefits and the costs of all fund styles excluding *i*, since it is conditional on a sure leave from *i*. Let f be a monotonically increasing function that maps $\Re \to \Re^+$. We define conditional probabilities consistent with the requirement (11) by

$$\lambda_t^{ij} = \frac{f(\pi_t^j - c_t^j)}{\sum_{k \neq i} f(\pi_t^k - c_t^k)}, \tag{14}$$

where $\pi - c$ represents the cost-adjusted profits, and $\pi, c \in \Re$. Costs may include transaction costs and management fees, and are defined as a constant fraction of portfolio returns.

$$c_t^j = \begin{cases} \bar{c}\pi_t^j & when\ \ j = A. \\ \underline{c}\pi_t^j & when\ \ j = P. \\ 0 & when\ \ j = X. \end{cases} \tag{15}$$

We further impose $\bar{c} > \underline{c} > 0$ to indicate that low-cost indexing still incurs some transaction expenses, and that investors pay higher management fees and commissions to invest in actively managed funds than indexing funds. When active portfolio management no longer outperforms others, this high entry cost encourages a shift to lower-cost passive management or even a market exit.

5. Steady State

Steady state concerns the long-term behaviour of a dynamic system. This section solves the steady-state solution for the Markovian dynamics (8) with the transition matrix given by (12), under the simplifying assumption of constant transition probabilities. That is, the tendency of capital moving from one investment style to another is assumed fixed over time. Although unrealistic, this assumption simplifies the calculation to a great extent.

In steady state, the Markov chain reaches a stationary distribution and Θ has the ergodic[4] property $\Theta^{'}\mathbf{M} = \Theta^{'}$. In addition, since Θ represents the capital ratios, it must satisfy $\Theta^{'}\mathbf{1}_3 = 1$ as given by (7). Thus, the steady-state solution is in fact the normalized left[5] eigenvector of M, corresponding to the eigenvalue unity. We solve for the steady-state solution and it is given by:

[4] Ergodicity requires the transition matrix to be irreducible and non-periodic. For a more detailed discussion on ergodicity, see, for example, Cox and Miller (1965).

[5] The right eigenvector of M corresponding to the eigenvalue unity is $\mathbf{1}_{3\times 1}$, since $M\mathbf{1}_{3\times 1} = \mathbf{1}_{3\times 1}$ as given by (8).

$$\theta^P = \frac{1}{\kappa}(1-\Pr^{XX})(1-\Pr^{AA})\big((1-\lambda^{XA})\lambda^{AX} + (1-\lambda^{AX})\big),$$

$$\theta^A = \frac{1}{\kappa}(1-\Pr^{XX})(1-\Pr^{PP})\big(1-(1-\lambda^{XA})\lambda^{PX}\big), \text{ and} \tag{16}$$

$$\theta^X = \frac{1}{\kappa}(1-\Pr^{AA})(1-\Pr^{PP})\big(1-(1-\lambda^{PX})(1-\lambda^{AX})\big), \text{ where}$$

$$\kappa=(1-\Pr^{XX})(1-\Pr^{AA})\big((1-\lambda^{XA})(\lambda^{AX})+(1-\lambda^{AX})\big)+(1-\Pr^{XX})(1-\Pr^{PP})\big(1-(1-\lambda^{XA})\lambda^{PX}\big)+(1-\Pr^{AA})(1-\Pr^{PP})\big(1-(1-\lambda^{PX})(1-\lambda^{AX})\big)$$

and $\Pr^{ii} < 1$, $i = P, A, X$.

We now apply the steady-state results to illustrate the limiting cases of the functional forms[6] given by (13). We consider both the cases when $\alpha = 0$ and when $\alpha \to \infty$. First, $\alpha = 0$ leads to $\Pr^{ii} = \frac{1}{2}$ and $\Pr^{ij} = \frac{\lambda^{ij}}{2}$, where $\sum_j \lambda^{ij} = 1$ for $i \neq j$. The steady-state capital ratios now become

$$\theta^P = \frac{1}{\kappa}\big[(1-\lambda^{XA})\lambda^{AX} + (1-\lambda^{AX})\big],$$

$$\theta^A = \frac{1}{\kappa}\big[1-(1-\lambda^{XA})\lambda^{PX}\big], \text{ and} \tag{17}$$

$$\theta^X = \frac{1}{\kappa}\big[1-(1-\lambda^{PX})(1-\lambda^{AX})\big], \text{ where}$$

$$\kappa = (1-\lambda^{XA})\lambda^{AX} + (1-\lambda^{AX}) + 1 - (1-\lambda^{XA})\lambda^{PX} + 1 - (1-\lambda^{PX})(1-\lambda^{AX}).$$

Moreover, it is interesting to observe that if all the conditional probabilities λ^{ij} are set to be $\frac{1}{2}$, the steady-state results (17) even reduce to $\theta^P = \theta^A = \theta^X = \frac{1}{3}$. Thus, when $\alpha = 0$ and $\lambda^{ij} = \frac{1}{2}$, we will have fixed and equal capital ratios among different fund

[6] Notice here we assume constant λ^{ij} since the steady-state results given by (16) are derived when transition probabilities are constant.

styles over time. We shall refer to this case as the static benchmark model. This extreme case of a small α is consistent with the discussion before, regarding the stickiness (or under-reaction) of investment when α is low.

On the other hand, when $\alpha \to \infty$, two situations arise. If $\pi^i > 0$, then $\lim_{\alpha \to \infty} \Pr^{ii} = 1$ and $\lim_{\alpha \to \infty} \Pr^{ij} = 0$ for $i \neq j$. If instead $\pi^i < 0$, then $\lim_{\alpha \to \infty} \Pr^{ii} = 0$ and $\lim_{\alpha \to \infty} \Pr^{ij} = \lambda^{ij}$, where $\sum_j \lambda^{ij} = 1$ for $i \neq j$. However a non-degenerate steady-state solution fails to exist in either of these situations, as the limiting transition matrices in both examples are reducible and do not satisfy the properties of ergodicity for the existence of a steady state of a Markov chain.

6. Comparative Statics

Steady state describes the long-term behaviour of capital movements, but it does not tell us how prices respond to capital movements caused by a change in transition probabilities. Price formation reflects the dynamics of investment flows characterized by transition probabilities. The impact on prices of a change in transition probabilities can be understood analytically by comparative statics and numerically by simulation. Simulation experiments are carried out in the next section. This section applies the analysis of comparative statics to examine the impact on steady-state prices due to the following three causes ranging from general to specific: first, a change in transition probabilities, second, a change in conditional probabilities on leaving the current state, and third, a change in the featuring factor of transition probabilities such as the responsiveness to profitability.

From market equilibrium condition (1), the price vector can be rewritten as

$$\mathbf{P}_t = K\sum_{i=1}^{N}\theta_t^i\,(\mathbf{w}_t^i \odot \mathbf{S}_t^{-1}). \tag{18}$$

It is convenient to define an $S \times 1$ vector $\mathbf{H}_t^i = \mathbf{w}_t^i \odot \mathbf{S}_t^{-1}$. The price vector can now be expressed as

$$\mathbf{P}_t = K\sum_{i=1}^{N}\theta_t^i\,\mathbf{H}_t^i. \tag{19}$$

Since $\mathbf{w}_t^i$ represents the portfolio weights on *S* stocks by strategy *i*, $\mathbf{H}_t^i$ can be simply understood as *demands per share*, which mainly reflects the strategy's expectation on future returns of different stocks. Suppose prices are in steady state denoted by $\mathbf{P}^*$. We obtain the following results of comparative statics. Their proofs are given in Appendix A.

$$\frac{\partial \mathbf{P}^*}{\partial \mathrm{Pr}^{sk}} = K\theta^s\left(\mathbf{H}^k + \sum_{j\neq k}\frac{\partial \mathrm{Pr}^{sj}}{\partial \mathrm{Pr}^{sk}}\mathbf{H}^j\right), \text{ where } \sum_{j\neq k}\frac{\partial \mathrm{Pr}^{sj}}{\partial \mathrm{Pr}^{sk}} = -1. \tag{20}$$

$$\frac{\partial \mathbf{P}^*}{\partial \lambda^{sk}} = K\theta^s\left(1-\mathrm{Pr}^{ss}\right)\left(\mathbf{H}^k + \sum_{j\neq k}\frac{\partial \lambda^{sj}}{\partial \lambda^{sk}}\mathbf{H}^j\right), \text{ where } \sum_{j\neq k}\frac{\partial \lambda^{sj}}{\partial \lambda^{sk}} = -1. \tag{21}$$

$$\frac{\partial \mathbf{P}^*}{\partial \alpha} = K\sum_i\left(\theta^i\sum_{j\neq i}\frac{\partial \mathrm{Pr}^{ij}}{\partial \alpha}(\mathbf{H}^j - \mathbf{H}^i)\right) \tag{22}$$

Suppose now α is fund-specific so it can be written as α^i. (22) now becomes

$$\frac{\partial \mathbf{P}^*}{\partial \alpha^s} = K\theta^s\sum_{j\neq s}\frac{\partial \mathrm{Pr}^{sj}}{\partial \alpha^s}\left(\mathbf{H}^j - \mathbf{H}^s\right). \tag{23}$$

In the case of logistic transition probabilities (12), we obtain

$$\frac{\partial \mathbf{P}^*}{\partial \alpha^s} = K\theta^s \Pr^{ss}(1-\Pr^{ss})\pi^s \left(\mathbf{H}^s - \sum_{j\neq s} \lambda^{sj} \mathbf{H}^j\right), \text{ where } \sum_{j\neq s} \lambda_t^{sj} = 1 . \tag{24}$$

An application of (20) is given as follows. Suppose now passive indexing becomes a popular investment approach, i.e. $k = passive\ indexing = P$ in equation (20). What is the resulting impact on stock prices? The sign will depend on the expectation of the indexed fund on future stock returns. More precisely, its impact on the price of a particular stock is positive, if the demand per share by indexers on the stock is greater the normalized sum of the demand per share by others, i.e. the second term on the RHS of equation (20) is greater than zero. An important implication of (20) is that a strategy's optimism on a particular stock can have a positive impact on the price of the stock if the strategy becomes popular.

Furthermore, since $\mathbf{H}^X = \mathbf{0}_S$, an application of (20) suggests that9 the tendency in staying out of the market (i.e. $k = non-investing = X$ in equation 20) always has a negative impact on prices. The resulting price falls can be easily understood as a consequence of a lack of investments.

Equation (21) tells us how prices are affected by a change in conditional probabilities. Its implication is similar to that of (20) by the same reasoning. (22) and (23) are better understood by their application (24) with logistic transition probabilities. As discussed before, α measures the responsiveness to profitability. Let us consider the case of active fund management, i.e. $s = active\ fund\ management = A$ in equation (24). Overreacting smart money or a higher α implies a significant increase in capital inflow when actively managed fund makes profits, i.e. $\pi^A > 0$. By equation (24), this results in a positive impact on the price of a particular stock if active management has a high expectation on the stock's returns (i.e. the last term on the RHS of equation 24 is greater than zero).

7. Simulation Experiments and Results Discussion

Computational simulation of market history provides a dynamic perspective on the impact of smart money movements. In this section, the pricing process is simulated for each trading period. The process can be understood as follows: stock prices determine fund profitability that in turn influences the flows of smart money; the dynamics of smart money then determines stock prices through market equilibrium, and the whole process repeats.

The simulation is based upon the equilibrium price equation (3). The portfolio allocations of passive and active fund managements are given by (4) and (5) respectively, with m set to be 10. Besides, f is set as an adjusted hyperbolic tangent function, $f(x) = \tanh(x/2) + 1$. To avoid the problem of non-existence of the inverse covariance matrix and for the sake of simplicity, we assume an identity matrix for $\mathbf{\Omega}_t$. The dynamics of smart money Θ_t is modelled by the Markov chain (8) with the transition matrix illustrated by (12). Further, transition probabilities are endogenized as functions of fund profitability and associated expenses, as given by (13), (14) and (15). The costs of active and passive fund managements as a fraction of their profitability, $\overline{c}$ and $\underline{c}$, are set to be 0.2 and 0.02 respectively. The pricing process is simulated for 2000 trading periods.

We consider different levels of investor sentiment in response to changes in fund profitability. The results are compared with the static benchmark model, i.e. when $\alpha = 0, \lambda^{ij} = \frac{1}{2}$, where there is no capital movement, and the capital ratios among non-investment, indexing, and actively managed funds are allocated fixedly and equally. The market dynamics resulting from these models are assessed in terms of both the Sharpe ratio[7] and market "drawdown". On one hand, it is reasonable in this study to use the Sharpe ratio as one criterion to measure the market impact of capital movements driven by fund selections. The logic behind is that the investment strategies applied to structure fund portfolios are designed based on the mean-variance criterion. Thus, the criterion used to assess the resulting market dynamics, namely, the Sharpe ratio, is consistent with the criterion that designs the portfolios and hence potentially influences the dynamics. On the other hand, the measure of market "drawdown" is particularly useful in understanding the downside market movement. As pointed out in the introduction, the concern of this paper lies in the potential association between the market behaviour and the investment flows driven by fund selection and investor sentiment. Specifically, the interest rests on how these return-chasing investment flows may contribute to extreme market events such as large price falls. On this regard, "drawdown" is a highly relevant measure of the resulting market dynamics in this study. In its simplest term, "drawdown" is a measure that quantifies uninterrupted falls in security prices. The benefits of using the measure of drawdown to assess the market impact have been discussed in the introduction. Below it is provided the definition of market drawdown. This study will consider three aspects of the drawdown measure, namely the duration, the depth, and the number of drawdowns,

[7] The author acknowledges the suggestion made by Mark Salmon and Shaun Bond that the standard Sharpe ratio could be a reasonable alternative measure of the resulting market dynamics in this paper.

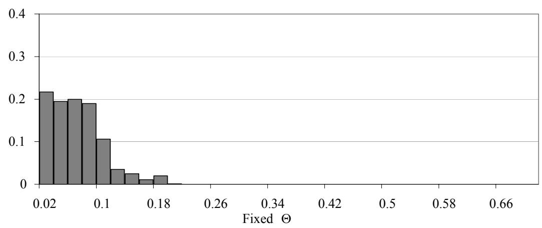
in both indexing and actively managed accounts.

For a price series p_t, a drawdown is simply defined as a sequence of $\{p_t\}_{t=d_0}^{d_0+d}$ for $d > 1$, where all the prices of the sequence fall down while there exists a price rise both immediately before and after the sequence; d is the duration or the length of the drawdown. In terms of a returns series $r_t = \frac{p_t}{p_{t-1}}$, a drawdown can be equivalently defined as a sequence of $\{r_t\}_{t=d_0+1}^{d_0+d}$ for $d > 1$, where $r_t < 1$ for $t = d_0 + 1,\ d_0 + 2, ..,$ $d_0 + d$, while $r_{d_0} > 1$ and $r_{d_0+d+1} > 1$. Without loss of generality, we assume $d_0 = 0$. The absolute depth of a drawdown is calculated by the difference between p_0 and p_d, i.e. $p_0 - p_d$, and the relative depth by the ratio $D_d = \frac{p_0 - p_d}{p_0}$. In this study, we compute the relative depth, D_d, that relates to the beginning position of a drawdown. D_d is calculated conditional on the preceding local peak, so that a drawdown is considered less severe if its preceding local peak is comparatively high. The relative depth of a drawdown can also be expressed in terms of returns by $1 - \prod_{t=1}^{d} r_t$. Finally, let N_d denote the number of drawdowns in a series. Note that N_d is bounded above by $(T+1)/2$ for a series of T trading periods. Table 1 reports the results of drawdown[8] calculated from index (or market) returns and also active investment returns. Figure 1 provides the distributions of the sizes of the drawdown.

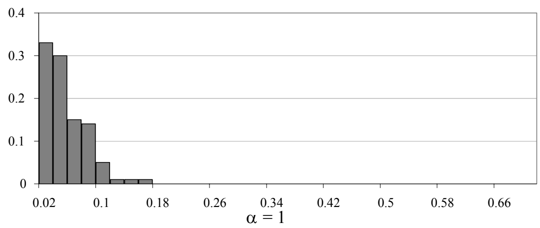
[8] The results of drawdown are calculated from portfolio returns π_t instead of simple returns r_t. As π_t given by (6) is just the weighted return in accordance with the underlying portfolio weights, the computation of the number, the duration, and the sizes of drawdown discussed above will still apply.

Table 1 Drawdown results of the investment returns from the market index fund and actively managed fund.

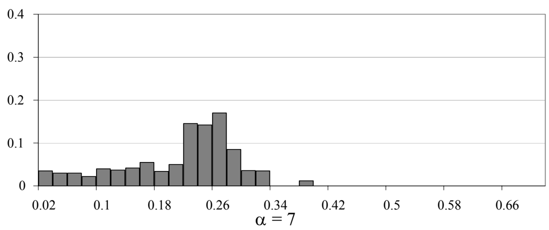
		Number of Period Loss (Frequency)	Number of Drawdown (Frequency)	Average Duration of Drawdown	Average Depth of Drawdown	Maximum Depth of Drawdown
Fixed Θ, $\theta^P = \theta^A = \theta^X = 1/3$	Index	806 (0.3224)	615 (0.246)	1.31057	0.0390024	0.18136
	Active	1694 (0.6776)	614 (0.2456)	2.75896	0.133933	0.677253
$\Theta'_{t+1} = \Theta'_t M_t$ $\Pr_t(\alpha = 1)$	Index	791 (0.3164)	622 (0.2488)	1.2717	0.0289949	0.404647
	Active	1709 (0.6836)	608 (0.2432)	2.81086	0.11926	0.618233
$\Theta'_{t+1} = \Theta'_t M_t$ $\Pr_t(\alpha = 7)$	Index	1139 (0.4556)	395 (0.158)	2.88354	0.187774	0.381794
	Active	1351 (0.5404)	388 (0.1552)	3.48196	0.352543	0.648755
$\Theta'_{t+1} = \Theta'_t M_t$ $\Pr_t(\alpha = 11)$	Index	1083 (0.4332)	363 (0.1452)	2.98347	0.463669	0.556648
	Active	1222 (0.4888)	359 (0.1436)	3.4039	0.573109	0.692364
$\Theta'_{t+1} = \Theta'_t M_t$ $\Pr_t(\alpha = 100)$	Index	826 (0.3304)	329 (0.1316)	2.51064	0.556938	0.653068
	Active	1110 (0.444)	326 (0.1304)	3.40491	0.598585	0.777556



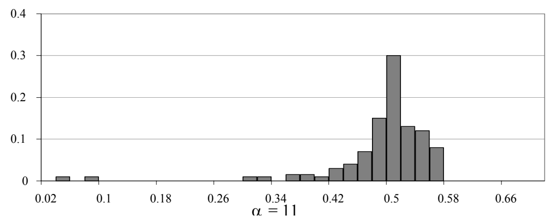
(a)



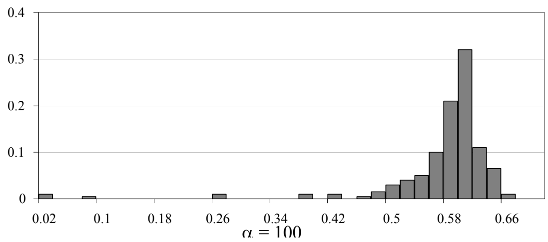
(b)



(c)



(d)



(e)

Figure 1 The distributions of the relative sizes of drawdown.

A number of patterns can be found from the results in Table 1 and Figure 1. These patterns can lead to rather different implications on investor sentiment. First, comparing only the results of varying levels of α excluding the static benchmark model, we find that overall the number of drawdown decreases but the average duration of drawdown increases as α gets larger. Furthermore, the average depth of drawdown also increases with α. The results suggest that overreaction aggravates both the duration and the size of drawdown in investment returns in the market index and active managed accounts. We may then tend to think that the static benchmark model with no investment movements would have smaller measures of drawdown. Surprisingly, the static model in fact leads to a larger drawdown size in average than the dynamic model with a small α. This observation implies the existence of a stabilizing force when there is a limited degree of capital movement.

The Sharpe ratios of the index fund returns and the actively managed fund returns, assuming a zero risk-free rate, are reported in Table 2. As the Sharpe ratio represents excess return per unit risk, it can also be regarded as a reward-to-risk efficiency measure. It is noticeable from the patterns of the reported Sharpe ratios in Table 2 that the simulated market characterized with a small degree of capital movement ($\alpha = 1$) has the highest level of reward-to-risk efficiency for both index and actively managed funds. The level of the reward-to-risk efficiency is lower in the static benchmark model where capital ratios remain equal and fixed among the set of possible investments. Moreover, the efficiency level decreases persistently as the parameter that reflects the degree of investor sensitivity to fund profitability increases. This efficiency decline is evident for both index and actively managed accounts.

The crucial question arises here is whether the results obtained using the two different measures, namely, “drawdown” and the Sharpe-ratio efficiency, will have the same implications or implications compatible with each other. Recall that there are two main findings from the drawdown results. First, investor overreaction reflected by speculative and drastic capital movement is found to have a significant contribution to large market falls. Second, although the destabilizing impact of investor overreaction is observed, investment movement and capital liquidity to some extent as opposed to the static case of capital immobility can in fact be stabilizing. Now, the first drawdown finding suggests a tendency for higher downside risk in the market characterized by overreacting investors chasing winning funds. As the risk gets higher, the reward-to-risk efficiency declines. Thus, the observation of a persistent decrease in the reward-to-risk efficiency as the level of overreaction exacerbates is in accordance with the first major finding from the drawdown results. On the other hand, the Sharpe-type efficiency level is indeed higher in the model with the smallest α than that in the static benchmark model. By the same token, this higher level of risk-adjusted efficiency is attributable to the stabilizing effect of capital liquidity and hence lower risk. Again, this efficiency increase is in line with the second major finding from drawdown results. Therefore, in summary, the overall pattern observed in the level of the reward-to-risk efficiency is consistent with the previous observations of downside market behaviour presented by the drawdown results.

There is an interesting observation that the Sharpe ratios of the actively managed fund are slightly lower than their counterparts of the index fund. The index fund outperforms the actively managed fund in terms of the reward-to-risk efficiency measure. This comparison of their risk-adjusted performance is consistent with one observation given in Table 1 that the number of period loss of the actively managed fund always exceeds its counterpart of the index fund. Although it is difficult to provide a rigorous, analytical explanation to the observed outperformance of the index fund both in terms of loss period and risk-adjusted reward, the outcomes may be considered as attributable to active fund management being inherently more risky.

In Table 2 it is also reported the means and variances of both index returns and active portfolio returns, together with the correlation coefficients between these two return series. Although overall it is seen that the mean returns of both management strategies[9] slightly increase with the investor sensitivity level α, their variances are found to exhibit an unproportionally large increase. This implies that overreaction adversely induces a volatile market. The observation of high volatility in the case of overreacting smart money is consistent with the results found in market drawdown.

Table 2 Summary statistics of the mean, variance, Sharpe ratio, and correlation of index returns and active managed fund returns.

		Mean	Variance	Sharpe Ratio	Correlation Coefficient
Fixed Θ, $\theta^P = \theta^A = \theta^X = 1/3$	Index	1.02608	0.00282958	19.29	-1
	Active	0.973918	0.00282958	18.31	
$\Theta'_{t+1} = \Theta'_t M_t$ $\Pr_t(\alpha = 1)$	Index	1.01962	0.00179315	24.08	-0.990231
	Active	0.976985	0.00241346	19.89	
$\Theta'_{t+1} = \Theta'_t M_t$ $\Pr_t(\alpha = 7)$	Index	1.02767	0.0114034	9.62	0.594797
	Active	0.975898	0.0145883	8.08	
$\Theta'_{t+1} = \Theta'_t M_t$ $\Pr_t(\alpha = 11)$	Index	1.04795	0.058714	4.32	0.921535
	Active	1.00201	0.0591932	4.12	
$\Theta'_{t+1} = \Theta'_t M_t$ $\Pr_t(\alpha = 100)$	Index	1.06738	0.0890357	3.58	0.888454
	Active	1.04008	0.092746	3.42	

[9] In the model, the mean index return exceeds the mean active investment return by an insignificant amount. We believe that this observation is highly model-specific and is largely attributed to the employed active trading strategy.

Now we turn to the discussion of the correlation outcomes. The profitability of these two investment strategies moves in the opposite directions in the benchmark model and also when α is low, i.e. when clients' money is sticky to the original investments. The reason to the observed negative return correlation can be grasped intuitively that when there is no source of investment inflow and the market capital size remains fixed, one strategy can only be profitable at the cost of the other. In the case of the static benchmark model, there is no investment entry or exit and the market capital ratio always remains $2/3$. Appendix 4B provides a proof that index returns and active investment returns sum to a constant, by imposing the constraint of fixed and equal capital ratios among different investment strategies. Since the sum of these two returns at each time period is fixed under certain assumptions, it becomes clear that one strategy is profitable at the expense of the other, and so their profitability moves in the reverse directions as shown by their correlation coefficient. The sum of these two returns remaining fixed also explains why their variances are virtually the same by a straightforward proof.

The return correlation however becomes positive when α increases. This observation implies a counter-intuitive situation when active and passive fund managements can be simultaneously profitable. Considering the price mechanism, this is not as surprising as it seems. In the presence of overreacting smart money, a profitable active fund management quickly attracts a vast investment inflow from not only index believers but also non-market participants. This pushes up the prices of the stocks on which active fund management puts more weights, and hence the overall market index price. Here the investment inflow from out of market ($\Pr^{XA}$) is crucial. If the investment inflow into active fund management comes merely from passive management, the weakened passive investment is likely to offset the push-up effect on the market index by the strong active investment.

The most straightforward example is found in bull markets, where various active fund managements can be profitable at the same time when the market index is soaring[10]. This can be grasped by that market conditions influence investor sentiments, and in particular, bull markets trigger massive new investment inflows that boost profitability. Even in the rare case when passive indexing has no investment inflow, the market price can still go up due to the push-up effect of a strong active investment.

8. Concluding Remarks and Discussions

This paper develops a Markovian model to capture the profit-chasing behaviour of "smart money" between two major fund management styles, namely, index fund and active portfolio fund. Market capital inflow and outflow are also considered in the model by allowing non-participation as one investment choice. In seeking the long-term capital allocations, the steady-state capital ratios among different investment strategies are derived under some simplifying assumption on transition probabilities. Both the analysis of comparative statics and computational simulation are applied. The paper studies the resulting market drawdown and reward-to-risk efficiency in this market characterized by the Markovian dynamics of smart money.

[10] For instance, during the bull market of 1998 – 1999, the Fidelity aggressive growth fund achieved returns of 190%.

One major finding suggests that market drawdown can be significantly attributed to overreacting smart money drastically moving from one fund style to another. Both the duration and the size of drawdown are considered in this study. However, it is also observed that capital liquidity and investment movement to some extent, as opposed to capital immobility, can in fact be stabilizing to the market. Moreover, using the criterion of the Sharpe ratio, it is demonstrated that the overall pattern observed in the level of this reward-to-risk efficiency consistently supports the observations of downside market behaviour presented by the drawdown results.

This paper also finds that when money is sensitive to fund performance, profitable active fund management is likely to trigger vast capital inflow that pushes up the asset prices of active portfolios and hence the overall index prices, given that there is no offsetting effect from possibly weakened passive investment. Therefore, performance sensitivity and new capital inflow make it possible that two much debated portfolio management styles, passive indexing and active fund management, can be simultaneously profitable. By the same token, a rapid investment withdraw triggered by overreacting investors in response to either bad news or underperformance can lead to active fund management being just as devastating as market index. On the contrary, if money is insensitive and investment capital remains immobile, the model is in fact a typical zero-sum game where one strategy will be profitable at the cost of the other.

Is the returns-chasing behaviour enabled by market liquidity socially desirable? Several implications can be drawn from the results. The returns-chasing behaviour induces a natural selection of investment funds, so that ill-performing funds are liquidated or merged while outperforming ones accumulate even more capital. From the viewpoint of seeking a valid investment tool, the increased competition level may enhance the effectiveness of asset managements. Besides, some degree of liquidity is desirable since it may work as a stabilizing force to the market as the results suggest. However, there are tradeoffs as well as benefits. The market with overreacting smart money chasing past winners and abandoning poor performing funds implies a higher downside risk. Particularly in bear markets, vast investment withdraws in a hasty fashion can exacerbate the already worsening market condition.

It is discussed in the introduction that the relevance and association between this study's context and international capital flows. The finding of this paper provides an alternative perspective to conventional knowledge regarding capital movement. It is established wisdom in mainstream economics that countries can prosper by opening up their economies to international trade and capital flows. There are many advantages that international capital can offer to a country; see e.g. Fernandez-Arias and Montiel (1996). The positive effects can be significant and they include lowering the cost of capital to creditworthy firms, complementing domestic savings to enable smooth consumption over time, and financing investment. Inflows of capital can stimulate economic growth and bring about convergence between the developing and developed world.

However, this perspective has been challenged by the recent crises in capital markets. Short term capital flows can on the other hand pose a number of potential threats to a country's macroeconomic health or even become a source of instability. Negative effects must be weighed against the benefits of capital account liberalization. For example, as discussed in the introduction, the level of portfolio flows has increased significantly in recent years. Portfolio flows across national boundaries are designed for greater risk diversification in world financial markets. However cyclical market condition change can influence investor sentiment which in turn poses a risk of sharp capital flow reversals and destabilization of the local economy. In countries with vulnerable financial sectors, capital movements purely driven by speculative purposes can have severely destabilizing effects.

On the other hand, in most countries capital inflows are often associated with widening current account deficits, largely owing to an increase in national investment and a decrease in national saving. Although economic activities may expand and GDP may grow faster, key macroeconomic variables can be pushed away from their long term equilibrium leading to weakened macroeconomic fundamentals at unsustainable levels. Calvo and Reinhart (1999) argued that if this circumstance is followed by abrupt capital flow reversals, there can be serious consequences of output collapses and currency and banking sector crises as what happened in several emerging markets throughout 1990s. The Institute of International Finance reports that the five hardest affected economies by the Asia Crisis, namely South Korea, Indonesia, Malaysia, Thailand, and Philippines, have experienced a reversal of capital movement from an inflow of 93 billion USD in 1996 to an estimated outflow of 12 billion USD in 1997. The total shift was 105 billion USD, a figure more than 10 percent of these economies' combined, pre-crisis GDP. This loss in capital account sustainability was connected with the fact that a large portion of these flows was short term in nature and can be easily withdrawn. With reference to the 1994 Mexican crisis, Edwards (1998) points out that the role played by large capital inflows, which at their peak surpassed 9 percent of Mexico's GDP, has been at the centre of almost every post mortem of the Mexican crisis.

What can be done to avoid or reduce the likelihood of problems associated with large volumes of short term capital flows? Short term capital controls can be implemented through mechanisms such as imposing tax on short term money or eliminating policy distortion that encourages speculative flows in the first place. Quoting Krugman (1998) these mechanisms are “curfews on capital flight” that should work to control short term hot money and encourage healthy and long term investment. For instance, quantitative restrictions were imposed in Malaysia in 1994 to prohibit inflows or outflows of funds. Another example was given by Chile. Chile has imposed tax-based restrictions, such as a tax on short-maturity loans, which make capital transfers more costly. Blondal and Christiansen (1999) found that inflow restrictions as a more commonly used measure is far more efficient in influencing capital movements than measures that are designed to restrain outflows.

There is certainly broad agreement that capital market liberalization is valuable in the long run, but in the short run it may be better for economies to follow more prudent liberalizations until they have reached a higher state of development. Economists such as Eichengreen (2000) argue that although it is important to avoid monetary and fiscal policy excesses, an economy should fulfil a set of conditions before capital liberalization is completed. These conditions should work to reduce the economy’s vulnerability on volatile capital flows. One crucial need in banking sector is to regulate the local lending practices of domestic banks to ensure prudent and effective credit-risk management. Another option is to reduce the reliance on short term lending. McKinnon (1991) argues that capital account liberalization should be the last step, after consolidation of other liberalizing measures and the strengthening of the domestic financial systems. Strengthening domestic financial systems can be implemented by reforms that create more transparent systems of corporate governance, robust financial infrastructures, and sound and stable policies for crisis prevention. Financial reforms may also help to enhance investor confidence for the long term and hence the economy's stabilization. Lastly but not leastly, these arguments in the context of international money flows can be mostly applied to capital movements in the fund market. Healthy markets should always be achieved before supporting free capital mobility.

Appendix A

<u>Proof A</u>

Suppose prices are in steady state and so is the ecology of capital ratios. Express the ergodic property for a steady-state ecology, $\Theta^{'} = \Theta^{'}\ \mathbf{M}$, in scalars,

$$\theta^j = \sum_i \theta^i \Pr^{ij} . \tag{A.1}$$

From (19) and (A.1), the steady-state price vector can be written as

$$\mathbf{P}^* = K\sum_j \sum_i \theta^i \Pr^{ij} \mathbf{H}^j . \tag{A.2}$$

By separating whether $i = s$ or $j = k$, (A.2) can be further decomposed into

$$\mathbf{P}^* = K\left\{ \theta^s \Pr^{sk} \mathbf{H}^k + \sum_{i \neq s} \theta^i \Pr^{ik} \mathbf{H}^k + \sum_{j \neq k} \theta^s \Pr^{sj} \mathbf{H}^j + \sum_{j \neq k} \sum_{i \neq s} \theta^i \Pr^{ij} \mathbf{H}^j \right\}. \tag{A.3}$$

From the equivalent of constraint (10), $\sum_j \Pr^{sj} = 1$, it is easy to obtain $\sum_{j \neq k} \frac{\partial \Pr^{sj}}{\partial \Pr^{sk}} = -1$.

Also we know that $\left.\frac{\partial \Pr^{ij}}{\partial \Pr^{sk}}\right|_{i \neq s} = 0$. Therefore, by partial differentiation of the steady-state price given by (A.3) with respect to the transition probability, we obtain

$$\frac{\partial \mathbf{P}^*}{\partial \Pr^{sk}} = K\theta^s \left(\mathbf{H}^k + \sum_{j \neq k} \frac{\partial \Pr^{sj}}{\partial \Pr^{sk}} \mathbf{H}^j \right), \text{ where } \sum_{j \neq k} \frac{\partial \Pr^{sj}}{\partial \Pr^{sk}} = -1 .$$

Proof B

We first decompose the steady state price (A.2) into four parts.

$$\mathbf{P}^* = K\left\{ \theta^s \Pr^{ss} \mathbf{H}^s + \theta^s \Pr^{sk} \mathbf{H}^k \Big|_{k\neq s} + \sum_{\substack{j\neq s \\ j\neq k}} \theta^s \Pr^{sj} \mathbf{H}^j + \sum_j \sum_{i\neq s} \theta^i \Pr^{ij} \mathbf{H}^j \right\} \tag{A.4}$$

Recall that λ^{ij} is not defined when $i = j$, and that the diagonal entries in the transition probability matrix $\mathbf{M}$ have no λ^{ij}, thus

$$\frac{\partial \Pr^{ss}}{\partial \lambda^{sk}} = 0 .$$

Also we know that $\Pr^{ij}$ is independent of λ^{sk} if $i \neq s$, thus

$$\left. \frac{\partial \Pr^{ij}}{\partial \lambda^{sk}} \right|_{i\neq s} = 0 .$$

The partial differentiation of (A.4) with respect to the conditional transition probability hence yields

$$\frac{\partial \mathbf{P}^*}{\partial \lambda^{sk}} = K\left\{ \theta^s \frac{\partial \Pr^{sk}}{\partial \lambda^{sk}} \mathbf{H}^k + \sum_{\substack{j\neq s \\ j\neq k}} \theta^s \frac{\partial \Pr^{sj}}{\partial \lambda^{sk}} \mathbf{H}^j \right\}. \tag{A.5}$$

Since the off-diagonal transition probabilities are defined by $\mathrm{Pr}^{sk}\big|_{k\neq s} = \lambda^{sk}(1-\mathrm{Pr}^{ss})$, it is straightforward that

$$\left.\frac{\partial \mathrm{Pr}^{sk}}{\partial \lambda^{sk}}\right|_{s\neq k} = 1-\mathrm{Pr}^{ss}.$$

By chain rule, $\left.\dfrac{\partial \mathrm{Pr}^{sj}}{\partial \lambda^{sk}}\right|_{\substack{j\neq s\\ j\neq k}} = \dfrac{\partial \mathrm{Pr}^{sj}}{\partial \lambda^{sj}}\dfrac{\partial \lambda^{sj}}{\partial \lambda^{sk}} = (1-\mathrm{Pr}^{ss})\dfrac{\partial \lambda^{sj}}{\partial \lambda^{sk}}$.

Also, from (11), $\sum_j \lambda^{sj} = 1$, it is easy to show that $\sum_{j\neq k}\dfrac{\partial \lambda^{sj}}{\partial \lambda^{sk}} = -1$.

Therefore, (A.5) reduces to

$$\frac{\partial \mathbf{P}^*}{\partial \lambda^{sk}} = K\theta^s\left(1-\mathrm{Pr}^{ss}\right)\left(\mathbf{H}^k + \sum_{j\neq k}\frac{\partial \lambda^{sj}}{\partial \lambda^{sk}}\mathbf{H}^j\right), \text{ where } \sum_{j\neq k}\frac{\partial \lambda^{sj}}{\partial \lambda^{sk}} = -1.$$

Proof C1

Suppose $\Pr^{ij} = \Pr^{ij}(\alpha)$.

From the constraint (10), we know that $\sum_j \Pr^{ij}(\alpha) = 1$, and hence $\sum_j \frac{\partial \Pr^{ij}(\alpha)}{\partial \alpha} = 0$.

Separating the diagonal entries from the off-diagonal ones yields

$$\frac{\partial \Pr^{ii}(\alpha)}{\partial \alpha} = -\sum_{j \neq i} \frac{\partial \Pr^{ij}(\alpha)}{\partial \alpha}. \tag{A.6}$$

From (A.2), we can obtain

$$\frac{\partial \mathbf{P}^*}{\partial \alpha} = K \sum_i \sum_j \theta^i \frac{\partial \Pr^{ij}}{\partial \alpha} \mathbf{H}^j = K \sum_i \left(\theta^i \frac{\partial \Pr^{ii}}{\partial \alpha} \mathbf{H}^i + \sum_{j \neq i} \theta^i \frac{\partial \Pr^{ij}}{\partial \alpha} \mathbf{H}^j \right). \tag{A.7}$$

Replacing (A.6) into (A.7) leads to

$$\frac{\partial \mathbf{P}^*}{\partial \alpha} = K \sum_i \left(\theta^i \sum_{j \neq i} \frac{\partial \Pr^{ij}}{\partial \alpha} (\mathbf{H}^j - \mathbf{H}^i) \right).$$

Proof C2

Now, suppose $\Pr^{ij} = \Pr^{ij}(\alpha^i)$, and $\alpha^i \neq \alpha^s$ if $i \neq s$. Thus, $\left.\frac{\partial \Pr^{ij}}{\partial \alpha^s}\right|_{i \neq s} = 0$.

Again from (A.2), we calculate the following.

$$\frac{\partial \mathbf{P}^*}{\partial \alpha^s} = K \sum_j \sum_i \theta^i \frac{\partial \Pr^{ij}}{\partial \alpha^s} \mathbf{H}^j$$

$$= K \left\{ \theta^s \frac{\partial \Pr^{ss}}{\partial \alpha^s} \mathbf{H}^s + \sum_{j \neq s} \theta^s \frac{\partial \Pr^{sj}}{\partial \alpha^s} \mathbf{H}^j + \sum_j \sum_{i \neq s} \theta^i \frac{\partial \Pr^{ij}}{\partial \alpha^s} \mathbf{H}^j \right\}$$

$$= K\left\{\theta^s \frac{\partial \Pr^{ss}}{\partial \alpha^s}\mathbf{H}^s + \sum_{j\neq s}\theta^s \frac{\partial \Pr^{sj}}{\partial \alpha^s}\mathbf{H}^j\right\}$$

From the same reasoning as (A.6), we know

$$\frac{\partial \Pr^{ss}}{\partial \alpha^s} = -\sum_{j\neq s}\frac{\partial \Pr^{sj}}{\partial \alpha^s}.$$

It therefore follows that

$$\frac{\partial \mathbf{P}^*}{\partial \alpha^s} = K\theta^s \sum_{j\neq s}\frac{\partial \Pr^{sj}}{\partial \alpha^s}\left(\mathbf{H}^j - \mathbf{H}^s\right). \tag{A.8}$$

An example is given below with the logistic transition probabilities (13).

We first obtain

$$\left.\frac{\partial \Pr^{sj}}{\partial \alpha^s}\right|_{j\neq s} = -\lambda^{sj}\Pr^{ss}(1-\Pr^{ss})\pi^s.$$

From (A.8),

$$\frac{\partial \mathbf{P}^*}{\partial \alpha^s} = K\theta^s \sum_{j\neq s}\frac{\partial \Pr^{sj}}{\partial \alpha^s}\left(\mathbf{H}^j - \mathbf{H}^s\right) = K\theta^s \Pr^{ss}(1-\Pr^{ss})\pi^s \sum_{j\neq s}\lambda^{sj}\left(\mathbf{H}^s - \mathbf{H}^j\right).$$

Since $\sum_{j\neq s}\lambda^{sj} = 1$ as given by (11), it follows that

$$\frac{\partial \mathbf{P}^*}{\partial \alpha^s} = K\theta^s \Pr^{ss}(1-\Pr^{ss})\pi^s\left(\mathbf{H}^s - \sum_{j\neq s}\lambda^{sj}\mathbf{H}^j\right), \text{ where } \sum_{j\neq s}\lambda_t^{sj} = 1.$$

Appendix B

Recall that market equilibrium at time t is given by (1) as

$$\mathbf{S}_t \odot \mathbf{P}_t = K \sum_{i=1}^{N} \theta_t^i \mathbf{w}_t^i ,$$

where $\mathbf{P}_t$, $\mathbf{S}_t$, and $\mathbf{w}_t^i$ are S-dimensional vectors of prices, outstanding shares, and portfolio weights of strategy i on S stocks. The scalars K and θ_t^i are the total capital size and the capital ratio allocated to investment fund i.

Rearrange the equilibrium condition (1) to obtain the price vector at time t

$$\mathbf{P}_t = K \sum_{i=1}^{N} \theta_t^i \mathbf{w}_t^i \odot \mathbf{S}_t^{-1} . \tag{B.1}$$

The static benchmark model assumes fixed and equal capital ratios among different investment funds. Imposing this assumption together with the simplified assumption of a constant number of outstanding shares, (B.1) becomes
$\mathbf{P}_t = K\theta \mathbf{S}^{-1} \odot \left(\sum_{i=1}^{N} \mathbf{w}_t^i \right)$, and similarly,

$$\mathbf{P}_{t-1} = K\theta \mathbf{S}^{-1} \odot \left(\sum_{i=1}^{N} \mathbf{w}_{t-1}^i \right).$$

Therefore, returns defined as the price ratios are given by the $S \times 1$ vector

$$\mathbf{R}_t = \mathbf{P}_t \odot \mathbf{P}_{t-1}^{-1} = \left(\sum_{i=1}^{N} \mathbf{w}_t^i \right) \odot \left(\sum_{i=1}^{N} \mathbf{w}_{t-1}^i \right)^{-1} . \tag{B.2}$$

Following the model, consider three investment strategies, i.e. three states, $i = P, A,$ and X. Notice that $\mathbf{w}_t^X = \mathbf{0}_S$ and $\mathbf{w}_t^{i}{}' \mathbf{1}_S = 1$ for $i = P, A$. Let $\mathbf{1}_S$ denote an $S \times 1$ vector of ones. The return vector (B.2) then becomes

$$\mathbf{R}_t = \left(\mathbf{w}_t^P + \mathbf{w}_t^A\right) \odot \left(\mathbf{w}_{t-1}^P + \mathbf{w}_{t-1}^A\right)^{-1}.$$

Fund profitability is measured by its portfolio return. Portfolio return is a scalar defined by (6) and is calculated as the summation of stock returns multiplied by the corresponding portfolio weights. The sum of index return and active investment return is then given by

$$\begin{aligned}
\pi_t^P + \pi_t^A &= (\mathbf{w}_{t-1}^P)'\mathbf{R}_t + (\mathbf{w}_{t-1}^A)'\mathbf{R}_t = (\mathbf{w}_{t-1}^P + \mathbf{w}_{t-1}^A)'\mathbf{R}_t \\
&= (\mathbf{w}_{t-1}^P + \mathbf{w}_{t-1}^A)' \left\{ \left(\mathbf{w}_t^P + \mathbf{w}_t^A\right) \odot \left(\mathbf{w}_{t-1}^P + \mathbf{w}_{t-1}^A\right)^{-1} \right\} \\
&= \mathbf{1}_S' \left(\mathbf{w}_t^P + \mathbf{w}_t^A\right).
\end{aligned}$$

Since $\mathbf{w}_t^{i}{}' \mathbf{1}_S = 1$ for $i = P, A$, thus

$$\pi_t^P + \pi_t^A = \mathbf{1}_S' \mathbf{w}_t^P + \mathbf{1}_S' \mathbf{w}_t^A = 2.$$

Therefore, the proof has shown that under the restriction of fixed and equal capital ratios among different investment strategies and also the assumption of a constant number of outstanding stock shares, the sum of index return and active investment return is a constant.

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